CS250: Discrete Math for Computer Science

L9: Tarski's Definition of Truth

Free and Bound Variables

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A formula says something about its **free** variables.

Terms in PredCalc

Def: Let Σ be a PredCalc vocabulary. A **term** $t \in \mathbf{term}(\Sigma)$ is a string of symbols that every world $W \in \mathrm{World}[\Sigma]$ must interpret as an element $t^W \in |W|$. Terms are defined recursively as follows:

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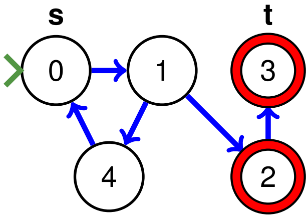
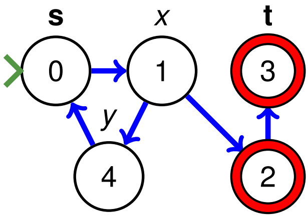
$$\begin{aligned}
\textbf{term}(\Sigma_{\text{Tarski}}) &= \mathrm{VAR} &&= \{x, y, z, u, v, w, x_1, \ldots\} \\
\textbf{term}(\Sigma_{\text{garst}}) &= \mathrm{VAR} \cup \{s, t\} &&= \{s, t, x, y, z, u, v, w, x_1, \ldots\}
\end{aligned}$$

term(Σ_{garst})



G_1



$G_1[1/x\ 4/y]$

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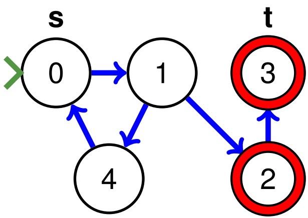

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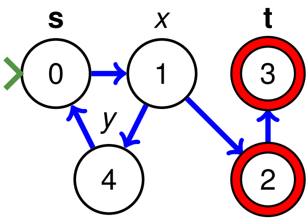

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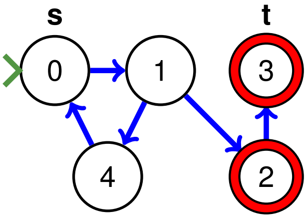

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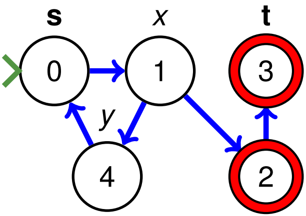

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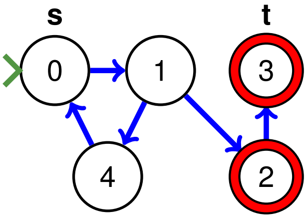

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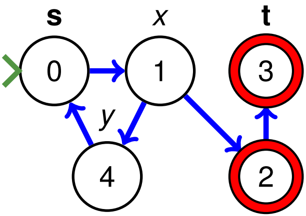

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$x^{G_1} = 0 \quad y^{G_1} = 0$

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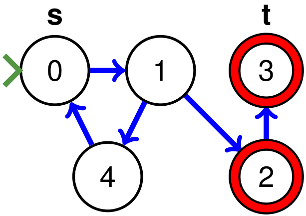

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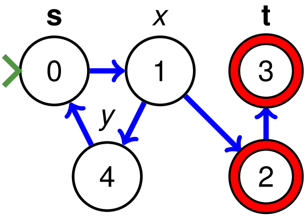

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$x^{G_1} = 0 \quad y^{G_1} = 0 \quad x^{G_1[1/x\ 4/y]} = 1$

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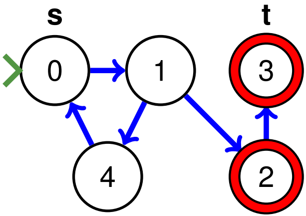

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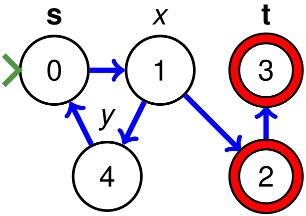

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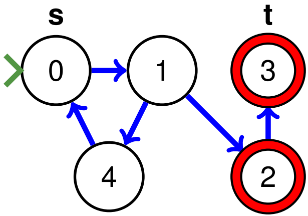

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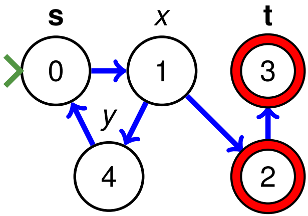

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$s^{G_1} = 0 \quad t^{G_1} = 3 \quad s^{G_1[1/x\ 4/y]} = 0 \quad t^{G_1[1/x\ 4/y]} = 3$

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$$G \models P(t_1, \ldots, t_a) \quad \text{iff} \quad (t_1^G, \ldots, t_a^G) \in P^G \qquad P^a \in \Sigma$$

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G	$\models$	$P(t_1, \ldots, t_a)$	iff	$(t_1^G, \ldots, t_a^G) \in P^G$	$P^a \in \Sigma$
G	$\models$	$\sim\alpha$	iff	$G \not\models \alpha$	PropCalc

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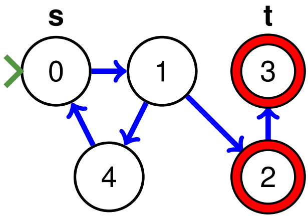
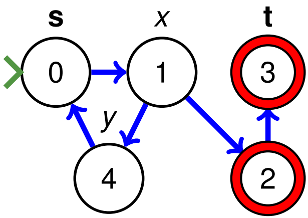
G	$\models$	$\forall x(\alpha)$	iff	**for all** $a \in \|G\|$ $G[a/x] \models \alpha$	
G	$\models$	$\exists x(\alpha)$	iff	**exists** $a \in \|G\|$ $G[a/x] \models \alpha$	

Examples using Tarski's Def. of Truth



G_1



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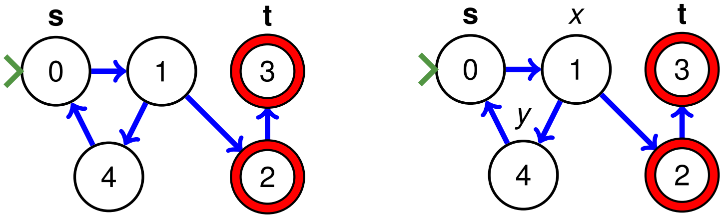

G_1 $\qquad$ $G_1[1/x\ 4/y]$

T $\quad G_1 \models s = x \qquad s^{G_1} = 0 = x^{G_1}$

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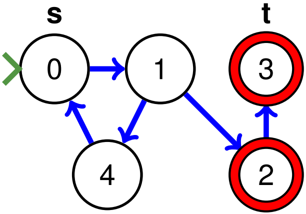

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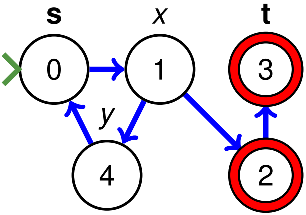

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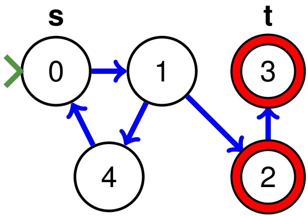
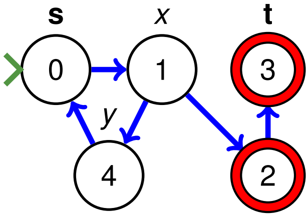
T/**F** $\quad G_1[1/x\ 4/y] \models s = x$

Examples using Tarski's Def. of Truth



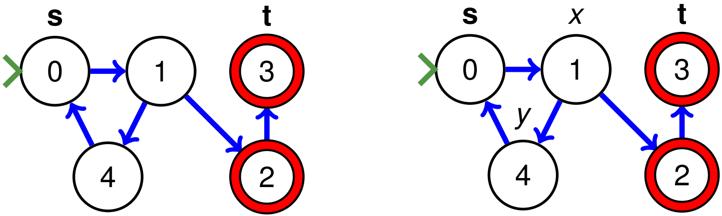
G_1



$G_1[1/x\ 4/y]$

T	$G_1 \models s = x$	$s^{G_1} = 0 = x^{G_1}$
F	$G_1[1/x\ 4/y] \models s = x$	$s^{G_1[1/x\ 4/y]} = 0 \neq 1 = x^{G_1[1/x\ 4/y]}$

Examples using Tarski's Def. of Truth



G_1 $\qquad\qquad$ $G_1[1/x\ 4/y]$

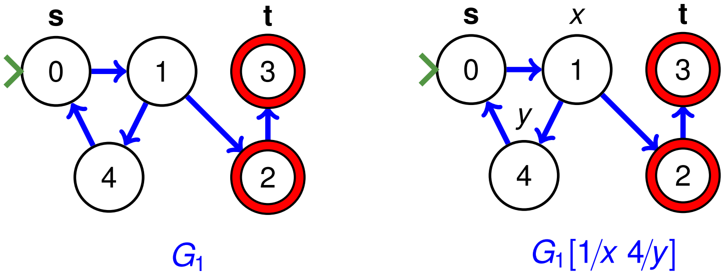
T $\quad G_1 \models s = x \qquad s^{G_1} = 0 = x^{G_1}$

F $\quad G_1[1/x\ 4/y] \models s = x \qquad s^{G_1[1/x\ 4/y]} = 0 \neq 1 = x^{G_1[1/x\ 4/y]}$

iClicker 9.1 T/F: $G_1 \models E(s, x)$

A: True $\qquad$ **B: False**

Examples using Tarski's Def. of Truth



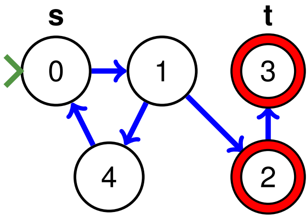
T $\quad G_1 \models s = x \qquad s^{G_1} = 0 = x^{G_1}$

F $\quad G_1[1/x\ 4/y] \models s = x \qquad s^{G_1[1/x\ 4/y]} = 0 \neq 1 = x^{G_1[1/x\ 4/y]}$

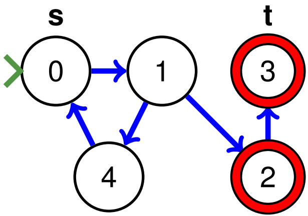
iClicker 9.2 T/F: $G_1[1/x\ 4/y] \models E(s, x)$

A: True **B: False**

Examples using Tarski's Def. of Truth



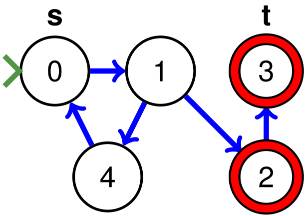
G_1



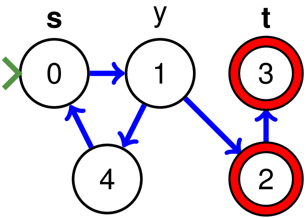
G_1

T/**F** $\quad G_1 \models \exists y\, E(x, y)$

Examples using Tarski's Def. of Truth



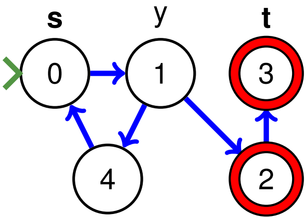
G_1



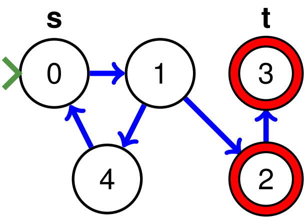
$G_1[1/y]$

T $\quad G_1 \models \exists y\, E(x,y) \quad G_1[1/y] \models E(x,y)$

Examples using Tarski's Def. of Truth



$G_1[1/y]$



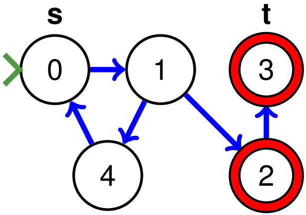
G_1

T $\quad G_1 \models \exists y\, E(x, y) \quad G_1[1/y] \models E(x, y)$

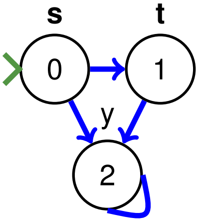
iClicker 9.3 T/F: $G_1[1/y] \models \exists x\, E(x, y)$

A: True **B: False**

Examples using Tarski's Def. of Truth



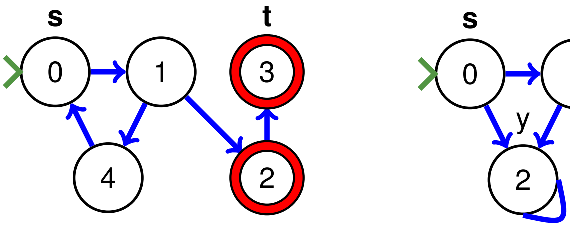
G_1



H_1

T/**F** $\quad G_1 \models (\forall x\, E(x, y)) \rightarrow y = t$

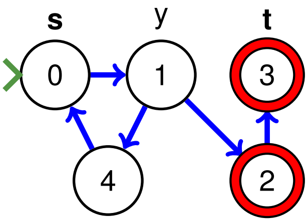
Examples using Tarski's Def. of Truth



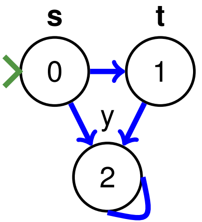
G_1 H_1

T $G_1 \models (\forall x\, E(x, y)) \rightarrow y = t \quad G_1 \models \sim\forall x\, E(x, y)$

Examples using Tarski's Def. of Truth



$G_1[1/y]$

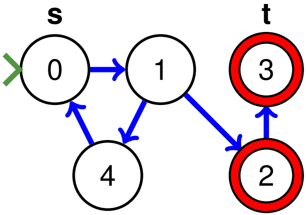


H_1

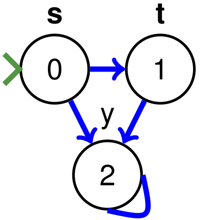
T $\quad G_1 \models (\forall x\, E(x, y)) \rightarrow y = t \quad G_1 \models \sim\forall x\, E(x, y)$

T/**F** $\quad H_1 \models (\forall x\, E(x, y)) \rightarrow y = t$

Examples using Tarski's Def. of Truth



G_1

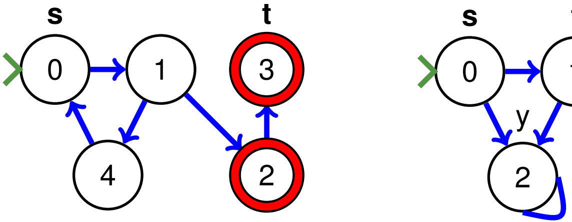


H_1

T $\quad G_1 \models (\forall x\, E(x,y)) \rightarrow y = t \quad G_1 \models \sim\forall x\, E(x,y)$

F $\quad H_1 \models (\forall x\, E(x,y)) \rightarrow y = t \quad H_1 \models (\forall x\, E(x,y)) \wedge y \neq t$

Examples using Tarski's Def. of Truth



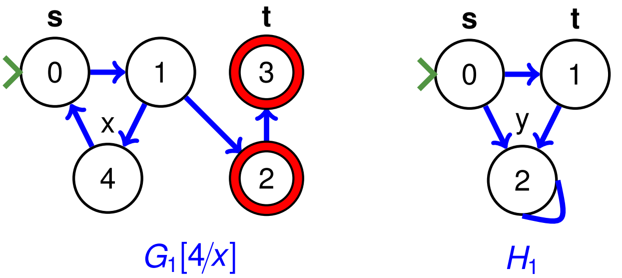
G_1 H_1

T $G_1 \models (\forall x\, E(x,y)) \rightarrow y = t$ $G_1 \models \sim \forall x\, E(x,y)$

F $H_1 \models (\forall x\, E(x,y)) \rightarrow y = t$ $H_1 \models (\forall x\, E(x,y)) \wedge y \neq t$

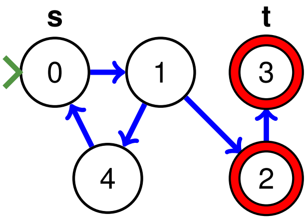
T/**F** $G_1 \models \forall x\, (E(x,y) \rightarrow y = t)$

Examples using Tarski's Def. of Truth

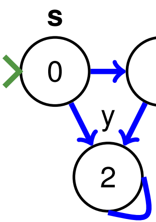


T	$G_1 \models (\forall x\, E(x,y)) \to y = t$	$G_1 \models \sim \forall x\, E(x,y)$
F	$H_1 \models (\forall x\, E(x,y)) \to y = t$	$H_1 \models (\forall x\, E(x,y)) \wedge y \neq t$
F	$G_1 \models \forall x\, (E(x,y) \to y = t)$	$G_1[4/x] \models \sim (E(x,y) \to y = t)$

Examples using Tarski's Def. of Truth



G_1



H_1

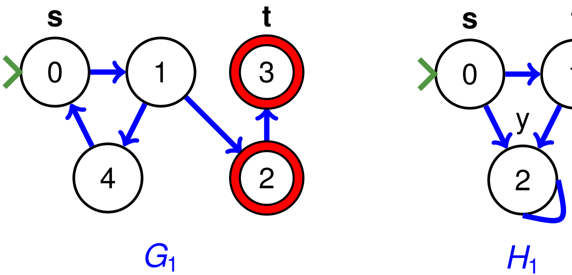
T $\quad G_1 \models (\forall x\, E(x,y)) \to y = t \qquad G_1 \models \sim \forall x\, E(x,y)$

F $\quad H_1 \models (\forall x\, E(x,y)) \to y = t \qquad H_1 \models (\forall x\, E(x,y)) \wedge y \neq t$

F $\quad G_1 \models \forall x\, (E(x,y) \to y = t) \qquad G_1[4/x] \models \sim (E(x,y) \to y = t)$

T/**F** $\quad H_1 \models \forall x\, (E(x,y) \to y = t)$

Examples using Tarski's Def. of Truth



T	$G_1 \models (\forall x\, E(x,y)) \rightarrow y = t$	$G_1 \models \sim \forall x\, E(x,y)$
F	$H_1 \models (\forall x\, E(x,y)) \rightarrow y = t$	$H_1 \models (\forall x\, E(x,y)) \wedge y \neq t$
F	$G_1 \models \forall x\,(E(x,y) \rightarrow y = t)$	$G_1[4/x] \models \sim (E(x,y) \rightarrow y = t)$
F	$H_1 \models \forall x\,(E(x,y) \rightarrow y = t)$	$H_1[1/x] \models \sim (E(x,y) \rightarrow y = t)$