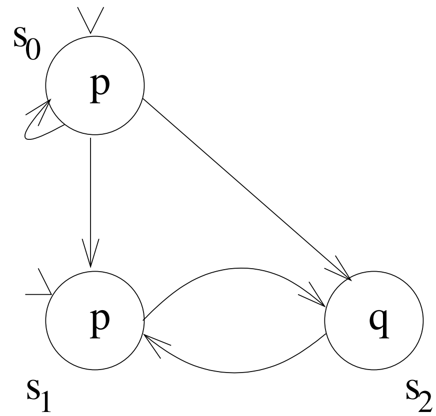
CS613:	**Homework 3**	Due in class Wednesday, Oct. 27, 2004

Reading: Please reach Chapter 6 of Huth and Ryan, plus the Emerson article on the mu calculus that I handed out.

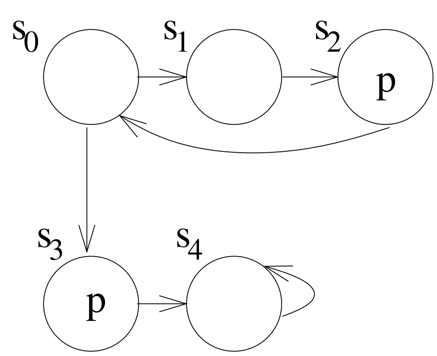
Problems:

1. By hand, please show how NuSMV would do explicit model checking on the Kripke structure, $\mathcal{K}_1$, shown on the right, with the LTL formula $\varphi \equiv \mathbf{G}[p\mathbf{U}q]$. Note that $\mathbf{G}[p\mathbf{U}q]$ is equivalent to $\neg[\top\mathbf{U}\neg[p\mathbf{U}q]]$. However, it will be easier to see what is going on if you use the formula $\mathbf{G}[p\mathbf{U}q]$. In this case, note that a "good" set, q, of formulas now has the additional self-consistency rule: If $\mathbf{G}\alpha \in q$ Then $\alpha \in q$. We also need the transition rules: If $\langle q, q' \rangle \in \delta$ and $\mathbf{G}\alpha \in q$ Then $\mathbf{G}\alpha \in q'$; and, if $\neg\mathbf{G}\alpha \in q$ and $\alpha \in q$ then $\neg\mathbf{G}\alpha \in q'$. Finally, we must add the acceptance condition that infinitely often we must enter a state where either $\mathbf{G}\alpha$ or $\neg\alpha$ is true. I would like to see the details here, i.e., tell me what the closure of φ is, draw the reachable part of the automaton $A_{\neg\varphi}$ as well as the new Kripke structure $\mathcal{K}_1 \times A_{\neg\varphi}$ on which NuSMV would do the fair model checking that would give us the answer. Please also try this in NuSMV and make sure that it gets the same answer that you did.



2. [Hint for using the mu-calculus: Shangzhu gave us a useful rule of thumb: pair $\vee$ with μ and $\wedge$ with ν.]

   (a) By hand, evaluate the mu-calculus formula $\nu Y \mu Z(\diamond((p \wedge Y) \vee Z))$ on the Kripke structure, $\mathcal{K}_2$, shown below. To do this, you would start by computing the fixed point for Z, when $Y^0 = S$. Call the steps $Z^0_0 = \emptyset$, Z^0_1, etc., and call this fixed point Z^0_∞. Then at the next round, you would set $Y^1 = Z^0_\infty$, and continue with $Z^1_0 = \emptyset$, Z^2_0, etc., until you reach the fixed point Y^∞.



   (b) Show that on all Kripke structures, $\mathbf{EGF}p \equiv \nu Y \mu Z(\diamond((p \wedge Y) \vee Z))$

(c) Using (b), the facts we showed in class, and the Emerson article, prove that every formula in Fair CTL can be expressed in $L\mu_2$, i.e. the mu calculus of alternation depth at most 2.

3. Prove that μ and ν are duals, i.e., show that $\nu Z(\varphi(Z)) \equiv \neg\mu Z(\neg\varphi(\neg Z))$.

4. Please email me a paragraph or more about what you might want to do your final project on. We need to discuss this and get settled on topics so that you can start doing reading, planning, etc. Ten of you need to present your projects in class. The last day of class is Dec. 13. I'd like to save two thirds of that class for summary and directions, etc. Thus, I'd like you to give your presentations say three per class time, i.e. 25 minute each on Dec. 6, Dec. 8, and Dec. 10. (Can everyone make an extra class from 9:05 to 10:20 on Friday Dec. 10?) Then one lucky person can go on the last day of class, Mon. Dec. 13. When you email me your paragraph, you can also tell me which of these four days you prefer to give your talk.