$LTL \subseteq CTL^*$  $CTL^*$ has both path formulas and state formulas.

**State Formulas:**

**base case:**  $(\mathcal{K}, s) \models p \iff p \in L(s)$

**inductive cases:** for any state formulas $\alpha, \beta$, $\neg \alpha$, $\alpha \land \beta$ are state formulas.

for any path formula, $\psi$, use Path Quantifiers: $A, E$ to construct the following state formulas:

$(\mathcal{K}, s) \models A \psi \iff$ for all $\pi$ such that $\pi_0 = s$, $(\mathcal{K}, \pi) \models \psi$

$(\mathcal{K}, s) \models E \psi \iff$ for some $\pi$ such that $\pi_0 = s$, $(\mathcal{K}, \pi) \models \psi$

**Path Formulas:**

**base case:** Every state formula is a path formula.

**inductive cases:** for any path formulas, $\psi, \varphi$, the following are path formulas:

$\neg \psi$, $\psi \land \varphi$, $F \psi$, $G \psi$, $X \psi$, $(\psi U \varphi)$

For example, for the below graph representing a Kripke structure $\mathcal{K}$ we have $(\mathcal{K}, 2) \models AF q$ and $(\mathcal{K}, 2) \models AGF q$.

**Emerson & Clarke:** $CTL$ has an efficient model checking algorithm.

In $CTL$, pair path quantifiers $(A, E)$ with temporal operators $(G, F, X, U)$. i.e. we only have state formulas. (And so $CTL \subseteq CTL^*$)

**Some examples:**

$(\mathcal{K}, s) \models EF p \iff$ there is some path from $s$ to a state which satisfies $p$.

$(\mathcal{K}, s) \models EG p \iff$ there is some path from $s$ along which $p$ always holds.

$AG(p \rightarrow EX q)$

$AG(Gr \rightarrow Fc) =$ weak fairness (expressible in $CTL$), “Always trying implies eventually succeeding.”

$A(GFr \rightarrow GFc) =$ strong fairness (not expressible in $CTL$, expressible in $CTL^*$), “Infinitely often trying implies infinitely often succeeding.”

**Theorem (Emerson & Clarke):** “Linear Time” CTL Model Checking.

There is an algorithm with input $\mathcal{K}, \varphi \in CTL$ and output $\{s \in S^\mathcal{K}|(\mathcal{K}, s) \models \varphi\}$ with running time $O(|\mathcal{K}||\varphi|)$.

(The running time for $LTL$ is $O(|\mathcal{K}|2^{|\varphi|}).$)