COMPSCI 311: Introduction to Algorithms **Fall 2018**

Homework 6

Released 11/27/2018 Due 12/11/2018 11:59pm in Gradescope

Instructions. You may work in groups, but you must individually write your solutions yourself. List your collaborators on your submission.

If you are asked to design an algorithm as part of a homework problem, please provide: (a) the pseudocode for the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submissions. Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. It will be very helpful if in your submission each question starts on a new page.

1. **(30 points) Max-Cut** In this problem we will prove that the MAX-CUT problem is NP-Complete. This may be surprising since we saw that MIN-CUT can be solved in polynomial time using Network Flows. MAX-CUT is the following problem: Given an undirected graph $G = (V, E)$ with nonnegative edge capacities $w_{u,v}$ for $(u, v) \in E$ and a number c, decide if there exists a cut in G with capacity at least c. Recall that a cut is a set of vertices $S \subset V$ and the capacity of the cut is $\sum_{(u,v),u\in S,v\notin S} w_{u,v}$. We also define a variant of SAT that is a useful intermediate problem for the reduction. NAE-k-SAT (not-all-equal) is the following problem: Given a boolean formula with exactly k terms in each clause, decide if there is an assignment such that each clause has at least one true term and one false term?
   (a) Prove that 3-SAT $\leq_P$ NAE-3-SAT (Hint: you may want to go through NAE-4-SAT).
   Hint: A conjunction of the form $(A \vee p) \wedge (B \vee \bar{p})$ is satisfiable iff $A \vee B$ is satisfiable (resolution rule).
   (b) Prove that NAE-3-SAT $\leq_P$ MAX-CUT.
2. **(15 points) Diverse Subset** (K&T Ch.8 Ex.2)
3. **(15 points) Monotone Sat with few True Variables** (K&T Ch.8 Ex.6)
4. **(20 points) Randomized Vertex Cover** (K&T Ch.13 Ex.18)
   Hint for (b): Consider the probabilities of exploring 1, 2, 3, ... edges incident to a vertex.
5. **(20 points) Feasible Subset** (K&T Ch.11 Ex.3)
6. **(0 points).** How long did it take you to complete this assignment?