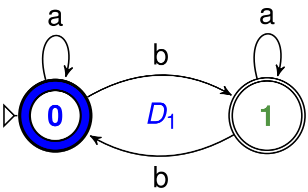
CS250: Discrete Math for Computer Science

L30: Intro to Finite Automata and Regular Expressions

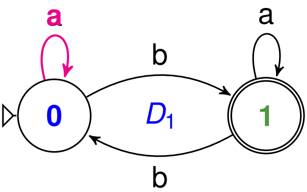
Deterministic Finite Automaton (DFA)



input string

state **0**

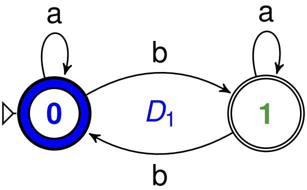
Deterministic Finite Automaton (DFA)



input string **a**

state **0**

Deterministic Finite Automaton (DFA)



input string **a**

state **0 0**

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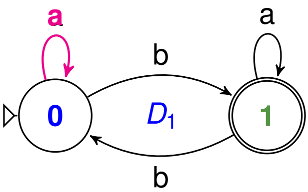

input string	**a**	**a**
state	**0**	**0**

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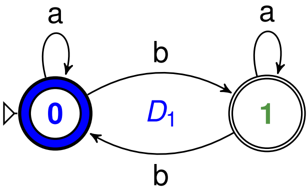
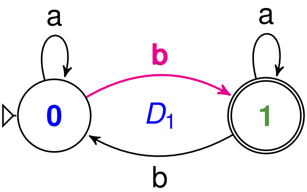


input string		**a**	**a**
state	**0**	**0**	**0**

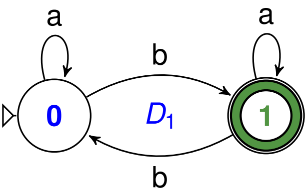
Deterministic Finite Automaton (DFA)



input string **a a b**

state **0 0 0**

Deterministic Finite Automaton (DFA)



input string	**a**	**a**	**b**	
state	**0**	**0**	**0**	**1**

$\mathcal{L}(D_1) = \{$**aab**,

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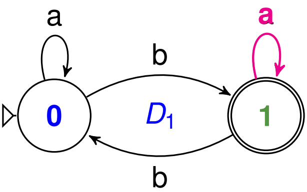

input string	**a**	**a**	**b**	**a**
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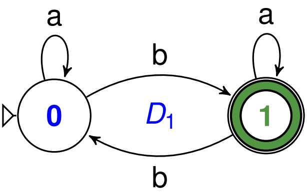

input string	**a**	**a**	**b**	**a**	
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$\mathcal{L}(D_1) \quad = \quad \{$**aab**, **aaba**,

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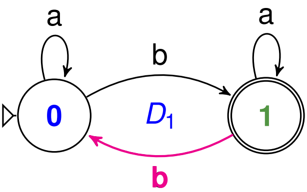

input string **a a b a b**

state **0 0 0 1 1**

$$\mathcal{L}(D_1) \quad = \quad \{\textbf{aab}, \textbf{aaba},$$

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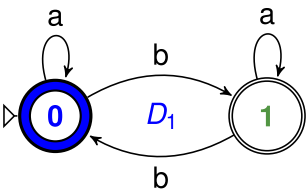

input string	**a**	**a**	**b**	**a**	**b**	
state	**0**	**0**	**0**	**1**	**1**	**0**

$\mathcal{L}(D_1) \quad = \quad \{$**aab**, **aaba**,

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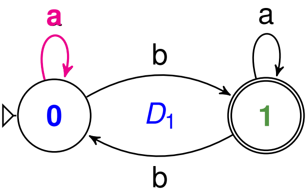

input string	**a**	**a**	**b**	**a**	**b**	**a**
state	**0**	**0**	**0**	**1**	**1**	**0**

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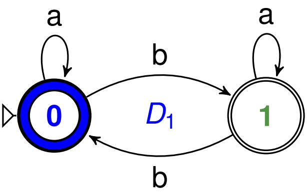

input string		**a**	**a**	**b**	**a**	**b**	**a**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**

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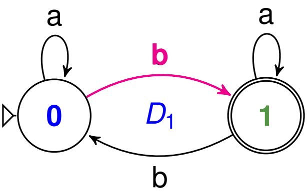

input string	**a**	**a**	**b**	**a**	**b**	**a**	**b**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**

$\mathcal{L}(D_1) \quad = \quad \{$**aab**, **aaba**,

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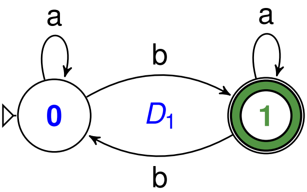

input string		**a**	**a**	**b**	**a**	**b**	**a**	**b**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**

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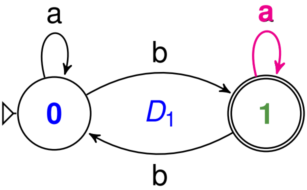

input string	**a**	**a**	**b**	**a**	**b**	**a**	**b**	**a**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**

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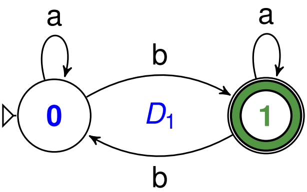

input string		**a**	**a**	**b**	**a**	**b**	**a**	**b**	**a**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**

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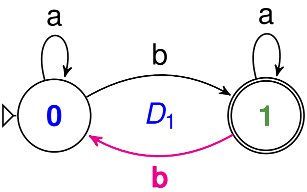

input string a a b a b a b a b

state 0 0 0 1 1 0 0 1 1

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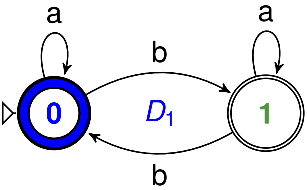

input string		**a**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**

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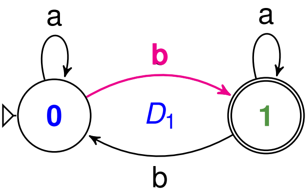

input string	**a a b a b a b a b b**
state	**0 0 0 1 1 0 0 1 1 0**

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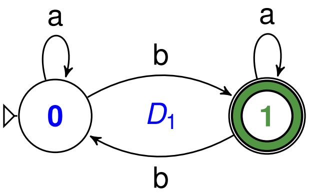

input string	**a a b a b a b a b b**
state	**0 0 0 1 1 0 0 1 1 0 1**

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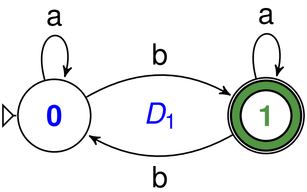

input string	**a a b a b a b a b b**
state	**0 0 0 1 1 0 0 1 1 0 1**

$\mathcal{L}(D_1) = \{$**aab**, **aaba**, **aababab**, **aabababa**, **aabababab**, $\ldots\}$

$$\mathcal{L}(D_1) = \left\{ w \in \{a, b\}^\star \;\middle|\; \#_b(w) \equiv 1 \pmod 2 \right\}$$

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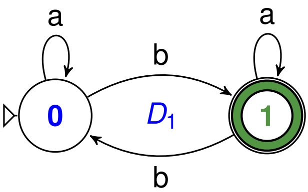

input string	**a a b a b a b a b b**
state	**0 0 0 1 1 0 0 1 1 0 1**

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D_1 **ignores a's** and **counts b's mod 2**

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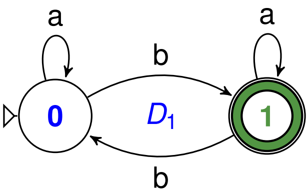

input string		**a**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**b**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**1**

$\mathcal{L}(D_1) = \{$**aab**, **aaba**, **aababab**, **aabababa**, **aabababab b**$, \ldots\}$

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D_1 **ignores a's** and **counts b's mod 2**

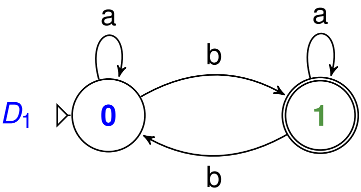
D_1 has **2 states** aka **1 bit of memory**

Deterministic Finite Automaton DFA

Q	finite set of **states**
Σ	finite **alphabet**
$\delta : Q \times \Sigma \to Q$	**transition function**
$s \in Q$	**start state**
$F \subseteq Q$	**final** (accept) **states**

$$D = (Q, \Sigma, \delta, s, F)$$

$$D_1 = (\{\mathbf{0}, \mathbf{1}\}, \{a, b\}, \delta_1, \mathbf{0}, \{\mathbf{1}\})$$



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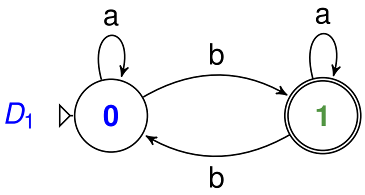

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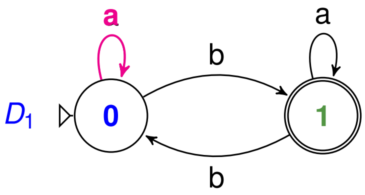
$\delta_1 : \{\mathbf{0}, \mathbf{1}\} \times \{a, b\} \to \{\mathbf{0}, \mathbf{1}\}$

$\delta_1(\mathbf{0}, a) = \mathbf{0}$



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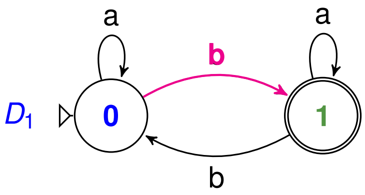

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$\delta_1(\mathbf{0}, a) = \mathbf{0} \quad \delta_1(\mathbf{0}, b) = \mathbf{1}$

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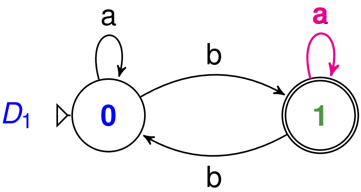

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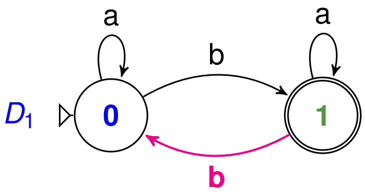

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Kleene Star $\star$

alphabets are finite sets of **letters**

$$\Sigma_{\text{bin}} = \{0, 1\}, \quad \Sigma_a = \{a\}, \quad \Sigma_{ab} = \{a, b\}, \quad \Sigma_{abc} = \{a, b, c\}$$

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Use a, b, c, d, e, f, g, h to denote letters

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a **string** is a finite sequence of letters

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$\Sigma^\star$ is the set of strings from alphabet Σ

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$$\Sigma^\star_{\mathrm{bin}} \quad = \quad \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$$

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$$\Sigma^\star_a = \{\epsilon, a, aa, aaa, aaaa, \ldots\}$$

Kleene Star $\star$

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$$\Sigma^\star_a = \{\epsilon, a, aa, aaa, aaaa, \ldots\}$$

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Kleene Star $\star$

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$$\Sigma_a^\star = \{\epsilon, a, aa, aaa, aaaa, \ldots\}$$

$$\Sigma_{ab}^\star = \{\epsilon, a, b, aa, ab, ba, bb, aaa \ldots\}$$

Use u, v, w, x, y, z to denote strings

Kleene Star *

concatenation is the multiplication operation for strings.

Kleene Star $\star$

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$$w \cdot x \quad = \quad wx \quad = \quad wx$$

Kleene Star $\star$

concatenation is the multiplication operation for strings.

$$\begin{aligned} w \cdot x &= wx &= wx \\ aa \cdot bc &= (aa)(bc) &= aabc \end{aligned}$$

Kleene Star $\star$

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$$
\begin{aligned}
w \cdot x &= wx &= wx \\
aa \cdot bc &= (aa)(bc) &= aabc \\
\epsilon \cdot w &= w \cdot \epsilon &= w
\end{aligned}
$$

Kleene Star $\star$

concatenation is the multiplication operation for strings.

$$\begin{aligned} w \cdot x &= wx &= wx \\ aa \cdot bc &= (aa)(bc) &= aabc \\ \epsilon \cdot w &= w \cdot \epsilon &= w \end{aligned}$$

concatenation also defined on **sets of strings**

Kleene Star $\star$

concatenation is the multiplication operation for strings.

$$
\begin{aligned}
w \cdot x &= wx &= wx \\
aa \cdot bc &= (aa)(bc) &= aabc \\
\epsilon \cdot w &= w \cdot \epsilon &= w
\end{aligned}
$$

concatenation also defined on **sets of strings**

$$S \cdot T \stackrel{\text{def}}{=} \{x \cdot y \mid x \in S, y \in T\}$$

Kleene Star *

concatenation is the multiplication operation for strings.

$$
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S \cdot T &\stackrel{\text{def}}{=} \{x \cdot y \mid x \in S, y \in T\} \\
\Sigma_{\text{bin}} \cdot \Sigma_{abc} &= \{0a, 0b, 0c, 1a, 1b, 1c\}
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Kleene Star $\star$

concatenation is the multiplication operation for strings.

$$
\begin{aligned}
w \cdot x &= wx &&= wx \\
aa \cdot bc &= (aa)(bc) &&= aabc \\
\epsilon \cdot w &= w \cdot \epsilon &&= w
\end{aligned}
$$

concatenation also defined on **sets of strings**

$$
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S \cdot T &\stackrel{\text{def}}{=} \{x \cdot y \mid x \in S, y \in T\} \\
\Sigma_{\text{bin}} \cdot \Sigma_{abc} &= \{0a, 0b, 0c, 1a, 1b, 1c\} \\
S^2 &\stackrel{\text{def}}{=} S \cdot S \\
S^n &\stackrel{\text{def}}{=} \underbrace{S \cdot \cdots \cdot S}_{n}
\end{aligned}
$$

Kleene Star $\star$

concatenation is the multiplication operation for strings.

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concatenation also defined on **sets of strings**

$$\begin{aligned} S \cdot T &\stackrel{\text{def}}{=} \{x \cdot y \mid x \in S, y \in T\} \\ \Sigma_{\text{bin}} \cdot \Sigma_{abc} &= \{0a, 0b, 0c, 1a, 1b, 1c\} \\ S^2 &\stackrel{\text{def}}{=} S \cdot S \\ S^n &\stackrel{\text{def}}{=} \underbrace{S \cdot \dots \cdot S}_{n} \\ S^0 &\stackrel{\text{def}}{=} \{\epsilon\} \end{aligned}$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

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$$S^\star \quad \stackrel{\text{def}}{=} \quad \bigcup_{i=0}^{\infty} S^i \quad = \quad S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S.

Kleene Star $\star$

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$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

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$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$|aba| \quad = \quad 3$$

Kleene Star $\star$

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$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$|aba| = 3$$
$$|ab| = 2$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$|aba| = 3$$

$$|ab| = 2$$

$$|a| = 1$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$|aba| = 3$$

$$|ab| = 2$$

$$|a| = 1$$

$$|\epsilon| = 0$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$\begin{aligned} |aba| &= 3 \\ |ab| &= 2 \\ |a| &= 1 \\ |\epsilon| &= 0 \\ w^n &= \underbrace{w \cdot w \cdots w}_{n} \end{aligned}$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$\begin{aligned} |aba| &= 3 \\ |ab| &= 2 \\ |a| &= 1 \\ |\epsilon| &= 0 \\ w^n &= \underbrace{w \cdot w \cdots w}_{n} \\ 0^5 &= 00000 \end{aligned}$$

Kleene Star $\star$

$$S^\star \stackrel{\text{def}}{=} \bigcup_{i=0}^{\infty} S^i = S^0 \cup S^1 \cup S^2 \cup \cdots$$

$S^\star$ is the **set of all strings** from S. $|w| \stackrel{\text{def}}{=}$ **length** of string w

$$\begin{aligned}
|aba| &= 3 \\
|ab| &= 2 \\
|a| &= 1 \\
|\epsilon| &= 0 \\
w^n &= \underbrace{w \cdot w \cdots w}_{n} \\
0^5 &= 00000 \\
(ab)^3 &= ababab
\end{aligned}$$

Kleene Star $\star$

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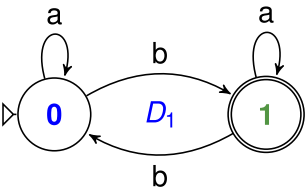
|aba| &= 3 \\
|ab| &= 2 \\
|a| &= 1 \\
|\epsilon| &= 0 \\
w^n &= \underbrace{w \cdot w \cdots w}_{n} \\
0^5 &= 00000 \\
(ab)^3 &= ababab \\
a^0 &= \epsilon
\end{aligned}$$

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \rightarrow Q$

extend δ to: $\delta^\star : Q \times \Sigma^\star \rightarrow Q$

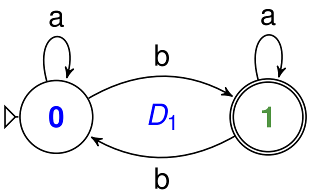


$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \to Q$

extend δ to: $\delta^\star : Q \times \Sigma^\star \to Q$

base: $\delta^\star(q, \epsilon) = q$



$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

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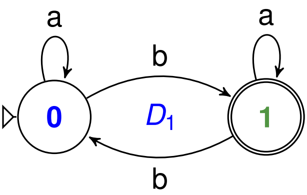
extend δ to: $\delta^\star : Q \times \Sigma^\star \rightarrow Q$

base: $\delta^\star(q, \epsilon) = q$

inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$



$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

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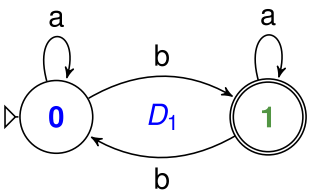
inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$

start at q, **read** w, thus at $\delta^\star(q, w)$



$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

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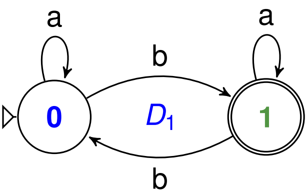
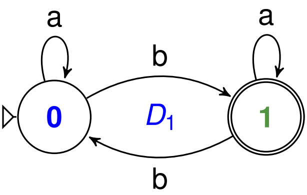


$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

iClicker 30.1 What is $\delta_1^*(0, \epsilon)$?

A: 0 **B:** 1

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \to Q$



extend δ to: $\delta^\star : Q \times \Sigma^\star \to Q$

base: $\delta^\star(q, \epsilon) = q$

inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$

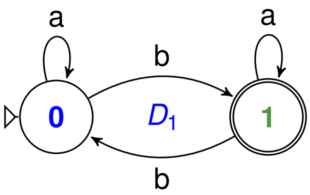
start at q, **read** w, thus at $\delta^\star(q, w)$

$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

iClicker 30.2 What is $\delta_1^*(0, aba)$?

A: 0 **B:** 1

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \to Q$



extend δ **to:** $\delta^\star : Q \times \Sigma^\star \to Q$

base: $\delta^\star(q, \epsilon) = q$

inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$

start at q, **read** w, thus at $\delta^\star(q, w)$

$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

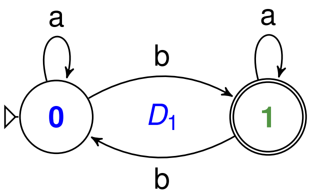
Def. (**language accepted** by D) $\stackrel{\text{def}}{=} \{w \in \Sigma^\star \mid \delta^\star(s, w) \in F\}$

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \to Q$

extend δ to: $\delta^\star : Q \times \Sigma^\star \to Q$

base: $\delta^\star(q, \epsilon) = q$

inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$



start at q, **read** w, thus at $\delta^\star(q, w)$

$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

Def. (**language accepted** by D) $\stackrel{\text{def}}{=} \{w \in \Sigma^\star \mid \delta^\star(s, w) \in F\}$

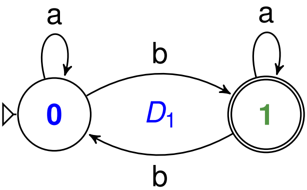
$\mathcal{L}(D) \stackrel{\text{def}}{=}$ **set of strings** taking D from s to F.

$D = (Q, \Sigma, \delta, s, F) \quad \delta : Q \times \Sigma \to Q$

extend δ to: $\delta^\star : Q \times \Sigma^\star \to Q$

base: $\delta^\star(q, \epsilon) = q$

inductive: $\delta^\star(q, wa) = \delta(\delta^\star(q, w), a)$



start at q, **read** w, thus at $\delta^\star(q, w)$

$D_1 = (Q_1, \Sigma_{ab}, \delta_1, 0, \{1\})$

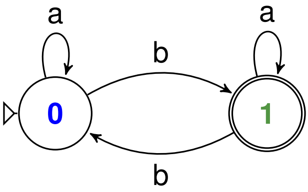
Def. (**language accepted** by D) $\stackrel{\text{def}}{=} \{w \in \Sigma^\star \mid \delta^\star(s, w) \in F\}$

$\mathcal{L}(D) \stackrel{\text{def}}{=}$ **set of strings** taking D from s to F.

iClicker 30.3 Is $aba \in \mathcal{L}(D_1)$?

A: yes **B:** no

DFA D_1

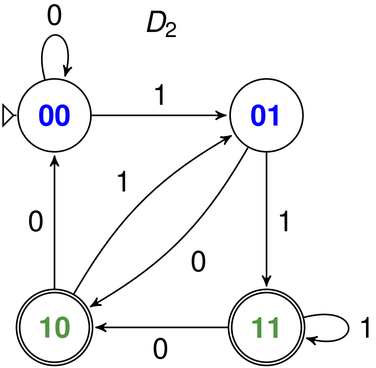


input string		**a**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**b**
state	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**1**	**1**	**0**	**1**

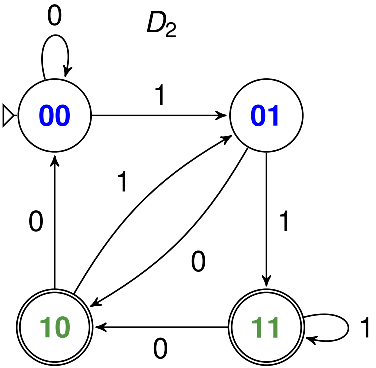
$$\mathcal{L}(D_1) = \{\textbf{aab}, \textbf{aaba}, \textbf{aabababab}, \textbf{aababaaba}, \textbf{aabababbabb}, \ldots\}$$

$$\mathcal{L}(D_1) = \{ w \in \{a, b\}^\star \mid \#_b(w) \equiv 1 \pmod 2 \}$$

$D_2 = (\Sigma^2_{\text{bin}}, \Sigma_{\text{bin}}, \delta_2, 00, \{10, 11\})$

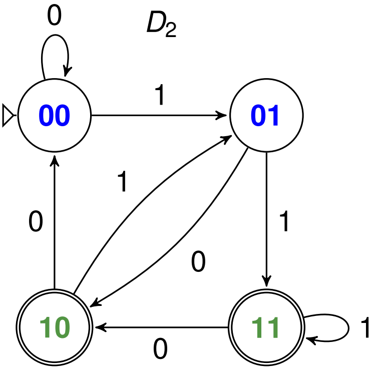


$D_2 = (\Sigma_{\text{bin}}^2, \Sigma_{\text{bin}}, \delta_2, 00, \{10, 11\})$



$$\mathcal{L}(D_2) = \left\{ w \in \Sigma_{\text{bin}}^{\star} \;\middle|\; \text{next to last letter of } w \text{ is } 1 \right\}$$

$D_2 = (\Sigma_{\mathrm{bin}}^2, \Sigma_{\mathrm{bin}}, \delta_2, 00, \{10, 11\})$

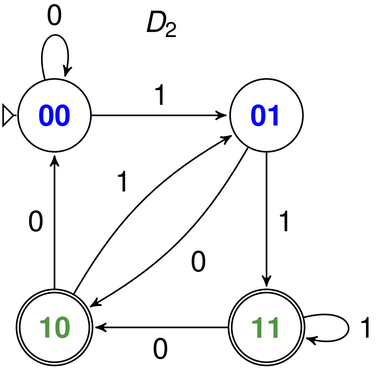


$$\begin{aligned}\mathcal{L}(D_2) &= \left\{ w \in \Sigma_{\mathrm{bin}}^\star \;\middle|\; \text{next to last letter of } w \text{ is } 1 \right\} \\ &= \Sigma_{\mathrm{bin}}^\star \cdot \{1\} \cdot \Sigma_{\mathrm{bin}} \quad = \quad \mathcal{L}((0 \cup 1)^* 1 (0 \cup 1))\end{aligned}$$

$D_2 = (\Sigma_{\text{bin}}^2, \Sigma_{\text{bin}}, \delta_2, 00, \{10, 11\})$

$$\delta_2(ab, c) = bc$$

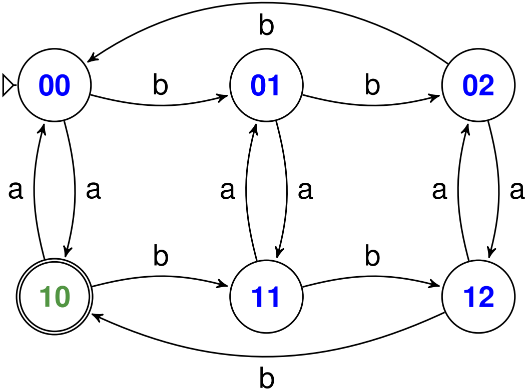
D_2 remembers the last two bits it has seen.



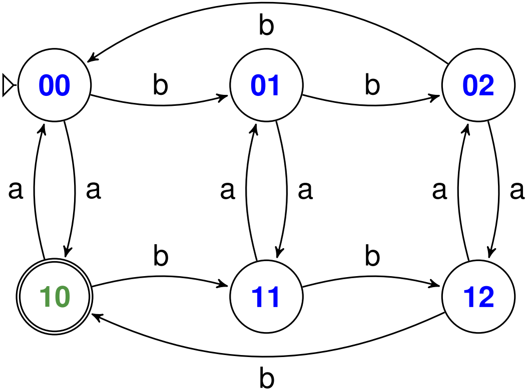
$$\mathcal{L}(D_2) = \{w \in \Sigma_{\text{bin}}^{\star} \mid \text{next to last letter of } w \text{ is } 1\}$$

$$= \Sigma_{\text{bin}}^{\star} \cdot \{1\} \cdot \Sigma_{\text{bin}} = \mathcal{L}((0 \cup 1)^* 1 (0 \cup 1))$$

$D_3 = (\Sigma_{\text{bin}} \cdot \{0, 1, 2\}, \Sigma_{ab}, \delta_3, 00, \{10\})$

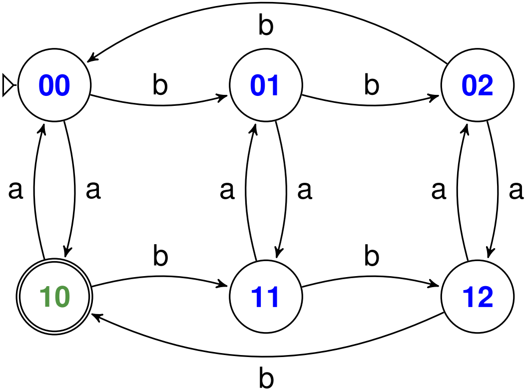


$D_3 = (\Sigma_{\mathrm{bin}} \cdot \{0, 1, 2\}, \Sigma_{ab}, \delta_3, 00, \{10\})$



$$\mathcal{L}(D_3) \quad = \quad \left\{ w \in \Sigma_{ab}^{\star} \;\middle|\; \#_a(w) \equiv 1 \pmod 2 \wedge \#_b(w) \equiv 0 \pmod 3 \right\}$$

$D_3 = (\Sigma_{\text{bin}} \cdot \{0, 1, 2\}, \Sigma_{ab}, \delta_3, 00, \{10\})$



$$\mathcal{L}(D_3) \quad = \quad \{ w \in \Sigma_{ab}^{\star} \mid \#_a(w) \equiv 1 \pmod 2 \wedge \#_b(w) \equiv 0 \pmod 3 \}$$

$$\delta_3^*(\mathbf{00}, w) \quad = \quad \#_a(w)\%2 \cdot \#_b(w)\%3$$