COMPSCI 311: Introduction to Algorithms

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slides credit: Dan Sheldon, Akshay Krishnamurthy, Andrew McGregor

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COMPSCI 311: Introduction to Algorithms

- **Instructor**: Marius Minea
- **Where**: ILC S140 (Integrative Learning Center)
- **When**: Mon/Wed 4:00-5:15pm
- **Discussion Sections**: Fri 9:05-9:55 and 10:10-11:00, Flint 201
- **TA**: Jesse Lingeman, Karine Tung, Raghavendra Addanki, Subhojyoti Mukherjee

Lectures similar to Section 1 (Dan Sheldon);
same TAs, homeworks, quizzes, midterms, Piazza, Gradscope;
different finals

What is Algorithm Design?

How do you write a computer program to solve a complex problem?

- Computing similarity between DNA sequences
- Routing packets on the Internet
- Scheduling final exams at a college
- Assign medical residents to hospitals
- Find all occurrences of a phrase in a large collection of documents
- Finding the smallest number of gas stations that can be built in the US such that everyone is within 20 minutes of a gas station.

DNA sequence similarity

- **Input**: two strings s_1 and s_2 of length n
  - s_1 = AGGCTACC
  - s_2 = CAGGCTAC
- **Output**: minimum number of insertions/deletions to transform s_1 into s_2
- **Algorithm**: ????
- Even if the objective is precisely defined, we are often not ready to start coding right away!

What is Algorithm Design?

- **Step 1**: **Formulate** the problem precisely
- **Step 2**: **Design** an algorithm
- **Step 3**: **Prove** the algorithm is correct
- **Step 4**: **Analyze** its running time

Important: this is an iterative process
Sometimes we don't get the algorithm right on the first try
Sometimes we'll redesign the algorithm to prove correctness easier or to make it more efficient

Usually, two steps:

- getting to a (mathematical) clean core of the problem
- identify the appropriate algorithm design techniques

Course Goals

- Learn how to apply the algorithm design *process*... by practice!
- Learn specific algorithm design techniques
  - Greedy
  - Divide-and-conquer
  - Dynamic Programming
  - Network Flows
- Learn to communicate precisely about algorithms
  - Proofs, reading, writing, discussion
- Prove when no exact efficient algorithm is possible
  - Intractability and NP-completeness

Prerequisites: CS 187 and 250

- Algorithms use data structures
- Familiarity
  - at programming level (lists, stacks, queues, ...)
  - with mathematical objects (sets, lists, relations, partial orders)
    precise statement of algorithm is in terms of such objects
- Two key notions to revisit:
  - Recursion: many algorithm classes are recursive
    so are most relations for computing algorithmic complexity
  - Proofs: for algorithm correctness; by induction, contradiction, ...

Proofs Are Important!

- Need to make sure algorithm is correct
- Think of special / corner cases
- Case in point: Timsort sorting algorithm was broken!
  - developed in 2002 (Python), adopted as standard sort in Java
  - tries to find and extend segments that are already sorted
  - uses stack to track segments and their lengths
  - *loop invariant* was not correctly reestablished
  - thus computed worst case stack size was wrong!
  - crash for array $>$ 67M elements
  - bug found and fixed in 2015 by theorem proving

Grading Breakdown

- **Participation (10%):** Discussion section, in-class quizzes (iClicker)
- **Homework (25%):** (every two weeks, usually due Thursday)
- **Moodle Quiz (5%):** (due every Monday).
- **Midterm 1 (20%):** Focus on first third of lectures. 7pm, Thu Feb 21
- **Midterm 2 (20%):** Focus on second third of lectures. 7pm, Thu Apr 11
- **Final (20%):** Covers all lectures. 3:30pm, Mon May 6

Course Information

Course websites:

people.cs.umass.edu/~marius/class/cs311/	Course information, slides, homework, **pointers to all other pages**
moodle.umass.edu	Quizzes, solutions, grades
piazza.com	Discussion forum, contacting instructors and TA's
gradescope.com	Submitting and returning homework

Announcements: Check your UMass email / Piazza regularly for course announcements.

Homeworks and Quizzes

- *Online Quizzes:* Quizzes must be submitted before 8pm Monday.
  No late quizzes allowed but we'll ignore your lowest scoring quiz.

- *Homework:* Submit via Gradescope, by 11:59 pm of due date.
  50% penalty for homework that is late up to 24 hours.
  Homework that is late by more than 24 hours receives no credit.
  One homework may be up to 24 hours late without penalty.

Collaboration and Academic Honesty

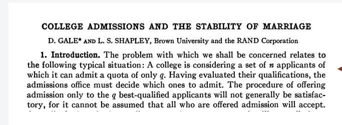
- *Homework:* Collaboration OK (and encouraged) on homework.
  But: you should read and attempt on your own first.
  The writeup and code **must** be your own.
  Looking at written solutions that are not your own is considered cheating. There will be formal action if cheating is suspected.
  You must list your collaborators and any sources (printed or online) at the top of each assignment.
- *Online Quizzes:* Should be done entirely on your own.
  You may consult the book and slides as you do the quiz.
  Again, there will be formal action if cheating is suspected.
- *Discussions:* Groups for the discussion section exercises will be assigned at the start of each session.
  You must complete the exercises with your assigned group.
- *Exams:* Closed book and no electronics.
  Cheating will result in an F in the course.
- If in doubt whether something is allowed, ask!

Stable Matching

- Real-life scenario
  - matching student interns to companies
  - or medical residents to hospitals
- Both students and companies have preferences / ranking lists
- If not properly managed, can become chaotic
  (assume participants are selfish, act in their own self-interest)
  - student may get better offer and reject current one
  - student may actively call company, see if they are preferred over the current status

2012 Nobel Prize in Economics

Lloyd Shapley. Stable matching theory and Gale–Shapley algorithm.



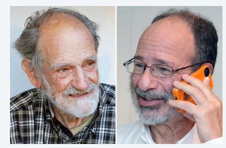
COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE

D. GALE* AND L. S. SHAPLEY, Brown University and the RAND Corporation

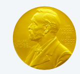
1. Introduction. The problem with which we shall be concerned relates to the following typical situation: A college is considering a set of n applicants of which it can admit a quota of only q. Having evaluated their qualifications, the admissions office must decide which ones to admit. The procedure of offering admission only to the q best-qualified applicants will not generally be satisfactory, for it cannot be assumed that all who are offered admission will accept.

original applications: college admissions and opposite-sex marriage

Alvin Roth. Applied Gale–Shapley to matching med-school students with hospitals, students with schools, and organ donors with patients.



Lloyd Shapley Alvin Roth



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slide credit: Kevin Wayne / Pearson

Stable Matching and College Admissions

- Suppose there are n colleges $c_1, c_2, \ldots, c_n$ and n students $s_1, s_2, \ldots, s_n$.
- Each college has a ranking of all the students that they could admit and each student has a ranking of all the colleges.
  To simplify, suppose each college can only admit one student.
- What other simplification(s) have we made?
- n students, n colleges – could potentially match one-to-one
- *Matching*: a set of pairs (c, s) such that every college and every student appears in at most one pair
- *Perfect* matching: every student and college is matched

Defining Stability

- Can we match students to colleges such that everyone is *happy*?
  - Not necessarily, e.g., if UMass was everyone's top choice.
- Can we match students to colleges such that matching is *stable*?
  - Need to precisely define stability
- *(In)stability*: Don't want to match (c, s) and (c', s') if c and s' would prefer to switch and be matched with each other.
- *Unstable pair*: A pair (c, s) is *unstable* if c prefers s to matched student and s prefers c to matched college
- Are the two wordings equivalent?
  It follows that an *unstable* pair is not part of matching

Problem Formulation

- **Input**: preference lists for n colleges and n students
- **Output?** need definitions first
- **Matching**: set M of college-student pairs, each college/student participate in at most one pair.
- **Perfect matching**: each college/student in *exactly* one pair
- **Instability** or **unstable pair** (with respect to matching M): a pair $(c, s) \notin M$ such that
  - $(c, s') \in M$ but c prefers s to s'
  - $(c', s) \in M$ but s prefers c to c'
- **Stable matching**: perfect matching with no instabilities
- **Output**: a stable matching

Clicker Question 1

	1st	2nd	3rd
Atlanta	**Xavier**	Yvette	Zeus
Boston	Yvette	Xavier	**Zeus**
Chicago	Xavier	**Yvette**	Zeus

	1st	2nd	3rd
Xavier	Boston	**Atlanta**	Chicago
Yvette	Atlanta	Boston	**Chicago**
Zeus	Atlanta	**Boston**	Chicago

Which is an unstable pair with respect to the matching {A - X, B - Z, C - Y}? (marked in **bold** above)

A: A - Y

B: B - X

C: C - Y

D: none of the above

Examples

Do stable matchings always exist? Are they unique? Let's see. . .

- **Example 1**: universal prefs

Colleges

a:	1	2
b:	1	2

Students

1:	a	b
2:	a	b

- $M = \{(a,1),(b,2)\}$? stable
- $M = \{(a,2),(b,1)\}$? not stable

Examples

- **Example 2**: inconsistent prefs

Colleges

a:	1	2
b:	2	1

Students

1:	b	a
2:	a	b

Clicker Q2: You are given an arbitrary set of preferences. Does it have more than one stable matching?

A. Yes
B. No
C. It depends on the preference lists

- $M = \{(a,1),(b,2)\}$? stable
- $M = \{(a,2),(b,1)\}$? stable

Propose-and-Reject (Gale-Shapley) Algorithm

```
Initially all colleges and students are free
while some college is free and hasn't proposed to every student do
    choose such a college c
    let s = highest ranked student to whom c has not proposed
    if s is free then
        c and s become matched
    else if s is matched to c' but prefers c to c' then
        c' becomes unmatched
        c and s become matched
    else                                          ▷ s prefers c'
        s rejects c and c remains free
    end if
end while
```

Analyzing the Algorithm

- Some natural questions:
  - Can we guarantee the algorithm terminates?
  - Can we guarantee the every college and student gets a match?
  - Can we guarantee the resulting allocation is stable?
- These questions are non-obvious
- Answer may differ if we slightly change problem
- Does the following setup differ, and if so, how?

Stable roommate problem

Q. Do stable matchings always exist?
A. Not obvious a priori.

Stable roommate problem.
- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

no perfect matching is stable

A–B, C–D ⇒ B–C unstable
A–C, B–D ⇒ A–B unstable
A–D, B–C ⇒ A–C unstable

Observation. Stable matchings need not exist.

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slide credit: Kevin Wayne / Pearson

Analyzing the Algorithm

- Some initial observations:
  - (F1) Once matched, students stay matched and only "upgrade" during the algorithm.
  - (F2) College propose to students in order of college's preferences.

Can we guarantee the algorithm terminates?

- Yes! Proof...
  - In every round, some college proposes to some student that they haven't already proposed to.
  - n colleges and n students $\implies$ at most n^2 proposals
  - $\implies$ at most n^2 rounds of the algorithm

Can we guarantee all colleges and students get a match?

- Yes! Proof by contradiction...
  - Suppose not all colleges and students have matches. Then there exists unmatched college c and unmatched student s.
  - s was never matched during the algorithm (by F1)
  - But c proposed to every student (by termination condition)
  - When c proposed to s, she was unmatched and yet rejected c. Contradiction!

Clicker Question 3

Depending on the problem instance, which of the following can happen during a run of the Gale-Shapley algorithm?

A: Each student accepts their first offer and never switches.

B: Some student switches their choice more than once during a run.

C: A and B, including for the same problem instance.

D: A and B, but only for different problem instances.

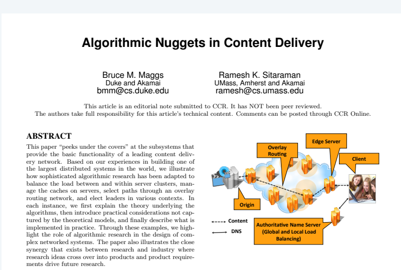
Can we guarantee the resulting allocation is stable?

- Yes! Proof by contradiction
  - Suppose there is an instability (c, s)
    - c is matched to s' but prefers s to s'
    - s is matched to c' but prefers c to c'
  - Did c offer to s?
    Yes, by (F2), since c offered to s' who is ranked lower
  - Did s accept offer from c?
    Maybe initially, but s must *eventually* reject c for another college, and, by (F1), s prefers final college c' to c
  - Contradiction!

A modern application

Content delivery networks. Distribute much of world's content on web.

User. Preferences based on latency and packet loss.
Web server. Preferences based on costs of bandwidth and co-location.
Goal. Assign billions of users to servers, every 10 seconds.



Algorithmic Nuggets in Content Delivery

Bruce M. Maggs, Duke and Akamai, bmm@cs.duke.edu
Ramesh K. Sitaraman, UMass, Amherst and Akamai, ramesh@cs.umass.edu

This article is an editorial note submitted to CCR. It has NOT been peer reviewed. The authors take full responsibility for this article's technical content. Comments can be posted through CCR Online.

ABSTRACT

This paper "peeks under the covers" at the subsystems that provide the basic functionality of a leading content delivery network. Based on our experiences in building one of the largest distributed systems in the world, we illustrate how sophisticated algorithmic research has been adapted to balance the load between and within server clusters, manage the caches on servers, select paths through an overlay routing network, and elect leaders in various contexts. In each instance, we first explain the theory underlying the algorithms, then introduce practical considerations not captured by the theoretical models, and finally describe what is implemented in practice. Through these examples, we highlight the role of algorithmic research in the design of complex networked systems. The paper also illustrates the close synergy that exists between research and industry where research ideas cross over into products and product requirements drive future research.

slide credit: Kevin Wayne / Pearson

Things To Do

- Think about:
  - Would it be better or worse for the students if we ran the algorithm with the students proposing?
  - Can a student get an advantage by lying about their preferences?
- Read: Chapter 1, course policies
- Enroll in Piazza, log into Moodle, and visit the course webpage.