Second-Order Constraints in Dynamic Invariant Inference

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Invariants on Program Behaviour

- Invariants are formal documentation:
  - Pre-/postconditions, class invariants
  - Effect specifications
  - Dependency specifications

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- Formal semantics: amenable to formal methods, testing

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Example:

ArrayStack.peek() **requires** preconditions:

```
 storageArray != null
 topOfStackIndex >= 0
 topOfStackIndex < storageArray.length
...
```

Limitations of current invariant-based approaches

- Invariants sometimes hard to write
  Partly addressed by invariant inference
  (e.g., Daikon, DIDUCE, DySy, Heureka, ...)
- Can bury important information
  E.g., some Daikon-inferred specifications for simple methods have dozens of axioms
- Can involve redundancy
- Can be inconsistent with higher-level knowledge

ArrayStack.peek() **requires**:

```
array != null
topOfStack >= 0
topOfStack < array.length
...
```

ArrayStack.pop() **requires**:

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array != null
topOfStack >= 0
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Our approach: *constraints between sets of invariants*

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- if we meet all preconditions of ‘peek’ we also meet all preconditions of ‘pop’:
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Examples:

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- 'pop' is safe to call after 'push':
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- ‘pop’ is safe to call after ‘push’:
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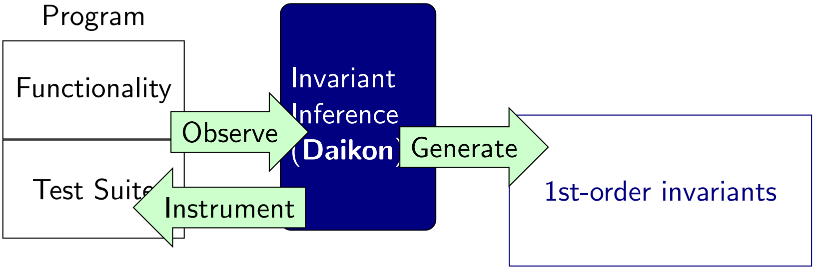
- ‘invert3x3Matrix’ works like ‘invertMatrix’, but may have stronger preconditions:
  CONCORD(invertMatrix, invert3x3Matrix)

Applying second-order constraints to Daikon



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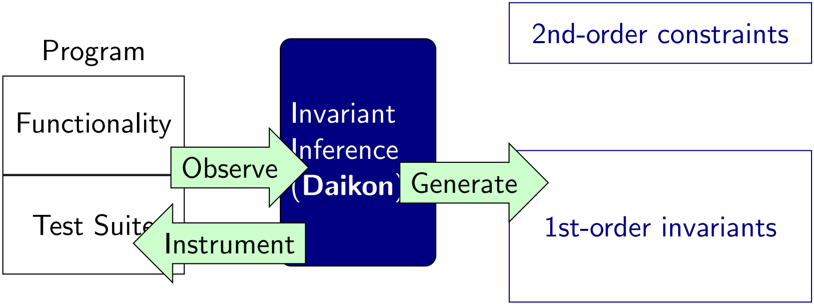
- Hand-written documentation



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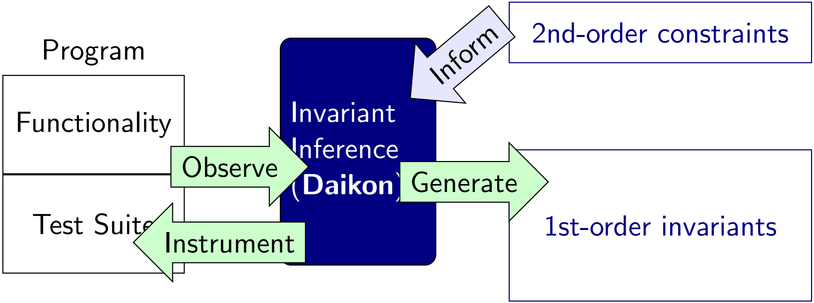
  - A Hints for first-order invariant inference



Applying second-order constraints to Daikon

- Hand-written documentation
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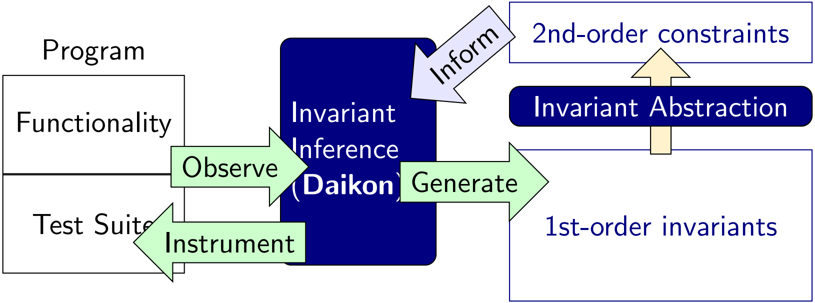
  - A Hints for first-order invariant inference
  - B *Automatically inferred* higher-level documentation



A catalogue of second-order constraints

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  'pop' is applicable whenever 'peek' is

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Other useful properties are conceivable

Logical structure of our constraint catalogue

requires	**ensures**
pre (A)	post (A)
pre (B)	post (B)

Logical structure of our constraint catalogue

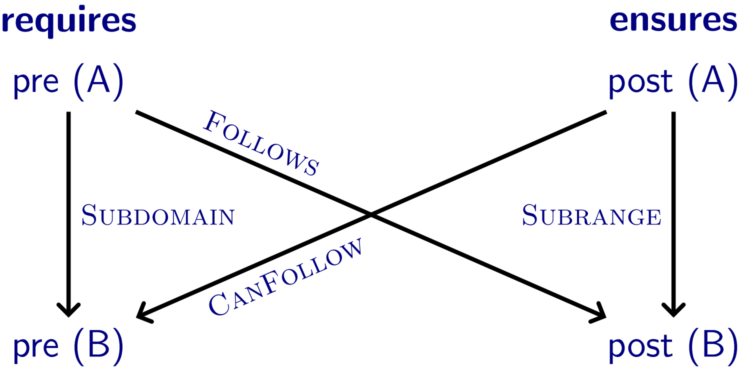
requires	**ensures**
pre (A)	post (A)
$\downarrow$ Subdomain	
pre (B)	post (B)

Logical structure of our constraint catalogue

requires | **ensures**

$$\begin{tikzcd} \text{pre }(A) \arrow[d, "\textsc{Subdomain}"] & & \text{post }(A) \arrow[d, "\textsc{Subrange}"'] \\ \text{pre }(B) & & \text{post }(B) \end{tikzcd}$$

Logical structure of our constraint catalogue



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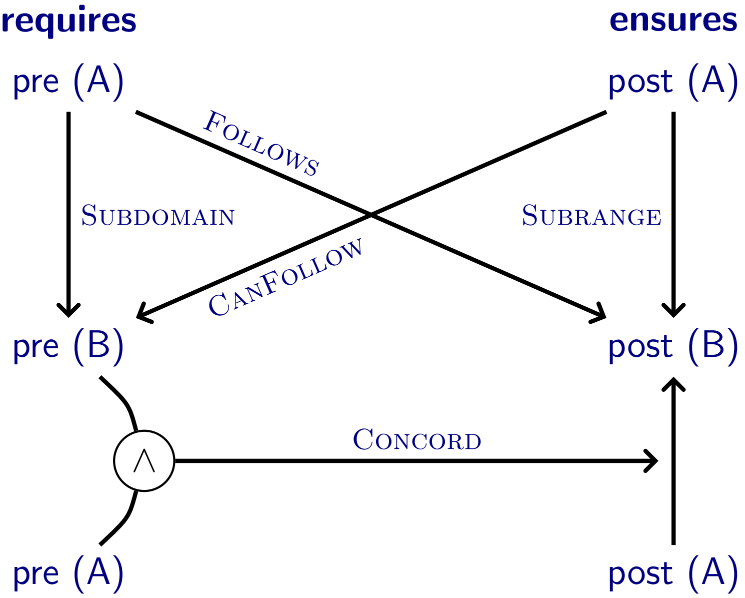
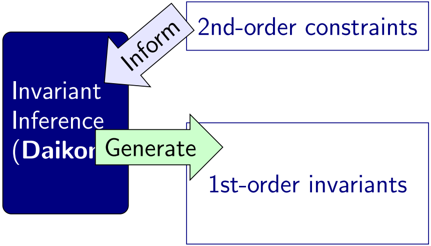


Inference with Second-Order Constraints

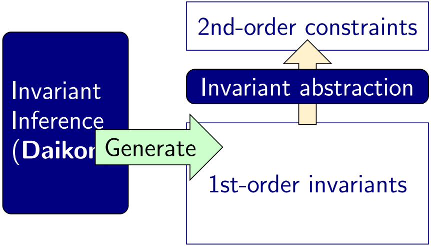
Ⓐ Refined First-Order Inference

- User provides hand-written second-order constraints



Ⓑ Second-Order Inference

- System infers second-order constraints



Refined First-Order Inference

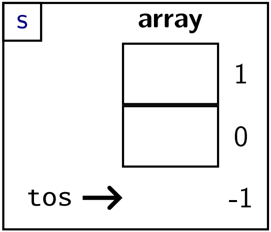
```
ArrayStack s = new ArrayStack();
s.push("bar");
s.push("foo");
assert s.peek() == "foo";
assert s.pop() == "foo";
assert s.peek() == "bar";
```

Daikon

`ArrayStack.peek()` **requires**

`ArrayStack.pop()` **requires**

Refined First-Order Inference




```
⇒ ArrayStack s = new ArrayStack();
   s.push("bar");
   s.push("foo");
   assert s.peek() == "foo";
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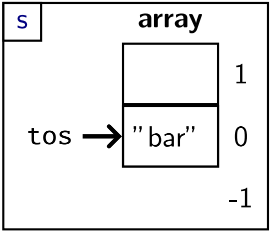
```

Daikon

`ArrayStack.peek()` **requires**   `ArrayStack.pop()` **requires**

Refined First-Order Inference

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ArrayStack s = new ArrayStack();
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assert s.peek() == "foo";
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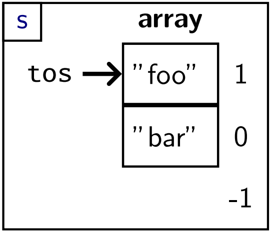
```

Daikon

`ArrayStack.peek()` **requires** `ArrayStack.pop()` **requires**

Refined First-Order Inference

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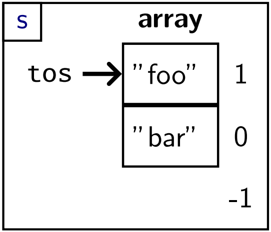
Daikon

`ArrayStack.peek()` **requires** `ArrayStack.pop()` **requires**

Refined First-Order Inference

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s.push("foo");
⇒ assert s.peek() == "foo";
assert s.pop() == "foo";
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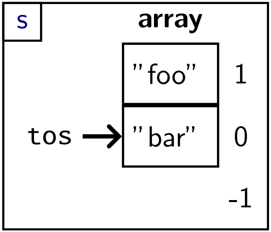
```

Daikon

`ArrayStack.peek()` **requires**
`array != null`
`tos = 1`

`ArrayStack.pop()` **requires**

Refined First-Order Inference

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⇒ assert s.pop() == "foo";
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Daikon

```
ArrayStack.peek() requires
 array != null
 tos = 1
```

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 array != null
 tos = 1
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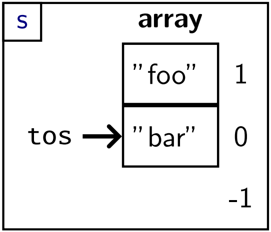
Refined First-Order Inference

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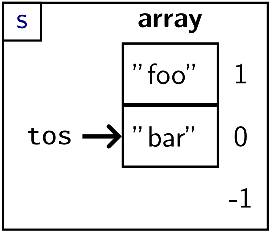
Daikon

```
ArrayStack.peek() requires
 array != null
 tos = 1...?
```

```
ArrayStack.pop() requires
 array != null
 tos = 1
```

Refined First-Order Inference

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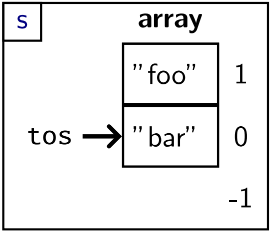
Daikon

```
ArrayStack.peek() requires
 array != null
 tos >= 0
```

~~tos = 1~~

```
ArrayStack.pop() requires
 array != null
 tos = 1
```

Refined First-Order Inference

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SUBDOMAIN(peek, pop)

```
ArrayStack.peek() requires
  array != null
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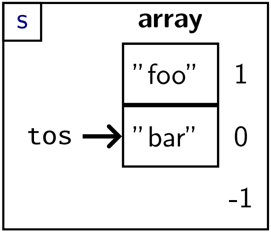


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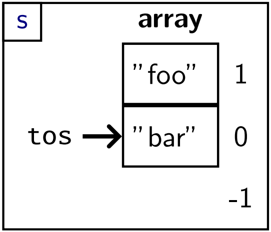


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Daikon

```
ArrayStack.peek() requires        ArrayStack.pop() requires
  ar
tos
tos
```

Each second-order constraint propagates information across Daikon

Second-Order Inference

- Hypothesise all possible 2nd-order constraints per class
- Compute fraction of overlap (using automated theorem prover): determine a *confidence* metric on 2nd-order constraint

Example hypothesis: SUBRANGE(ArrayStack.pop, ArrayStack.peek)

peek() **ensures**
  return.class == String
  this has only one value

pop() **ensures**
  return.class == String
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$post(peek) \Rightarrow$ `return.class == String`?

$$confidence = \frac{1}{1}$$

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$post(peek) \Rightarrow$ `this has only one value?`

$$confidence = \frac{1+1}{1+1}$$

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post(*peek*) ⇒ *return* has only one value?

$$confidence = \frac{1+1+0}{1+1+1}$$

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- Hypothesise all possible 2nd-order constraints per class
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- Incorporate per-invariant **Daikon Confidence** metrics

Example hypothesis: SUBRANGE(ArrayStack.pop, ArrayStack.peek)

DC `peek()` **ensures**

.9	`return.class == String`
.8	*this* has only one value

DC `pop()` **ensures**

.7	`return.class == String`
.6	*this* has only one value
.5	*return* has only one value

$$\mathit{confidence} =$$

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post(*peek*) $\Rightarrow$ *return* has only one value?

$$\textit{confidence} = \frac{.9 \times .8(.7 + .6 + 0)}{1 + 1 + 1}$$

Evaluation (overview)

A Refined First-Order Inference

- Evaluated changes to inferred first-order invariants
- Second-order constraints written by hand for:
  - Daikon's StackAr
  - 18 Apache Commons Collections classes
  - 7 AspectJ classes

B Second-Order Inference

- Evaluated generated second-order constraints
- For all hand-written second-order constraints
- For random classes:
  - 2 Apache Commons Collections
  - 2 AspectJ
- Confidence threshold: 0.75

Evaluation (overview)

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Evaluation: Refined Invariant Inference (1)

StackAr: Daikon's stack-based array implementation

- int topOfStack: top-of-stack location
- int[] theArray: stack representation
- pop(): remove top element
- top(): peek at top element value
- topAndPop(): remove top element and return it

Evaluation: Refined Invariant Inference (1)

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SUBDOMAIN(POP, TOP)
SUBDOMAIN(TOP, TOPANDPOP)
SUBDOMAIN(TOPANDPOP, POP)

- Removed 5 false invariants:
  - `this has only one value`
  - `theArray has only one value`
  - `theArray.length == 100`
  - `theArray[topOfStack] != null`
  - `topOfStack < theArray.length-1`

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$$\textsc{Subdomain}(\textsc{pop}, \textsc{top})$$
$$\textsc{Subdomain}(\textsc{top}, \textsc{topAndPop})$$
$$\textsc{Subdomain}(\textsc{topAndPop}, \textsc{pop})$$

- Removed 5 false invariants:
- Added 2 new invariants:
  - `this.topOfStack >= -1`
  - `DEFAULT_CAPACITY != theArray.length-1`

Evaluation: Refined Invariant Inference (1)

StackAr: Daikon's stack-based array implementation

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- Removed 5 false invariants:
- Added 2 new invariants:
- Refined 1 overly-specific invariants:

```
this.topOfStack < size(this.theArray[])-1
```

$$\Downarrow$$

```
this.topOfStack <= size(this.theArray[])-1
```

Evaluation: Refined Invariant Inference (2)

Experiment	2nd-order Constraints Written	1st-order Invariants Added	Removed	Refined
StackAr #1	3	2	5	1
StackAr #2	3	3	4	0
Apache Commons Collections	26	25 + 1	35	5
AspectJ	26	12 + 1	12	3

- Most changes positive
- Only two incorrect invariants introduced
  - Both due to nonmonotonicity in the underlying first-order invariant inference mechanism

Evaluation: Refined Invariant Inference (2)

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StackAr #2	3	3	4	0
Apache Commons Collections	26	25 + 1	35	5
AspectJ	26	12 + 1	12	3

- Most changes positive
- Only two incorrect invariants introduced
  - Both due to nonmonotonicity in the underlying first-order invariant inference mechanism

Overwhelmingly positive effects

Evaluation (overview)

Ⓑ Second-Order Inference

- Evaluated generated second-order constraints
- For all hand-written second-order constraints
- For random classes:
  - 2 Apache Commons Collections
  - 2 AspectJ
- Confidence threshold: 0.75

Evaluation: Second-Order Inference

1. Confirming our manual annotations:
   Of our 64 original manual annotations:
   - 37 we inferred
   - 27 we did *not* infer, of which:
     - 12 we had wrongly annotated
     - 7 lacked any supporting data samples
     - 6 we rejected due to 'noise' first-order invariants
     - 2 were **Concord**

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2. Finding new second-order constraints:

	inferred	**incorrect**
Apache Commons: AbstractMapBag	2	0
Apache Commons: SingletonMap	806	0
AspectJ Reflection:	30	0
AspectJ LstBuildConfigManager:	112	5

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IMMUTABLE class: suggests even higher-order invariants!

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Aspects Reflection:	[illegible]	0
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$\boxed{\text{Incomplete unit tests} \Rightarrow \text{Poor Daikon invariants}}$

Conclusions

Second-order constraints:

- Permit high level of discourse about program properties
- Refine the quality of detected first-order invariants
- Can be detected automatically
- Are easy to use and powerful