There are very few natural problems that are:

- Known to be in NP, and
- Not known to be NP-complete, and
- Not known to be in P

Examples:

- Factoring natural numbers
- Graph Isomorphism
- Model Checking the μ-Calculus

Theorem 11.1 (Ladner) *If* P $\neq$ NP *then there exists an intermediate problem* $I \in \text{NP} - \text{P}$ *that is not* NP *complete.*

Proof: Assume that P $\neq$ NP.

We will construct I by a method called "delayed diagonalization".

The construction will make sure that:

- I is **not hard**: $\quad \text{SAT} \not\leq I. \qquad R_1, R_3, R_5, \ldots$
- I is **not easy**: $\quad I \notin \text{P}. \qquad R_2, R_4, R_6, \ldots$

R_{2k+1} : "M_k isn't a DSPACE$[k \log n]$ reduction from SAT to I"

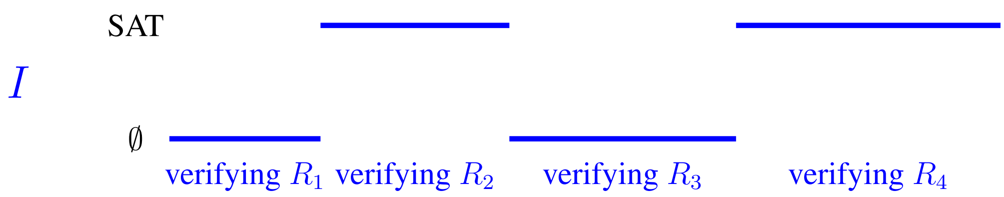
R_{2k+2} : "M_k isn't a DTIME$[kn^k]$ recognizer of I"

Observation: If all the R_i's are met, then we're done.

Conditions to Satisfy: $R_i, \quad i = 1, \ldots \infty$

R_{2k+1} : "M_k isn't a DSPACE$[k \log n]$ reduction from SAT to I"

R_{2k+2} : "M_k isn't a DTIME$[kn^k]$ recognizer of I"



On input w, recursively $I(w)$ does following:

1. **do** for a total of $|w|$ steps {
2.   **for** $i = 1 \ldots \infty$ **do** {
3.   **for** $x = 1 \ldots \infty$ **do** {
4.   **if** (R_i verified on input x) **then** next i
5. } } }
6. **if** (i is even and $w \in$ SAT) **then ACCEPT**
7. **else** REJECT

Note: In line 4, I simulates itself **deterministically**. Thus, to check if an input is in SAT it might need exponential time. Thus, it might only find out exponentially later that condition R_i has been met. That's why this method is called **delayed diagonalization**. The key idea, is that if i is even we are simulating SAT, so if we do this long enough we cannot be in P, whereas if i is odd then we are rejecting all inputs, so if we do this long enough we cannot be NP complete. $\square$