NAME: ____________________

COMPSCI 250
Introduction to Computation
Second Midterm Fall 2019

M. Minea 7 November 2019

DIRECTIONS:

• Answer the problems on the exam pages.

• There are 4 problems on pages 2–6, some with multiple parts, for 100 points + 10 extra credit. Probable scale is around A=95, C=65, but will be determined after we grade the exam.

• Justify your answers and show your work. This may help with assigning partial credit.

• If you need extra space use a blank page.

• No books, notes, calculators, or collaboration.
Question 1 (20):
Let \( C_n \) be the number of strings of length \( n \) over \( \Sigma = \{a, b, c\} \) that do not contain either \( aa \) or \( ba \).

(a) Find a recurrence for \( C_n \) (i.e., a relation using previous terms of the sequence).

(b) Show by induction that \( C_n = \frac{(1 + \sqrt{2})^{n+1} + (1 - \sqrt{2})^{n+1}}{2} \).
Question 2 (20):

(a) Consider the sequence given by \(a_0 = 0, a_1 = 1, a_n = 2a_{n-1} + a_{n-2}\) for \(n > 1\).
State and prove a theorem that tells for exactly which values of \(n\) the value \(a_n\) is divisible by 5.

(b) Consider the directed graph \(G_n\) (\(G_3\) is depicted), with all edges going up, right, or down and right. More precisely, \(G_n\) has all nodes \((x, y)\) with \(0 \leq x, y \leq n\) and \(x + y \leq n\), and edges \((x, y) \rightarrow (x + 1, y)\), \((x, y) \rightarrow (x, y + 1)\) and \((x, y + 1) \rightarrow (x + 1, y)\) (if both endpoints belong to \(G_n\)).
Find and prove a recurrence and then a formula for the number of directed paths from node \((0, 0)\) to the rightmost node \((n, 0)\). Justify your arguments completely and rigorously.
Question 3 (40p) In your graph searches, use a closed list. Show the evolution of the open list. When you need to decide which node to explore first, choose alphabetical order.

(a) In the given directed graph, carry out a DFS from node 00 without a goal node. Draw the DFS tree, and identify the type of any non-tree edges.

(b) In the given undirected graph, carry out a BFS from node 12 without a goal node. Draw the BFS tree, and also show any non-tree edges.
For the following two questions, the cost of diagonal edges is 1.5, all other edges have cost 1.

(c) In the given undirected graph, perform a UCS from node 03 with goal 30.

(d) In the given undirected graph, carry out an A* search from node 03 with goal node 30. The heuristic function for node $xy$ is $h(xy) = (y + 3 - x)/2$. 
Question 4 (30p)

The following are fifteen true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

a. The following is not a well-defined recursive function on binary strings: \( f(\lambda) = 1, f(u0) = f(u), f(u11) = f(u), f(u01) = 1 - f(u). \)

b. If \( P(0), P(1) \) and \( P(2) \) are true, and for all \( n > 3, (P(n - 4) \rightarrow P(n)) \lor (P(n - 3) \rightarrow P(n)) \) then \( P(n) \) is true for all \( n \).

c. If \( P(0) \) holds, and \( (P(j) \land P(k)) \rightarrow P(2^k(2j + 1)) \) for all \( j, k \geq 0 \), then \( P(n) \) holds for all \( n \geq 0 \).

d. Consider the relation \( D \) on naturals, so that \( D(0,0) \) holds and \( D(S(x), S(S(y))) \leftrightarrow D(x, y) \), where \( S \) means successor. Then \( D(x, y) \) holds iff \( y = 2x \).

e. Let \( f \) be a function on strings, so that \( f(\lambda) = \lambda \) and \( f(u) = (f(u^R))^R \), where \( R \) is string reversal. Then \( f \) is the identity function.

f. If nodes \( u \) and \( v \) are in different strongly connected components of a directed graph, then \( P(u,v) \oplus P(v,u) \), where \( P \) is the path predicate.

g. By concatenating a shortest \( u \leadsto v \) path with a shortest \( v \leadsto w \) path we get a shortest path \( u \leadsto w \).

h. For any arithmetic expression with at least two operators, either the prefix form or the postfix form contains two consecutive operators.

i. If we have a sequence of \( n \) binary operators and \( n \) operands, there are at most \( n \) ways to insert another operand and make it a valid postfix expression string.

j. If an undirected graph with \( n \) nodes has a simple cycle containing all nodes, then any DFS tree will have depth \( n - 1 \).

k. In an undirected graph, if using a closed list, the number of times a node is reached is the same in BFS and DFS from the same starting node.

l. In a BFS of a directed graph, no graph edge links nodes that are more than one level apart.

m. During uniform cost search, any node \( u \) that has an edge from the start node \( s \) will be placed on the queue only once.

n. If the heuristic \( h \) is admissible, when we take \( (u, \text{prio}(u)) \) off the queue, we might put on a neighbor of \( u \) with a lower value.

o. In a game tree with two choices at each step, which terminates in three moves (W-B-W), White might have a winning strategy even if only 2 of the 8 leaves are winning.

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