1. Motivation

A/B testing is a powerful and widely used method for performing causal inference in real-world settings. These experiments are difficult to reason over in networks, where treatment effects may spill over to individuals assigned to control.

Graph cluster randomization [1] provides a framework for A/B testing in networks which reduces bias from interference between units.

However, little is known about the interaction between the underlying graph topology, the clustering method, and the error of the final causal estimate.

2. Graph Cluster Randomization (GCR)

For a given graph $G$ and clustering algorithm:
1. Cluster the graph $G$.
2. Randomly assign treatment to each cluster, i.e., all nodes within a cluster share treatment status.
3. Estimate the causal effect.

Assigning treatment to entire clusters provides an approximation to the social behavior of nodes under global treatment or global control and limits spillover effects.

GCR produces an unbiased estimator of average treatment effect (ATE) irrespective of the graph partitioning [1]. However, poorly chosen clusters may increase the variance of the estimator.

We characterize the relationship between graph topologies and causal effect estimation. We show that modularity, a commonly used metric for measuring the quality of community detection algorithms, can be used as a reliable proxy for error in treatment effect arising from the clustering method.

3. Experimental design and topology

Estimation using GCR is influenced by local graph topology, clustering technique, and exposure model. This introduces a large space of variation within the GCR framework.

**Outcome models:**
The form of the response function of an individual according to its number or proportion of treated friends.

**Random graphs:**
- (a) small-world networks
- (b) scale-free networks
- (c) stochastic block models (SBMs)

**Modularity and effect estimation**

Modularity ($Q$) is a measure of the division of the network into clusters. It is calculated from the number of edges between nodes in the same cluster and the number of edges between nodes in different clusters.

$$Q = \frac{1}{2m} \sum_{vw} \left( A_{vw} - \frac{k_v k_w}{2m} \right) s_v s_w + \frac{1}{2}$$

Ugander et al. [1] show the variance of the effect estimator is deeply linked to the exposure probabilities of each node.

Exposure probabilities of a node depend directly on the treatment status of its neighbors. These are assigned by cluster, meaning the exposure probabilities are upper bounded by the modularity of the clustering.

4. Modularity as variance bounds

Reducing variance bounds for an unbiased estimator increases confidence in the ATE estimate.

Exposure probabilities also depend on the true outcome model, which is not known in practice. Modularity bounds the variance of the estimator according only to the graph clustering.

Clusterings with high modularity reduce the variability of the average treatment effect estimate due to choices in the experimental design.

**Experimental setup:**
1. Generate random graphs using four different graph generation algorithms, sweeping across parameter settings.
2. Construct graph clusterings using the 3-net algorithm.
3. For each clustering, randomly assign clusters to treatment or control.
4. Estimate the treatment effect as a function of treatment assignment.
5. For each outcome model, calculate the actual treatment effect as a function of treatment assignment, and determine the ATE error of the estimate.

Our results show that modularity functions as a proxy for the variance in average treatment effect error induced by the graph topology.