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COMPSCI 250
Introduction to Computation
First Midterm Spring 2019

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are 4 problems on pages 2–7, some with multiple parts, for 100 points + 10 extra credit. Probable scale is around A=95, C=65, but will be determined after we grade the exam.
- Justify your answers and show your work. This may help with assigning partial credit.
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.

Question 1 (35):

a. (10p) Translate the following statements, using predicates with arguments over a set D of dogs. Clearly define what your predicates mean.

(3p) (A1) Every dog is happy if all its offspring can play in the snow.

(1p) (A2) Thick-furred dogs can play in the snow.

(3p) (A3) A dog is thick-furred if it is the offspring of at least one thick-furred dog.

(2p) (A4) Every thick-furred dog is the offspring of some thick-furred dog.

(1p) (C) All thick-furred dogs are happy.

b. (15p) Assuming statements A1–A3 are all true, prove that statement (C) is true. Make it clear every time you use a quantifier proof rule.

c. (6p) Does A3 imply A4? Does A4 imply A3? Prove each statement or give a counterexample (a set of dogs with corresponding values for the *offspring* and *thick-furred* predicates).

d. (4p) Do A1, A2 and A4 together imply C? Prove or give a counterexample.

Question 2 (20):

Consider the set of strings over the alphabet of lowercase letters. Define the relation $R(u, v)$ if $v = uw$, and w contains no letters that come before any of the letters of u in the alphabet.
For example, $R(ba, bab)$ holds, but not $R(ba, baac)$.

a. (10XC) Prove that R is a partial order on strings. You need not do quantifier proofs, but use notation that makes your reasoning as clear and rigorous as possible.

b. (10p) Now consider the same relation, but over the set of strings of at most three letters with the alphabet $\{a, b\}$. Draw the Hasse diagram for the relation R over this set.

Question 3 (30):

The following are fifteen true/false or multiple choice questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

a. The proposition $(p \leftrightarrow q) \vee (r \wedge s)$ is true in how many of the 16 lines of the truth table?
a) 3 b) 9 c) 10 d) 12

b. If assuming P we prove Q, and assuming Q we prove P, then we can derive $P \wedge Q$.

c. Consider the propositions p: "It is cloudy", and q: "it is raining".
Which of the following expresses the sentence "It is not cloudy unless it is raining"?
a) $\neg p \wedge q$ b) $\neg p \vee q$ c) $\neg q \vee p$ d) $\neg p \vee \neg q$

d. The statement $\forall x : \exists y : \forall z (P(x, y) \wedge R(w, z) \wedge Q(z))$ is a predicate over how many free variables?
a) 1 b) 2 c) 3 d) 4 e) 5

e. The statements $(\exists x : P(x)) \vee (\exists x : Q(x))$ and $\exists x : \exists y : (P(x) \vee Q(y))$ are not equivalent.

f. If $(\forall x : \exists y : P(x, y)) \rightarrow (\exists x : \forall y : P(x, y))$ then there exists a value a such that $P(a, y)$ holds for all y.

g. $(\exists x : \forall y : P(x, y)) \rightarrow (\exists y : \forall x : P(y, x))$ is a tautology.

h. The number of binary relations that can be constructed on a set with two elements is
a) 4 b) 8 c) 16 d) 64

i. Consider any surjective function $f : A \rightarrow B$. If f is not injective, there are no injective functions from A to B.

j. Let $f, g : \mathbb{N} \rightarrow \mathbb{N}$ be polynomials with natural coefficients. Then $f \circ g = g \circ f$.

k. A binary relation is *well-defined* if and only if its inverse is *one-to-one*.

l. Let R and S be two symmetric relations on a set A. The relation $U(x, y) = R(x, y) \vee S(x, y)$ may fail to be symmetric.

m. A partial order on a set A is also a total relation on $A \times A$.

n. Which of the following relations over the set of all people is an equivalence relation?
a) $\{(x, y) \mid x$ and y are siblings $\}$ c) $\{(x, y) \mid x$ and y have the same age $\}$
b) $\{(x, y) \mid x$ and y are married $\}$ d) none of the three

o. The relation on naturals defined by "x and y have some common divisor" is an equivalence relation.

Question 4 (25):

Consider the following compound proposition:
$(r \to u) \land ((p \to s) \to (q \lor r)) \land \neg(p \land r) \land (u \to (p \lor s)) \land ((u \lor p) \to \neg q) \land (r \lor (s \land u))$

a. (5p) Convert it to conjunctive normal form (a conjunction of clauses, which are disjunctions of propositions or their negations).

b. (10p) Find an assignment to atomic propositions that makes it true, or show there is none.

c. (10p) Consider the sets $D = (A \Delta B) \cap C$ and $E = (B \cap (A \cup C)) \setminus (A \cap C)$. Use propositional rules (equational proofs) or set identities to express $D \setminus E$ and $D \cap E$ in the simplest way using A, B, C, and their complements $\bar{A}$, $\bar{B}$, $\bar{C}$.

1	/35
2	/20
3	/30
4	/25
Total	/110