COMPSCI 311: Introduction to Algorithms	Spring 2019

Homework 5

Released 3/29/2019 — Due 4/8/2019 11:59pm in Gradescope

Instructions. You may work in groups, but you must write solutions yourself. List collaborators on your submission.

If you are asked to design an algorithm, please provide: (a) the pseudocode or precise description in words of the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submissions. Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. Please assign pages to questions in Gradescope.

1. **(10 points) Flows and Cuts.** Figure 1 shows a flow network on which an (s, t) flow has been computed. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers specifically, the four edges of capacity=3 have no flow being sent on them.)

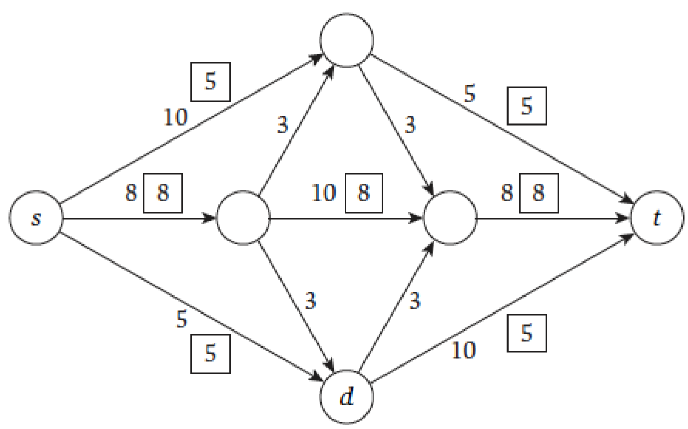

Figure 1: Flows and Cuts

   (a) What is the value of this flow? Is this a maximum (s, t) flow in this graph?

   (b) Find a minimum (s, t) cut in the flow network pictured in Figure 1, and also say what its capacity is.

2. **(10 points) K&T Ch7 Ex4.** Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   (a) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e. If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i.e., for all edges e out of s, we have $f(e) = c_e$).

   (b) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A, B) be a minimum s-t cut with respect to these capacities $c_e : e \in E$. Now suppose we add 1 to every capacity; then (A, B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

3. **(20 points) Updating Flows.** Let $G = (V, E)$ be a unit-capacity flow network with source s and sink t. We are also given an integer maximum flow for G. Give an algorithm to efficiently update the maximum flow in G.

   (a) a new edge with unit capacity is *added* to E;

   (b) an edge is *deleted* from E.

   Note : The algorithm you provide should be faster than recomputing the maximum flow in the updated graph.

4. **(20 points) K&T Ch7 Ex7.** Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible base stations. We'll suppose there are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also k base stations; the position of each of these is specified by (x, y) coordinates as well.

   For each client, we wish to connect it to exactly one of the base stations. Our choice of connections is constrained in the following ways.There is a range parameter r such that a client can only be connected to a base station that is within distance r. There is also a load parameter L such that no more than L clients can be connected to any single base station.

   Your goal is to design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station, subject to the range and load conditions in the previous paragraph.

5. **(0 points)** How long did it take you to complete this assignment?