Problems:

1. Let EMPTY-DFA $= \{D \mid D$ a deterministic finite automaton and $\mathcal{L}(D) = \emptyset\}$ and
   EMPTY-NFA $= \{N \mid N$ a nondeterministic finite automaton and $\mathcal{L}(N) = \emptyset\}$
   and
   EQIV-DFA $= \{(D, E) \mid D, E$ DFAS; $\mathcal{L}(D) = \mathcal{L}(E)\}$
   Show that EMPTY-DFA, EMPTY-NFA and EQIV-DFA are all NL complete.

2. A Horn formula is a CNF formula such that each clause has at most one positive literal. For example, the following clauses are Horn clauses: (x_1), $(\overline{x_1})$, $(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$, $(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee x_4)$; but the following are not: $(x_1 \vee x_2)$, $(\overline{x_1} \vee x_2 \vee x_3)$, $(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \vee x_4 \vee x_5)$

   Show that HORNSAT is P complete.

   [Hint: there is a linear-time algorithm for Horn-Sat. For a given Horn formula, φ, start with the truth assignment that makes all the variables false. If this doesn't satisfy φ, then there must be a unit, positive clause, (x_i). Assign x_i true, thus forever satisfying this clause which you may now delete. Continue.

   For the hardness of HornSAT, look at the analogy of Fagin's Theorem where the TIME$[n^k]$ TM is deterministic. What's different? Do something similar to the proof of Cook's Theorem from Fagin's Theorem.]

3. Show that the following problem is PSPACE-complete:

$$\Sigma^\star\text{NFA} \quad = \quad \{N \mid N \text{ is an NFA; } \mathcal{L}(N) = \Sigma^*\}$$