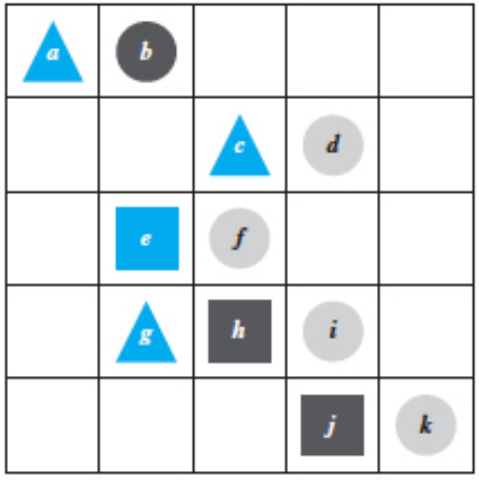
CS250: PredCalc Vocabularies

$T_1 =$



$$
\begin{aligned}
\text{Circle}^{T_1} &= \{b, d, f, i, k\} \\
\text{Square}^{T_1} &= \{e, h, j\} \\
\text{Triangle}^{T_1} &= \{a, c, g\} \\
\text{Blue}^{T_1} &= \{a, c, e, g\} \\
\text{Black}^{T_1} &= \{b, h, j\} \\
\text{Gray}^{T_1} &= \{d, f, i, k\} \\
\text{Above}^{T_1} &= \{(a, c), \ldots, (a, k), \ldots, (i, k)\} \\
\text{LeftOf}^{T_1} &= \{(a, b), \ldots, (a, k), \ldots, (j, k)\}
\end{aligned}
$$

Tarski's World vocabulary: $\Sigma_{\text{Tarski}} =$

$(\text{Circle}^1, \text{Square}^1, \text{Triangle}^1, \text{Blue}^1, \text{Black}^1, \text{Gray}^1, \text{Above}^2, \text{LeftOf}^2;)$

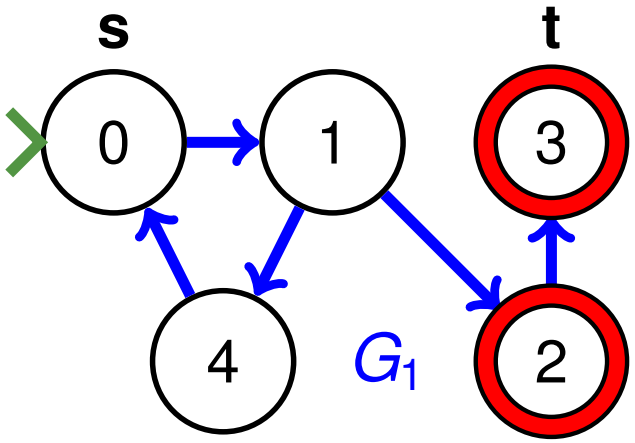
Σ_{Tarski} has six **predicate symbols** of **arity** 1, and two of arity 2.

A **world**, $T_1 \in \text{World}[\Sigma_{\text{Tarski}}]$ has:

A **non-empty universe**, $|T_1| = \{a, b, c, d, e, f, g, h, i, j, k\}$

and a predicate of the correct arity over its universe for each predicate symbol in Σ_{Tarski}.

vocabulary of graphs $\Sigma_{\text{garst}} = (A^1, R^1, E^2; s, t)$



$$
\begin{aligned}
|G_1| &= \{0,1,2,3,4\} \\
A^{G_1} &= \{0\} \\
R^{G_1} &= \{2,3\} \\
E^{G_1} &= \{(0,1),(1,2),(1,4),(2,3),(4,0)\} \\
s^{G_1} &= 0 \\
t^{G_1} &= 3
\end{aligned}
$$

vocabulary of sets $\Sigma_{\text{set}} = (\in^2[\text{infix}]; \emptyset)$

Examples of $\text{PredCalc}(\Sigma_{\text{set}})$

$$\sigma_{\emptyset} \stackrel{\text{def}}{=} \forall x\,(\sim x \in \emptyset)$$

$$\sigma_{\text{extensionality}} \stackrel{\text{def}}{=} \forall xy\,((\forall z\,(z \in x \leftrightarrow z \in y)) \to x = y)$$

$$x \subseteq y \hookrightarrow \forall z\,(z \in x \to z \in y)$$

Vocabulary of Number Theory

$$\Sigma_{\#\text{thy}} \stackrel{\text{def}}{=} (\leq^2 [\text{infix}]; 0, 1, +^2[\text{infix}], \cdot^2[\text{infix}])$$

$\mathbf{N},\ \mathbf{Z},\ \mathbf{Q},\ \mathbf{R} \in \text{World}[\Sigma_{\#\text{thy}}]$

Examples of $\text{PredCalc}(\Sigma_{\#\text{thy}})$

$$\text{“1 is id for } \cdot\text{”} \stackrel{\text{def}}{=} \forall x\ x \cdot 1 = x$$

$$x < y \hookrightarrow x \leq y \wedge x \neq y$$

$$x|y \hookrightarrow \exists z\ (x \cdot z = y)$$

$$\text{prime}(x) \hookrightarrow 1 < x \wedge \forall y\ (1 < y \wedge y|x \rightarrow y = x)$$

$\mathbf{N}, \mathbf{Z}, \mathbf{Q}, \mathbf{R} \models$ “1 is id for $\cdot$”