Reading Please read Sections 2.1 through 2.3 by the time this assignment is due. I suggest that you only read one section at a sitting and that you think about all of the exercises – assigned or not – to help make sure that you are understanding all the concepts as you read.

Problems

1. Do the following two resolution exercises:

   (a) exercise 33, p. 36.
   (b) exercise 34, p. 36.

2. This problem is meant to reinforce the idea of the inductive step in the proof of the completeness of resolution for PropCalc which we did in class and which is also in the text. Consider the following CNF formula represented as a set of clauses:

$$\varphi = \{\{-3, 1\}, \{-1, 2\}, \{-2, -3\}, \{3, -1\}, \{1, -2\}, \{2, 3\}\}$$

   (a) Compute $\varphi_0 = \varphi[x_3/\bot]$ and $\varphi_1 = \varphi[x_3/\top]$
   (b) Give a resolution derivation of $\varphi_0 \vdash \square$.
   (c) Give a resolution derivation of $\varphi_1 \vdash \square$.
   (d) Now show using the above two that $\varphi \vdash \{3\}$ and $\varphi \vdash \{-3\}$.

   Thus, you get a proof of $\varphi \vdash \square$.

3. Exercise 42, p. 43: Let free(φ) be the set of all variables that occur freely in φ. Define free(φ) formally by induction on terms and then on formulas.

4. Exercise 45, p. 49: Consider the following sentences ρ, σ, τ which express that the predicate E is reflexive, symmetric, and transitive:

$$\begin{aligned}
\rho &= \forall x E(x, x) \\
\sigma &= \forall xy(E(x, y) \to E(y, x)) \\
\tau &= \forall xyz(E(x, y) \land E(y, z) \to E(x, z))
\end{aligned}$$

   Show that these sentences are independent by constructing three structures that satisfy each possible pair of the sentences but not the third.

   [Note that $\land$ and $\lor$ bind more tightly than $\to$, so I didn't use extra parentheses in the definition of τ.]

5. Exercise 55, p. 52: show that the following pairs of formulas are not equivalent by constructing structures that satisfy one of them but not the other:

(a) $\alpha \overset{\text{def}}{=} (\forall x A(x) \vee \forall x R(x))$ $\quad \beta \overset{\text{def}}{=} \forall x (A(x) \vee R(x))$

(b) $\gamma \overset{\text{def}}{=} (\exists x A(x) \wedge \exists x R(x))$ $\quad \delta \overset{\text{def}}{=} \exists x (A(x) \wedge R(x))$

(c) For some extra credit: is one of the following formulas valid? $(\alpha \rightarrow \beta)$, $(\beta \rightarrow \alpha)$? If so, say which one and argue why it is valid.