NAME: ____________________________

COMPSCI 250
Introduction to Computation
First Midterm Fall 2019

M. Minea 10 October 2019

DIRECTIONS:

- Answer the problems on the exam pages.
- There are 5 problems on pages 2–7, some with multiple parts, for 100 points + 10 extra credit. Probable scale is around A=95, C=65, but will be determined after we grade the exam.
- Justify your answers and show your work. This may help with assigning partial credit.
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.

Question 1 (25):

On their morning walks, Cardie and Duncan always visit Simple Gifts Farm, home of the busy farm dogs Danish (d) and Scout (s). In one recent four-day period, each of the farm dogs in the set $F = \{d, s\}$ was assigned zero or more tasks from the set T each day, so that the relation A is a subset of $F \times T \times D$ and $A(x, y, z)$ means "farm dog x was assigned task y on day z".

The set T consists of the tasks Deter Vermin (DV), Greet Cardie and Duncan (GCD), Supervise Chicken Feeding (SCF), and Supervise Squash Harvest (SSH). The set D consists of Monday, Tuesday, Wednesday, and Thursday.

a. (10p) Translate the following sentences to quantified statements in predicate logic.

1) There is a task that was assigned to both farm dogs every day.

2) Scout was never assigned SCF, and Danish was never assigned SSH.

3) SSH was assigned to a farm dog on exactly the days that were not Tuesday,
and SCF was assigned to a farm dog on exactly the days that were not Wednesday.

4) On any two different days, there exists a farm dog that was assigned GCD on one day but not the other.

5) Every dog has a day on which they were assigned exactly three tasks.

b. (15p) Show that the four premises (1)-(4) together imply the goal (5).
You may use a combination of formulas and English, but make the use of quantifier rules clear.

Question 2 (15):

Consider the following compound proposition:

$(d \to \neg(c \to a)) \land (\neg d \to (c \to b)) \land (a \oplus b) \land ((a \lor c) \to \neg b) \land (c \to (a \land \neg d))$

a. (8p) Convert it to conjunctive normal form (a conjunction of clauses, which are disjunctions of propositions or their negations).

b. (7p) Find an assignment to atomic propositions that makes it true, or show there is none.

Question 3 (20):

Let C be a set of finite languages over an alphabet Σ. Define a relation on C as follows: $P(X, Y)$ if $X = Y$ or every string in X is a proper substring of some string in Y.

a. (10p) Show that P is a partial order on C.

b. (10p) Take C to be the set of all languages over $\{a, b\}$ that have exactly two strings, and the sum of their lengths is at most 2. Draw a Hasse diagram for the the order P on C.

Question 4 (20):

a. (10p) Perform the extended Euclidean algorithm on numbers 104 and 41, and find the inverse of 104 modulo 41 and the inverse of 41 modulo 104, if they exist. Clearly state your answer.

b. (10p) Solve the congruence system $x \equiv -12 \pmod{104}$, $x \equiv 13 \pmod{41}$.
Does it have a positive solution less than 1000? If so, which?

Question 5 (30):

The following are fifteen true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

a. $((p \oplus q) \oplus p) \leftrightarrow q$ is a tautology.

b. $(p \wedge (p \rightarrow q)) \leftrightarrow (p \wedge q)$ is a tautology.

c. The proposition $(p \rightarrow q) \vee (r \rightarrow s)$ has 9 true entries in the truth table.

d. If I assume P and prove Q; then separately I assume Q and prove R; then separately I assume $\neg R$ and prove $\neg P$, then P, Q and R are all equivalent.

e. The statement $(A \Delta B) \Delta (A \Delta C) = (B \Delta C) \setminus A$ is a set identity.

f. The statements $\exists x : P(x) \vee Q(x)$ and $(\exists x : P(x)) \vee (\exists x : Q(x))$ are equivalent.

g. If A is a set of strings, all of length k, B is a set of strings, all of length l, and $k \neq l$, then AB and BA have the same number of elements.

h. If a relation is total, symmetric and transitive, it must be reflexive.

i. If $f : A \rightarrow B$ is any function, then the sets $\{x \in A : f(x) = b\}$ for $b \in B$ define a partition of A.

j. If $f : A \rightarrow B$ is one-to-one and $g : B \rightarrow C$ is bijective, then $g \circ f$ is bijective.

k. The relation $R(u, v)$ defined as "u is a prefix of v or v is a prefix of u" is an equivalence relation over any language of strings.

l. If in Question 3, we do not require languages to be finite, then the statement at 3a may be false.

m. Let A be a set of natural numbers. Then the relation $R(x, y)$ defined as "x and y have the same number of divisors" is an equivalence relation but cannot be a partial order.

n. If p is a prime, then the relation $R(x, y)$ on naturals, defined as "x and y both have inverses modulo p" is an equivalence relation.

o. If m and n are distinct odd numbers with a common divisor greater than 1, then the system $x \equiv 3 \pmod{m}$ and $x \equiv 7 \pmod{n}$ always has no solution.

1	/25
2	/15
3	/20
4	/20
5	/30
Total	/110