| CS513: | **Homework 5** | Due at gradescope by midnight, 12/7/17 |
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**Note:** I will hand out solutions the morning of Dec. 11, so I cannot accept late hw5's after that.

## Problems

1. Given that SAT is NPC, show that CNF-SAT is also NPC. Hint: we already know that CNF-SAT is in NP. Show that SAT $\leq$ CNF-SAT by giving an easy-to-compute polynomial-time reduction, $f$ such that for all $\varphi \in \mathbf{P_{fmla}}$, $(\varphi \in \text{SAT}) \Leftrightarrow (f(\varphi) \in \text{CNF-SAT})$.

   [Hint: first transform $\varphi$ to $\varphi'$ in negation normal form, i.e. using only logical connectives $\wedge, \vee, \neg$, with the $\neg$'s pushed all the way inside.]

2. We have seen that FO-VALID is r.e. complete and FO-SAT is co-r.e. complete. Let FINITE-FO-VALID be the set of FO formulas that are true in all appropriate finite structures and FINITE-FO-SAT be the set of FO formulas that are true in some finite structure. Show that FINITE-FO-SAT is r.e. complete, and thus FINITE-FO-VALID is co-r.e. complete.

   [Hint: start with the fact that Post's Correspondence Problem (PCP) is equivalent to the Halting problem, i.e., it is r.e. complete.]

3. Show that the operators we used in the proof of the $O(|\mathcal{M}| \cdot |\varphi|)$ time CTL model checking algorithm, i.e., $\neg, \wedge$, **EX**, **EU**, and, **EG**, suffice to express all of CTL.

4. Recall that we showed in class that $\mathbf{EF}p \equiv \mu Z.(p \vee \mathbf{EX}Z)$. Show the following about the $\mu$ calculus:

   (a) $\mathbf{AF}p \;\equiv\; \mu Z.(p \vee \mathbf{AX}Z)$

   (b) $\mathbf{E}(p\mathbf{U}q) \;\equiv\; \mu Z.(q \vee (p \wedge \mathbf{EX}Z))$

   (c) Prove that $\mu$ and $\nu$ are duals, i.e., $\quad \neg\mu Y\,.\,\varphi(Y) \quad\equiv\quad \nu Z\,.\,\neg\varphi(\neg Z)$

   [Hint: you might want to use the Tarski Knaster Theorem which says that when $\varphi$ is monotone, $\mu Y\,.\,\varphi(Y) = \varphi^{\infty}(\bot)$, and, $\nu Y\,.\,\varphi(Y) = \varphi^{\infty}(\top)$. Here $\varphi^{\infty}(\bot)$ means $\varphi^{t}(\bot)$ with $t$ minimal such that $\varphi^{t}(\bot) = \varphi^{t+1}(\bot)$.]