NAME: _____

COMPSCI 250
Introduction to Computation
Second Midterm Fall 2019

M. Minea 7 November 2019

DIRECTIONS:

- Answer the problems on the exam pages.
- There are 4 problems on pages 2–6, some with multiple parts, for 100 points + 10 extra credit. Probable scale is around A=95, C=65, but will be determined after we grade the exam.
- Justify your answers and show your work. This may help with assigning partial credit.
- If you need extra space use a blank page.
- No books, notes, calculators, or collaboration.

Question 1 (20):

Let C_n be the number of strings of length n over $\Sigma = \{a, b, c\}$ that do not contain either aa or ba.

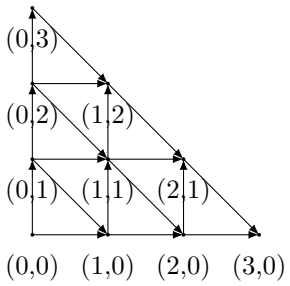
(a) Find a recurrence for C_n (i.e., a relation using previous terms of the sequence).

(b) Show by induction that $C_n = \left((1+\sqrt{2})^{n+1} + (1-\sqrt{2})^{n+1}\right)/2$.

Question 2 (20):

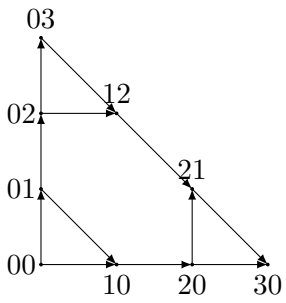
(a) Consider the sequence given by $a_0 = 0$, $a_1 = 1$, $a_n = 2a_{n-1} + a_{n-2}$ for $n > 1$.
State and prove a theorem that tells for exactly which values of n the value a_n is divisible by 5.

(b) Consider the directed graph G_n (G_3 is depicted), with all edges going up, right, or down and right. More precisely, G_n has all nodes (x, y) with $0 \leq x, y \leq n$ and $x + y \leq n$, and edges $(x, y) \to (x + 1, y)$, $(x, y) \to (x, y + 1)$ and $(x, y + 1) \to (x + 1, y)$ (if both endpoints belong to G_n).
Find and prove a recurrence and then a formula for the number of directed paths from node $(0, 0)$ to the rightmost node $(n, 0)$. Justify your arguments completely and rigorously.

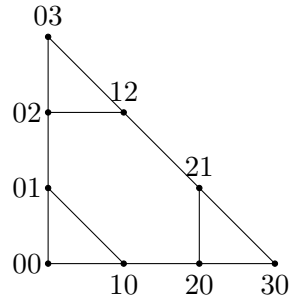


Question 3 (40p) In your graph searches, use a closed list. Show the evolution of the open list. When you need to decide which node to explore first, choose alphabetical order.

(a) In the given directed graph, carry out a DFS from node 00 without a goal node. Draw the DFS tree, and identify the type of any non-tree edges.

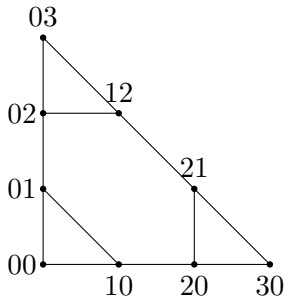


(b) In the given undirected graph, carry out a BFS from node 12 without a goal node. Draw the BFS tree, and also show any non-tree edges.

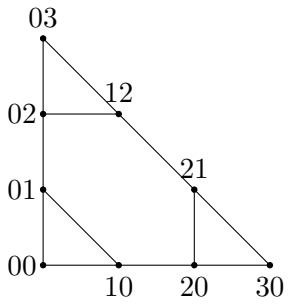


For the following two questions, the cost of diagonal edges is 1.5, all other edges have cost 1.

(c) In the given undirected graph, perform a UCS from node 03 with goal 30.



(d) In the given undirected graph, carry out an A* search from node 03 with goal node 30. The heuristic function for node xy is $h(xy) = (y + 3 - x)/2$.



Question 4 (30p)

The following are fifteen true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

a. The following is not a well-defined recursive function on binary strings: $f(\lambda) = 1$, $f(u0) = f(u)$, $f(u11) = f(u)$, $f(u01) = 1 - f(u)$.

b. If $P(0)$, $P(1)$ and $P(2)$ are true, and for all $n > 3$, $(P(n-4) \to P(n)) \vee (P(n-3) \to P(n))$ then $P(n)$ is true for all n.

c. If $P(0)$ holds, and $(P(j) \wedge P(k)) \to P(2^k(2j+1))$ for all $j, k \geq 0$, then $P(n)$ holds for all $n \geq 0$.

d. Consider the relation D on naturals, so that $D(0,0)$ holds and $D(S(x), S(S(y))) \leftrightarrow D(x,y)$, where S means successor. Then $D(x,y)$ holds iff $y = 2x$.

e. Let f be a function on strings, so that $f(\lambda) = \lambda$ and $f(u) = (f(u^R))^R$, where ^{R} is string reversal. Then f is the identity function.

f. If nodes u and v are in different strongly connected components of a directed graph, then $P(u,v) \oplus P(v,u)$, where P is the path predicate.

g. By concatenating a shortest $u \rightsquigarrow v$ path with a shortest $v \rightsquigarrow w$ path we get a shortest path $u \rightsquigarrow w$.

h. For any arithmetic expression with at least two operators, either the prefix form or the postfix form contains two consecutive operators.

i. If we have a sequence of n binary operators and n operands, there are at most n ways to insert another operand and make it a valid postfix expression string.

j. If an undirected graph with n nodes has a simple cycle containing all nodes, then any DFS tree will have depth $n - 1$.

k. In an undirected graph, if using a closed list, the number of times a node is reached is the same in BFS and DFS from the same starting node.

l. In a BFS of a directed graph, no graph edge links nodes that are more than one level apart.

m. During uniform cost search, any node u that has an edge from the start node s will be placed on the queue only once.

n. If the heuristic h is admissible, when we take $(u, prio(u))$ off the queue, we might put on a neighbor of u with a lower value.

o. In a game tree with two choices at each step, which terminates in three moves (W-B-W), White might have a winning strategy even if only 2 of the 8 leaves are winning.

1	/20
2	/20
3	/40
4	/30
Total	/110

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