COMPSCI 311: Introduction to Algorithms	**Fall 2018**

Homework 5

Released 11/1/2018 — Due Friday 11/16/2018 11:59pm in Gradescope

Instructions. You may work in groups, but you must individually write your solutions yourself. List your collaborators on your submission.

If you are asked to design an algorithm as part of a homework problem, please provide: (a) the pseudocode for the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submissions. Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. It will be very helpful if in your submission each question starts on a new page.

1. **(20 points) Updating flows** Let $G = (V, E)$ be a unit-capacity flow network with source s and sink t. We are also given an integer maximum flow for G. Describe how the maximum flow can be efficiently updated when **a)** a new edge with unit capacity is *added* to E; **b)** an edge is *deleted* from E.

2. **(20 points) Reduce Flows** (K&T Ch.7 Ex.12) You are given a flow network with unit-capacity edges: It consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for every $e \in E$. You are also given a parameter k. The goal is to delete k edges so as to reduce the maximum $s-t$ flow in G by as much as possible. In other words, you should find a set of edges $F \subset E$ so that $|F| = k$ and the maximum $s - t$ flow in $G' = (V, E \setminus F)$ is as small as possible subject to this. Give a polynomial-time algorithm to solve this problem.

3. **(15 points) Edge-disjoint paths** Given a directed graph $G = (V, E)$ with vertices $s, t \in V$, give an algorithm that finds the maximum number of edge-disjoint paths from s to t.

   Extra credit (10 points): Do the same for node-disjoint paths.

4. **(25 points) Escape problem** (K&T Ch.7 Ex.14)

   Consider a directed graph $G = (V, E)$ and two disjoint sets of nodes $X, S \subset V$. A set of *evacuation routes* is a set of paths so that (i) each node in X is the start of one path, (ii) each path ends at a node in S, and (iii) the paths do not share any edges.

   a) Show how to decide in polynomial time whether a set of evacuation routes exists.

   b) Do the same when (iii) reads "the paths do not share any nodes".

   c) Give an example with the same G, X and S when (a) is possible but (b) not.

5. **(20 points) Minimum Path Cover** (CLRS P. 26-2)

   A *path cover* of a directed graph $G = (V, E)$ is a set P of vertex-disjoint paths such that every vertex in V is included in exactly one path in P. Paths may start and end anywhere, and they may be of any length, including 0. A *minimum path cover* of G is a path cover containing the fewest possible paths.

   a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$. Hint: Assuming that $V = \{1, 2, \ldots, n\}$, construct and run a maximum flow algorithm on the graph $G' = (V', E')$, where $V' = \{x_0, x_1, ..., x_n\} \cup \{y_0, y_1, ..., y_n\}$, $E' = \{(x_0, x_i) : i \in V\} \cup \{(y_i, y_0) : i \in V\} \cup \{(x_i, y_j) : (i, j) \in E\}$.

   b) Does your algorithm work for directed graphs that contain cycles? Prove or give a counterexample.

6. **(0 points).** How long did it take you to complete this assignment?