COMPSCI 250 Spring 2019 - Homework 2

Due: Thursday February 21 at 11:59 pm in Gradescope

This assignment has 80 points plus 10 extra credit. The number in parentheses after each problem is its point value.

Please submit a single PDF file, with the problems in order (as below), and legible. Look at your PDF before submitting it – it is fine to scan or photograph a handwritten document but it the graders can't read it, they won't grade it.

Please assign pages to problems in Gradescope. Graders will click on the problem number. If no page shows up because it's not assigned, the assumption is you have not solved the problem.

Be sure you are doing Problems in the book and not Exercises: the numbers should start with P rather than E.

You are responsible for following the academic honesty guidelines on the Grading and Requirements page. This means that what you present must be your own work in presentation, and you must acknowledge all sources of aid other than course staff and the textbook.

P2.3.N (10) In CS 187, you've encountered big-O notation. Consider two functions with natural arguments and positive real values: $f, g : \mathbb{N} \to \mathbb{R}^+$. Consider the following propositions, where c ranges over positive reals, and n, n_0 range over the naturals:

$P : \exists c : \exists n_0 : \forall n : n \geq n_0 \to f(n) \leq cg(n)$ (this is the definition of "$f(n)$ is $O(g(n))$")

$Q : \forall n_0 : \exists c : \forall n : n \geq n_0 \to f(n) \leq cg(n)$

$R : \forall c : \exists n_0 : \forall n : n \geq n_0 \to f(n) \leq cg(n)$

a) Are any two of these propositions equivalent, for arbitrary choices of f and g? (prove it, or give examples of f and g for which they differ).

b) Is any of the propositions always true, whatever f and g?

c) Can you find two functions f and g for which R is true ?

P2.5.N (10): Let X, Y, Z be any three languages. Using the definition of language concatenation, write quantified expressions for the membership predicates of the languages $(X \cup Y)Z$ and $XZ \cup YZ$. Are the two languages equal? Give a quantifier proof, or a counterexample.
Do the same for the languages $(X \setminus Y)Z$ and $XZ \setminus YZ$.

P2.6.1 (10)

P2.6.4 (10)

P2.8.4 (10)

P2.9.3 (10)

P2.10.4 (10)

P2.10.XC (10) Let $A = \{a_1, a_2, ..., a_n\}$ be a finite set and P the set of all possible partitions of A. Your task is to define a partial order on P so that $\{\{a_1\}, \{a_2\}, ..., \{a_n\}\}$ is the maximal element and $\{\{a_1, a_2, ..., a_n\}\}$ is the minimal element. Prove that your solution is indeed a partial order. Hint: any partition represents an equivalence relation; define an order on such relations.

P2.11.5 (10)