

Natural Deduction Rules for PredCalc

| | introduction | elimination | proviso |
|---------------|--|---|---|
| \wedge | $\frac{\alpha \quad \beta}{\alpha \wedge \beta}$ | $\frac{\alpha \wedge \beta}{\alpha} \quad \frac{\alpha \wedge \beta}{\beta}$ | |
| \vee | $\frac{\alpha}{\alpha \vee \beta} \quad \frac{\beta}{\alpha \vee \beta}$ | $\frac{\alpha \vee \beta \quad \alpha \vdash \psi \quad \beta \vdash \psi}{\psi}$ | |
| \rightarrow | $\frac{\alpha \vdash \beta}{\alpha \rightarrow \beta}$ | $\frac{\alpha \rightarrow \beta \quad \alpha}{\beta} \quad \frac{\alpha \rightarrow \beta \quad \sim \beta}{\sim \alpha}$ | |
| F | $\frac{\alpha \quad \sim \alpha}{\mathbf{F}}$ | $\frac{\alpha \vdash \mathbf{F}}{\sim \alpha} \quad \frac{\sim \alpha \vdash \mathbf{F}}{\alpha}$ | |
| $\sim \sim$ | $\frac{\alpha}{\sim \sim \alpha}$ | $\frac{\sim \sim \alpha}{\alpha}$ | |
| $=$ | $\overline{t = t}$ | $\frac{t_1 = t_2 \quad \varphi[t_1/x]}{\varphi[t_2/x]}$ | |
| \forall | $\frac{\Gamma \vdash \varphi[x_0/x]}{\Gamma \vdash \forall x \varphi}$ | $\frac{\forall x \varphi}{\varphi[t/x]}$ | x_0 does not occur in $\Gamma \cup \{\varphi\}$ |
| \exists | $\frac{\varphi[t/x]}{\exists x \varphi}$ | $\frac{\Gamma \vdash \exists x \varphi \quad \Gamma, \varphi[x_0/x] \vdash \psi}{\Gamma \vdash \psi}$ | x_0 does not occur in $\Gamma \cup \{\varphi, \psi\}$ |