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COMPSCI 250
Introduction to Computation
Second Midterm Spring 2019

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are 4 problems on pages 2–7, some with multiple parts, for 100 points + 10 extra credit. Probable scale is around A=95, C=65, but will be determined after we grade the exam.
- Justify your answers and show your work. This may help with assigning partial credit.
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.

Question 1 (20):

a. (10p) Perform the extended Euclidean algorithm on numbers 103 and 40, and find the inverse of 103 modulo 40 and the inverse of 40 modulo 103, if they exist. Clearly state your answer.

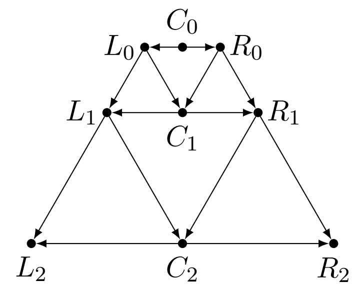
b. (10p) Solve the congruence system $x \equiv 3 \pmod{103}$, $x \equiv 2 \pmod{40}$.

Question 2 (20p)

a. (10p) A sequence a_n is defined recursively by $a_1 = 1$, $a_2 = 4$ and $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n > 2$. Prove that $a_n = n^2$ for all positive naturals n.

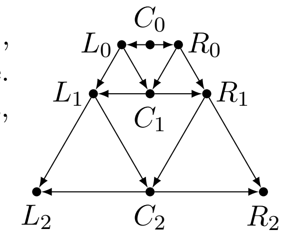
b. (10p) Find a recurrence relation for the number of strings of **0** and **1** with length $n \geq 0$ that do not contain two consecutive **0**s. (Hint: Consider the two choices for the first character).

Question 3 (50): Consider a directed graph G_n formed of successively larger equilateral triangles, as in the figure (which shows G_2). There are edges from C_k to L_k and R_k (for $k \leq n$), from L_{k-1} to C_k and L_k, and from R_{k-1} to C_k and R_k, if $k > 0$. The length of the segment L_0R_0 is 1.

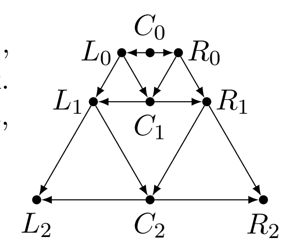


a. (10p) How many paths are there from C_0 to L_n?
Find and justify a recurrence. Then find a formula in terms of n, and prove it by induction.

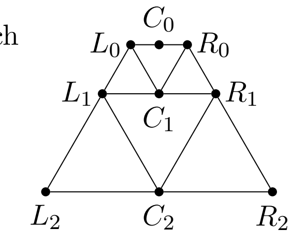
b. (10p) In the graph G_2, carry out a breadth-first search starting from L_0, without goal node, recognizing nodes as they come off the queue. Explore neighbors in alphabetical order. Show the search progress, draw the BFS tree, and identify the non-tree edges.



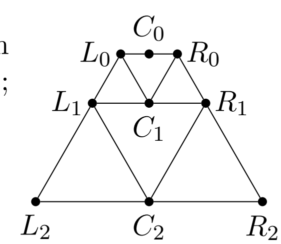
c. (10p) In the graph G_2, carry out a depth-first search starting from C_0, without goal node, recognizing nodes as they come off the stack. Explore neighbors in alphabetical order. Show the search progress, draw the DFS tree, and identify the type of any non-tree edges.



d. (10p) In the **undirected** version of G_2, carry out a uniform-cost search from L_0 with goal node L_2.
Edge costs are equal to their lengths. The length of L_0R_0 is 1.
At equal priority, order nodes alphabetically.



e. (10p) In the **undirected** version of G_2, carry out an A^* search from L_0 with goal node L_2. The heuristic function is: $h(L_i) = 2 - i$; $h(C_i) = h(L_i) + 1/2$; and $h(R_i) = h(L_i) + 1$.
Edge costs are equal to their lengths. The length of L_0R_0 is 1.
At equal priority, order nodes alphabetically.



Question 4 (20p)

The following are ten true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

a. The function $f(m,n) = 2^m(2n+1)$ is a one-to-one correspondence from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N} \setminus \{0\}$.

b. Let P be a property of strings over $\{0,1\}$. If $P(\lambda)$ is true, and for any string w, $P(w) \rightarrow P(w1)$, $P(w) \rightarrow P(w00)$ and $P(w0) \rightarrow P(w1)$, then $P(w)$ is true for all w.

c. Let P be a property of naturals. If $P(0)$ is true, $P(n)$ is true for all odd n, and $P(n) \rightarrow P(2n)$ for all n, then $P(n)$ is true for all n.

d. If the postfix string of an arithmetic expression contains two consecutive operators, then the prefix string of that expression also contains two consecutive operators.

e. It is possible to construct a game tree with a winning strategy for the first player if less than 1/4 of the leaf nodes are winning nodes.

f. The height of a rooted tree is the maximum distance from the root to a leaf. Then every rooted binary tree of height h has exactly 2^h leaves.

g. Consider the recursive algorithm to cover a 2^k by 2^k square, with one square missing, with L-shaped tiles. The base case of this algorithm is to cover a 1 by 1 square (with one square missing) by doing nothing. When we call this algorithm on a 2^k by 2^k square, the resulting call tree has height k and exactly 4^k leaves.

h. Since the number 250 satisfies the congruences $x \equiv 16 \pmod{39}$ and $x \equiv 55 \pmod{65}$, the next largest integer to satisfy these two congruences is $250 + 39 \cdot 65 = 2785$.

i. If in a connected graph G, after cutting an edge, the resulting graph is no longer connected, then G is a tree.

j. A forest is an undirected graph with no cycles. A tree is a connected forest. If F is a forest with five nodes and three edges, there is at most one node in F that is not the endpoint of any edge.

1	/20
2	/20
3	/50
4	/20
Total	/110