

1. Show that $\text{DSPACE}[n] \neq \text{NP}$. [Hint: use a Hierarchy Theorem.]
2. Let $\text{EMPTY-CFL} = \{G \mid G \text{ a context-free grammar and } \mathcal{L}(G) = \emptyset\}$. Prove that EMPTY-CFL is P-complete.

[Hint: to show that a given CFL, G , generates no strings, I suggest that you develop a linear-time marking algorithm which marks all nonterminals, N , as “useful” if there is a derivation from N to some $w \in \Sigma^*$. For example, if there is a rule $A \rightarrow a$, for $a \in \Sigma$, then you would mark A as useful. To show that EMPTY-CFL is P-hard, I suggest that you reduce $\overline{\text{MCVP}}$ to it.]

3. Let $A \subseteq \{0, 1\}^*$ be a random oracle in the sense that each string w is either in A or not in A with probability $1/2$, independent of all the other strings.

Prove that with this distribution of A 's, the probability that $\text{P}^A = \text{NP}^A$ is 0.

[Hint: this is somewhat similar to the construction we did in class of a B s.t. $\text{P}^B \neq \text{NP}^B$. Just consider unary languages, and now we'll look for a run of consecutive w 's that are in A . For a given length, the probability of the existence of such a run, should be $1/2$.]