Epp §1.2 Language of Sets

1. A **set** is a collection of objects.
2. **Axiom of Extension:** If sets A and B contain exactly the same elements, then $A = B$.
3. Let $A = \{1, 2, 3\}, B = \{3, 1, 2\}, C = \{1, 1, 3, 1, 2, 1, 3\}$
4. In **set-roster** notation, order and repetitions don't matter.
5. Each of A, B, C contain exactly the same three elements, so $A = B = C$, they are the same set.
6. Sets may have other sets as elements, e.g., $D = \{0\}$, $E = \{1\}$, $F = \{D, E\} = \{\{0\}, \{1\}\}$, $G = \{1, E\} = \{1, \{1\}\}$.

iClicker: Is $E = G$?
A: yes, they both have the same element
B: no, G has two elements and E has only one element

Important Sets: $\mathbf{N}, \mathbf{Z}, \mathbf{Q}, \mathbf{R}$

$\mathbf{N}$	$=$	$\{0, 1, 2, \ldots\}$	set of **natural numbers**
$\mathbf{Z}$	$=$	$\{n, -n \mid n \in \mathbf{N}\}$	set of **integers**
$\mathbf{Q}$	$=$	$\{\frac{p}{q} \mid p, q \in \mathbf{Z}, q \neq 0\}$	set of **rational numbers**
$\mathbf{R}$	$=$		set of **real numbers**

S^+	$=$	$\{s \in S \mid s > 0\}$	positive elements of S
S^-	$=$	$\{s \in S \mid s < 0\}$	negative elements of S
S^{nonneg}	$=$	$\{s \in S \mid s \geq 0\}$	non-negative elements of S

$$\mathbf{N} = \mathbf{Z}^{\text{nonneg}}$$

These will be used throughout CS250, **Please Memorize!**

Set-Builder Notation

$$S = \{t \in T \mid P(t)\}$$
$$= \text{the set of elements of } T \text{ that satisfy property } P$$

$$\mathbf{R}^+ = \{r \in \mathbf{R} \mid r > 0\}$$
$$\mathbf{Z}^{\text{nonneg}} = \{z \in \mathbf{Z} \mid z \geq 0\} = \mathbf{N} = \{0, 1, 2, \ldots\}$$
$$(-2, 1) = \{r \in \mathbf{R} \mid -2 < r < 1\} \quad \textbf{open interval}$$
$$[-.5, 4] = \{r \in \mathbf{R} \mid -.5 \leq r \leq 4\} \quad \textbf{closed interval}$$
$$H = \{z \in \mathbf{Z} \mid -.5 \leq z \leq 4\}$$

iClicker: How many elements does the set *H* have?
A: 3, B: 4, C: 5

Subsets

- **Def.** A is a **subset** of B ($A \subseteq B$) if every element of A is an element of B. We also say A is **contained** in B ($A \subseteq B$), and B **contains** A ($B \supseteq A$).
- A is a **proper subset** of B if $A \subseteq B$ and $A \neq B$.
- Write $A \not\subseteq B$ to mean that A is not a subset of B, i.e., there exists an element $b \in B$ such that $b \notin A$.

- $\mathbf{Z}^+ \subseteq \mathbf{N} \subseteq \mathbf{Z}^{\text{nonneg}} \subseteq \mathbf{Z} \subseteq \mathbf{Q} \subseteq \mathbf{R}$.

iClicker: Which of the above containments are proper?

A: all of them

B: all except the first

C: all except the second

Cartesian Product

- The **ordered pair** (a, b) consists of a first element, a, and a second element b. Two ordered pairs are equal just if their first elements are equal and their second elements are equal: $(a, b) = (c, d) \quad \Leftrightarrow \quad a = c$ and $b = d$.
- The **Cartesian product** $A \times B \ = \ \{(a, b) \mid a \in A; b \in B\}$.
- $A = \{a, b, c\}, \quad B = \{0, 1\}$
- $A \times B = \{(a, 0), (a, 1), (b, 0), (b, 1), (c, 0), (c, 1)\}$
- The **cardinality** of a set, S, ($|S|$), is the number of elements in the set. $|A| = 3$, $|B| = 2$, $|A \times B| = 6$,
- In general, $|A \times B| = |A| \cdot |B|$.
- The **empty set**, $\emptyset = \{\}$, is the set with no elements.
- $|\emptyset| = 0$
- What is $|\mathbf{N}|$?
- What is $|\mathbf{Q}|$?
- What is $|\mathbf{R}|$?

Answers to Quiz R2

True or False wrt the following sets:

$$
\begin{aligned}
A &= \mathbf{Z}^{\text{nonneg}} \\
B &= \{n \in \mathbf{Z} \mid -5 \le n \le 5\} \\
C &= \{2, 4, 6, 8, 10, \ldots\} \\
E &= \{z \in A \mid z \ \mathbf{mod}\ 2 = 0,\ \text{i.e, } z \text{ is even}\}
\end{aligned}
$$

1. $B \subseteq A$: False: $-5 \in B - A$
2. $E \subseteq A$: True: E contains exactly the even elements of A.
3. C is a proper subset of E: True: $C = E - \{0\}$
4. There exists $x \in E$ s.t. $x \in B$: True: $\{0, 2, 4\} \subseteq B \cap E$
5. $E \subseteq C$: False: $0 \in E - C$

6. $a \in \{a, b, c\}$: True: a is an element of $\{a, b, c\}$.
7. $a \subseteq a, b, c$: False: a is not a set.
8. $a \in \{\{a\}, \{b\}, \{c\}\}$: False: $\{\{a\}, \{b\}, \{c\}\}$ has three elements, all of them sets, so not equal to a.
9. $\{\{a\}\} \subseteq \{\{a\}, \{b\}\}$: True: $\{\{a\}\}$ has one element, $\{a\}$, which is also an element of $\{\{a\}, \{b\}\}$.
10. $\{a\} \subseteq \{\{a\}, \{b\}\}$: False: $\{a\}$ has one element, a, which is not an element of $\{\{a\}, \{b\}\}$.
11. How many elements are in the set $\{a, \{a, b\}, \{b, a\}\}$?: 2: note that $\{a, b\} = \{b, a\}$.
12. How many elements are in the set $\{a, \{a\}, \{\{a\}\}\}$?: 3: the listed elements are all distinct.

13. What is the first element of (2,0) ?: 2 is the first element of the ordered pair (2,0).
14. Is (2,0) = (0,2)?: No: the definition of an ordered pair says that $(a, b) = (c, d)$ exactly if $a = c$ and $b = d$.
15. Is $(1, 1) = \{1\}$?: No, ordered pairs of elements are different from sets of these elements. The book mentions Kuratowski's encoding of pairs as sets, (a, b) is encoded as $\{a, \{a, b\}\}$, but that will not concern us in CS250. (Under Kuratowski's encoding, (1,1) would be encoded by the set $\{1, \{1, 1\}\} = \{1, \{1\}\}$.)
16. For $A = \{a, b\}$ and $S = \{0, 1\}$, which of the following is **not** an element of $A \times S$? $(1, b), (a, 0), (a, 1), (b, 1)$: $A \times S = \{(a, 0), (a, 1), (b, 0), (b, 1)\}$, so $(1, b) \notin A \times S$.
17. For $A = \{a, b\}$ and $S = \{0, 1\}$, is $A \times S = S \times A$? No, for example $(a, 0) \in A \times S \ - \ S \times A$.