COMPSCI 311: Introduction to Algorithms **Fall 2018**

Homework 4

Released 10/18/2018 Due 11/1/2018 11:59pm in Gradescope

Instructions. You may work in groups, but you must individually write your solutions yourself. List your collaborators on your submission.

If you are asked to design an algorithm as part of a homework problem, please provide: (a) the pseudocode for the algorithm, (b) an explanation of the intuition for the algorithm, (c) a proof of correctness, (d) the running time of your algorithm and (e) justification for your running time analysis.

Submissions. Please submit a PDF file. You may submit a scanned handwritten document, but a typed submission is preferred. It will be very helpful if in your submission each question starts on a new page.

1. **(20 points) Maximum Subsequence Sum** Given an array of n arbitrary real numbers, write an $O(n)$ algorithm to find a contiguous subsequence with maximum sum.
   Hint: Consider whether a maximum subsequence can include a previous candidate in whole or in part.

2. **(20 points) Longest Increasing Subsequence** Give an $O(n \log n)$ algorithm to find the longest (not necessarily strictly) increasing subsequence of a sequence of n numbers.
   Hint: Store information to quickly find the longest sequence you can extend with the current element.

3. **(20 points) Inversions and Insertions**. Given a permutation of the numbers from 1 to n, give an algorithm that finds the minimal number of changes needed to put the numbers in sorted order. A change means taking a number out of the sequence and placing it somewhere else in the sequence (including at beginning or end).

4. **(20 points) Ordered Graphs** (K&T Ch.6 Ex.3). Consider a directed acyclic graph with nodes given in topological order, $v_1, \ldots, v_n$, where every node except v_n has some outgoing edge.

   a) Show that the following algorithm does not correctly determine the length of the longest path

   L = 0
   for ($u = v_1$; $u \neq v_n$; $u = v_j$, ++L)
   choose edge (u, v_j) with minimum j
   return L

   b) Give an efficient algorithm that finds the length of the longest path.

5. **(20 points) Game Strategy** You have $2n$ coins of arbitrary denominations in a row. You and an evil mathematician play a game where you alternate taking a coin from either of the ends. You start. For instance, with the row of coins 5, 3, 7, 10, you can take the 10, the mathematician might take the 7, then you can take the 5 and gain 15 in total, while the mathematician would only have 10.

   Write a dynamic programming algorithm that precomputes the moves to maximize your winnings. Assume the evil professor tries their best to stop you.

6. **(0 points).** How long did it take you to complete this assignment?