CS250: Discrete Math for Computer Science

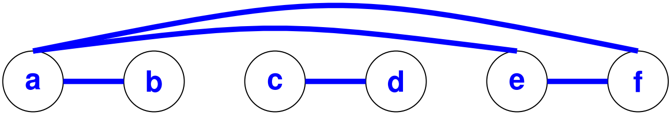
L29: DFS on Directed Graphs

Depth First Search (undirected graphs)

```
DFSmain(G)
 for each u in V :
   color[u] = white
   parent[u] = NULL
 time=0
 for each u in V :
   if (color[u] == white):
     DFSVisit(u)
```

```
DFSVisit(u)
  color[u] = red               // in process
  d[u] = ++time                // discover
  for each v in Adj(u) :
    if (color[v] == white) :   // unseen
      parent[v] = u            // tree edge
      DFSVisit(v)
  color[u] = black             // done
  f[u] = ++time                // finish
```

black Tree edge brown Back edge

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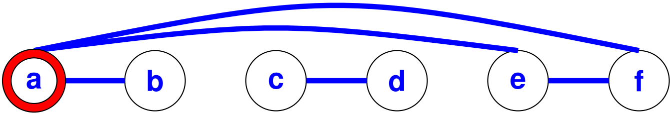


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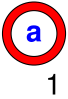

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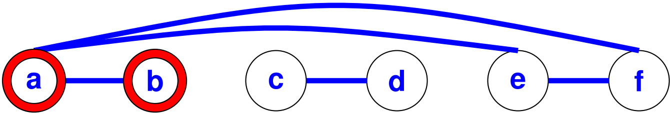


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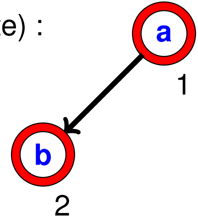

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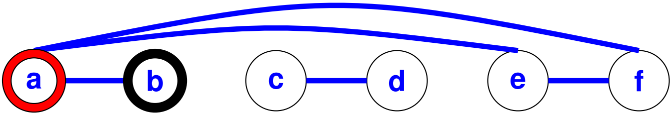


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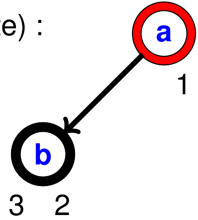

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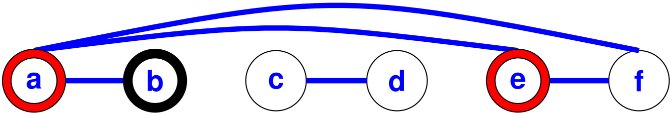


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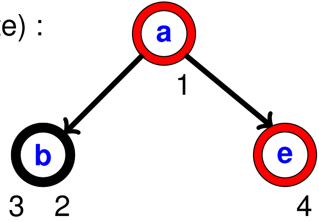

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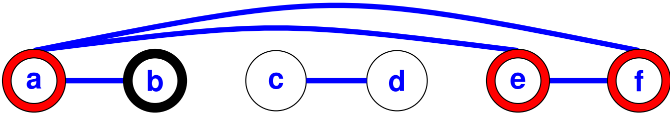


DFSVisit(u)

color[u] = **red**

d[u] = ++time

for each v in Adj(u) :

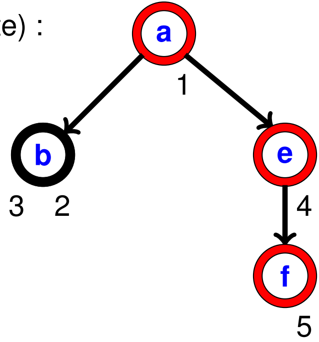
if (color[v] == white) :

parent[v] = u

DFSVisit(v)

color[u] = **black**

f[u] = ++time



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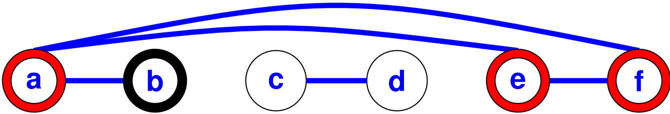


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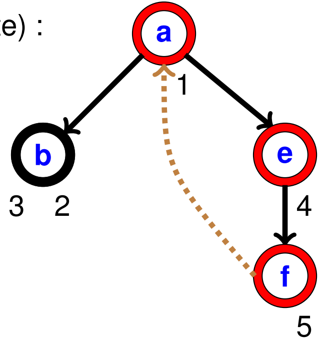

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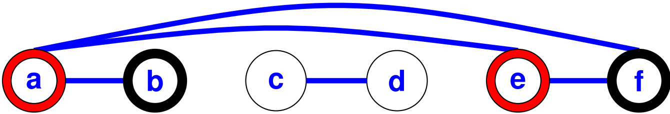

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color[u] = **red**

d[u] = ++time

for each v in Adj(u) :

if (color[v] == white) :

parent[v] = u

DFSVisit(v)

color[u] = **black**

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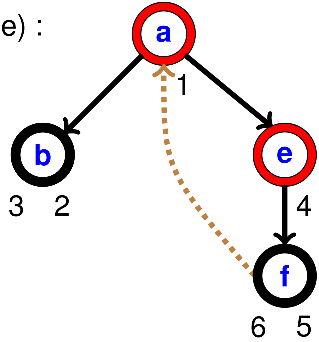

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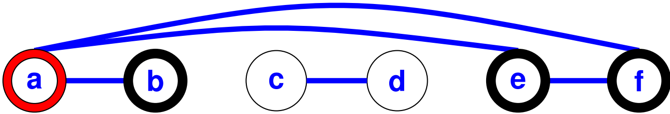

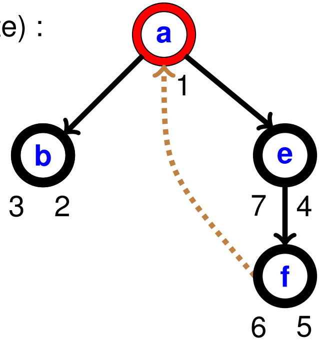

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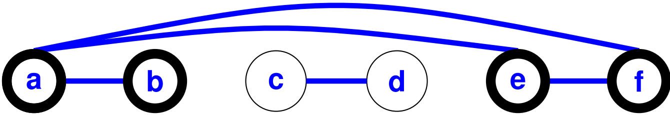


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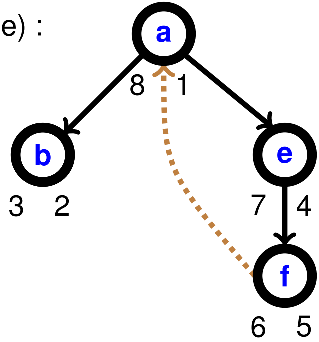

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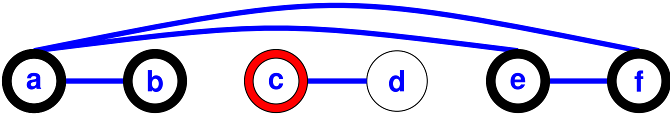

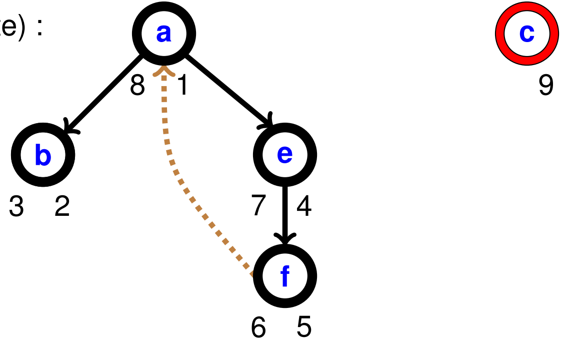

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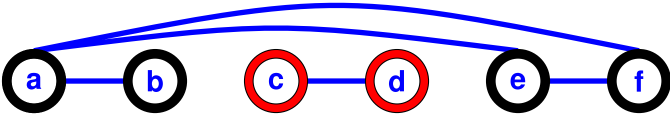


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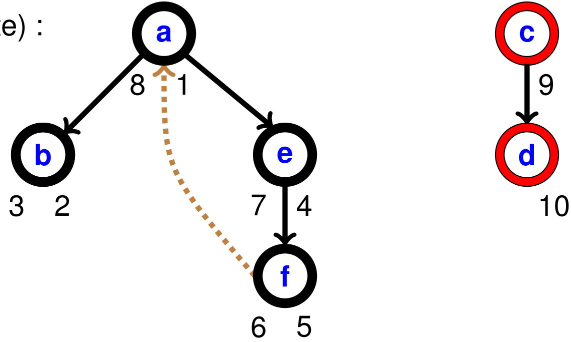

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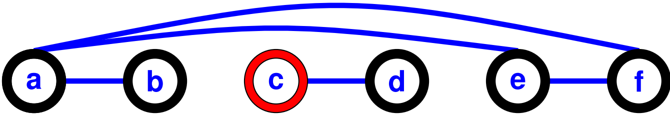


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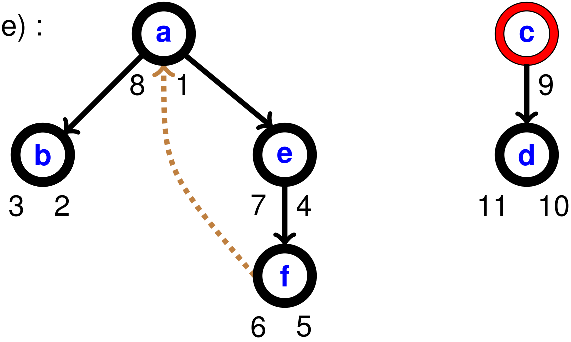

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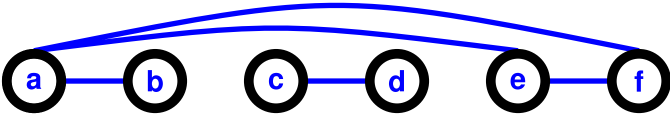


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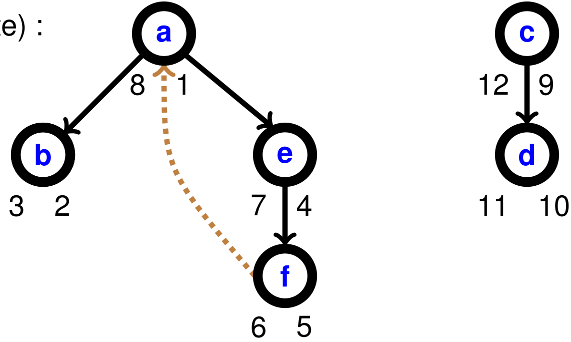

Depth First Search (undirected graphs)

```
DFSmain(G)
 for each u in V :
   color[u] = white
   parent[u] = NULL
 time=0
 for each u in V :
   if (color[u] == white):
     DFSVisit(u)
```

```
DFSVisit(u)
 color[u] = red                    // in process
 d[u] = ++time                     // discover
 for each v in Adj(u) :
   if (color[v] == white) :        // unseen
     parent[v] = u                 // tree edge
     DFSVisit(v)
 color[u] = black                  // done
 f[u] = ++time                     // finish
```

Thm. (Properties of DFS on Undirected Graphs)
Let G be an undirected graph with n vertices and m edges.
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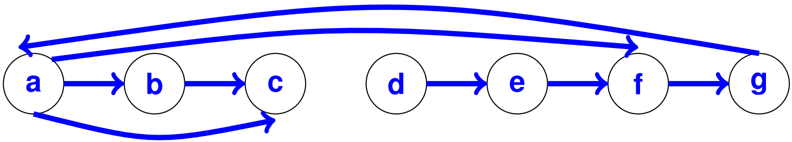
Then:

1. DFS(G) runs in **linear time**, i.e., $O(n+m)$.
2. DFS computes **connected components** of G.
3. DFS determines which of these components is cyclic: a component is **cyclic** iff it has a **backedge**.

black	brown	cyan	fuchsia
tree edge	back edge	cross edge	forward edge



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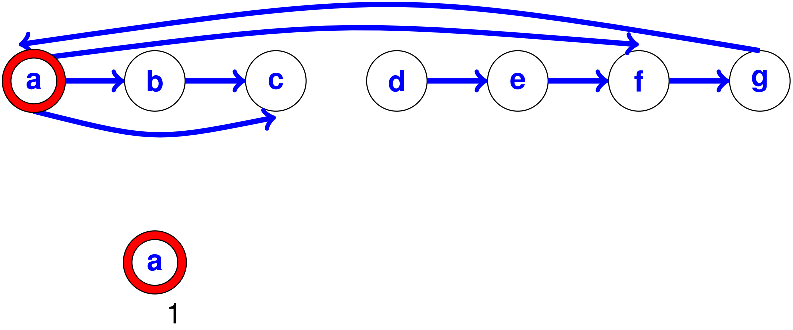

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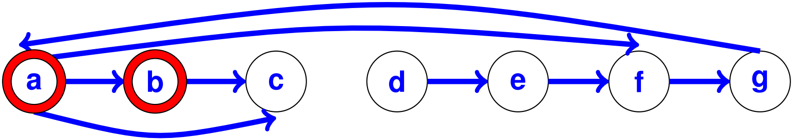

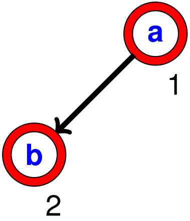

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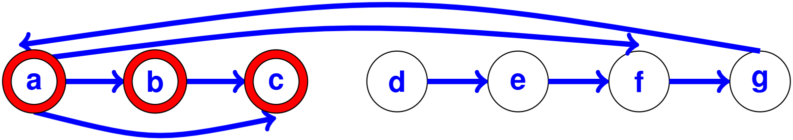

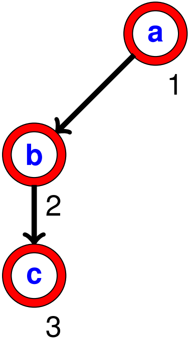

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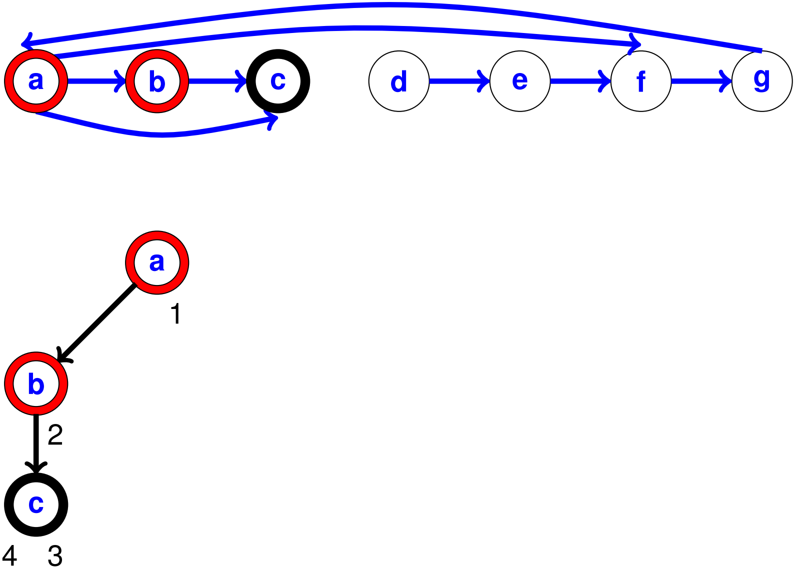

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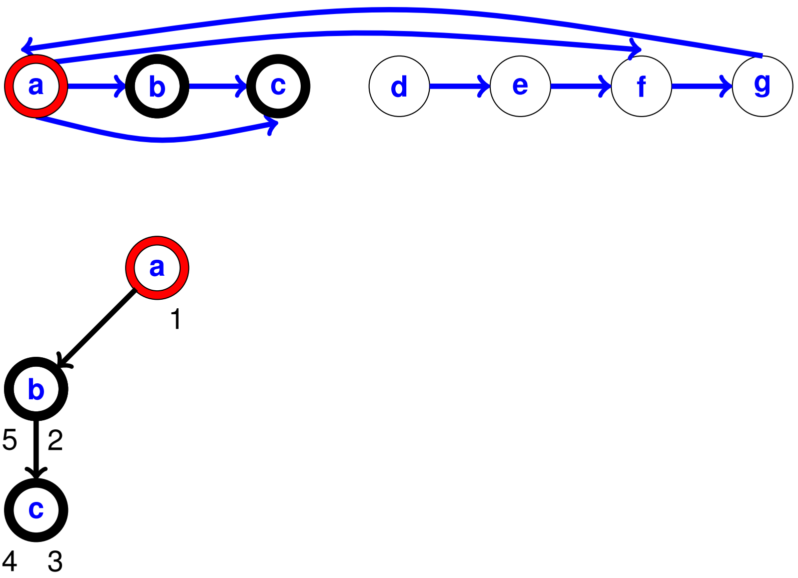

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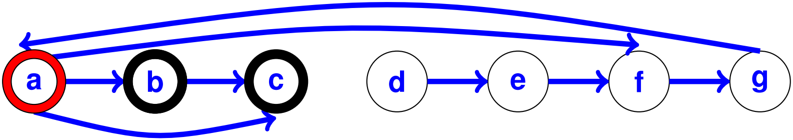

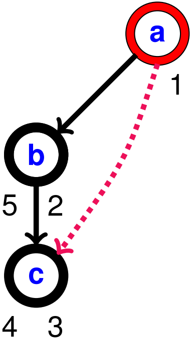

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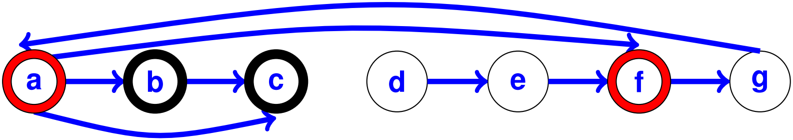

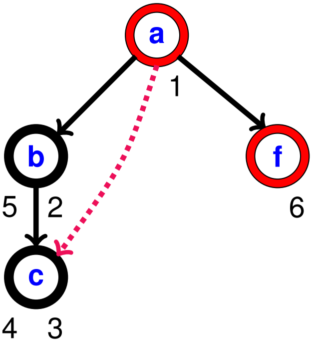

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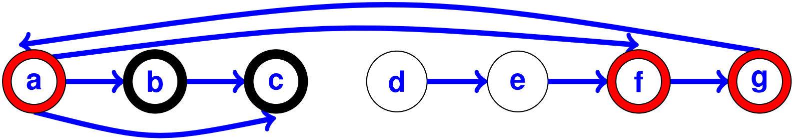

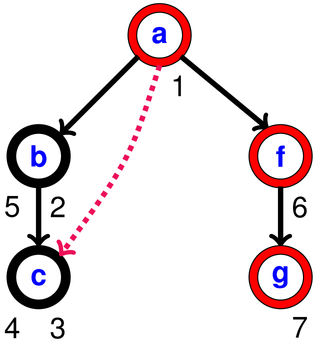

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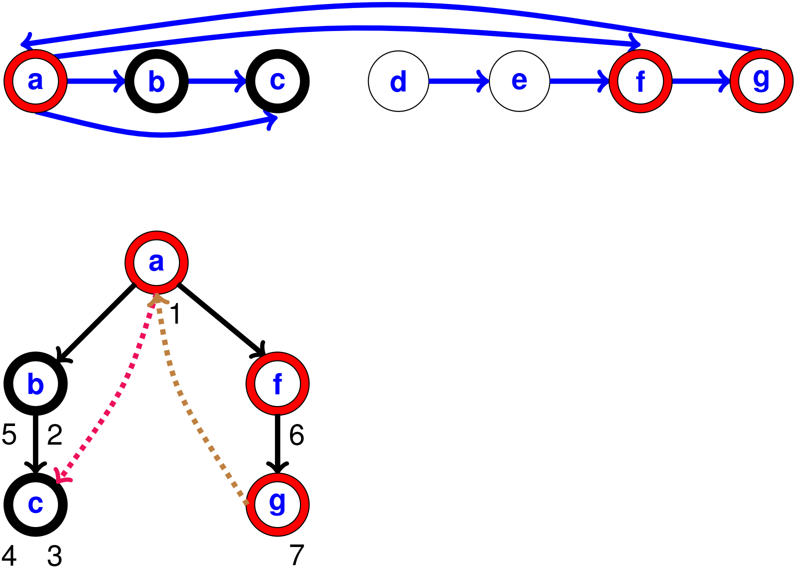

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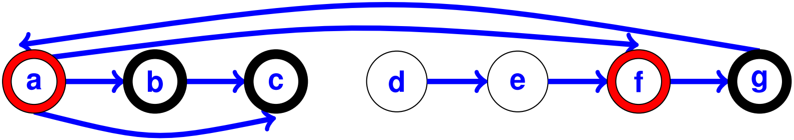

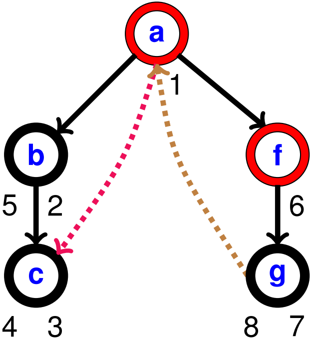

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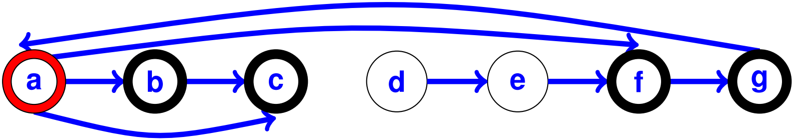

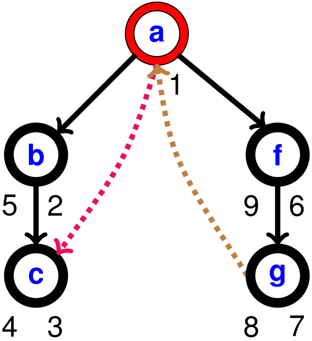

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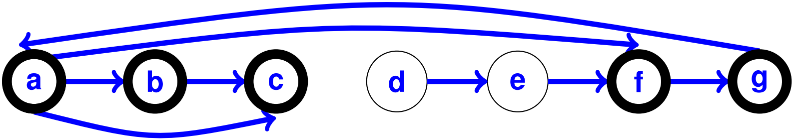

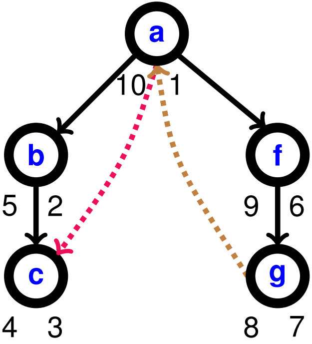

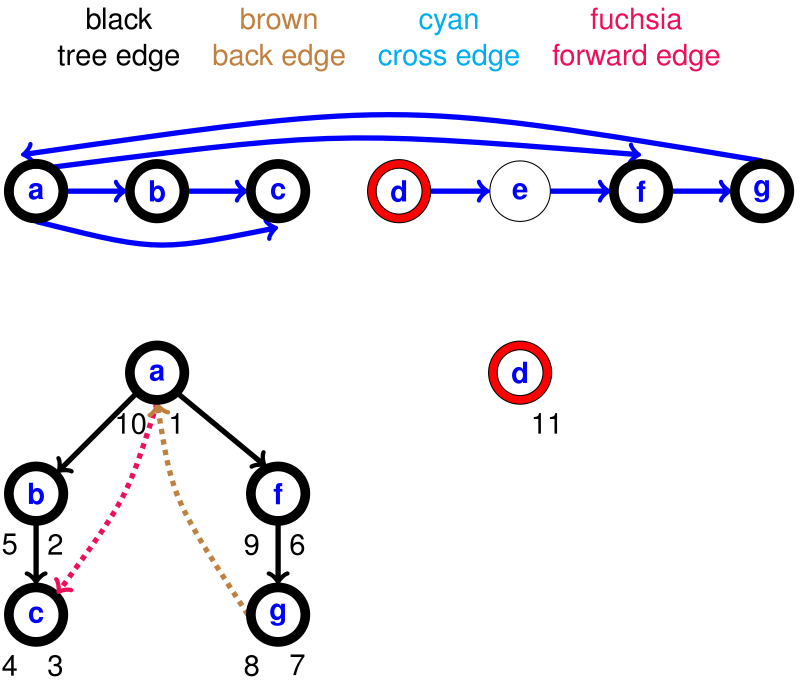
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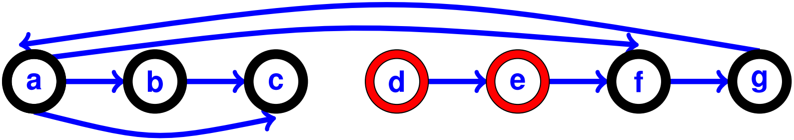

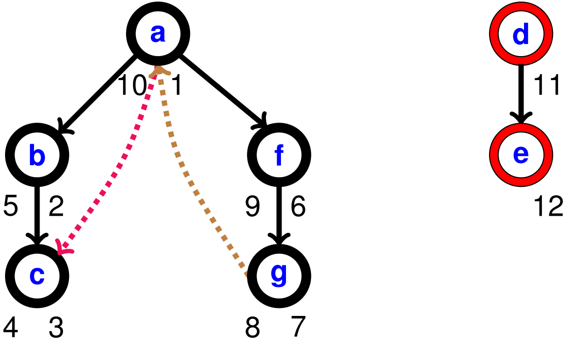

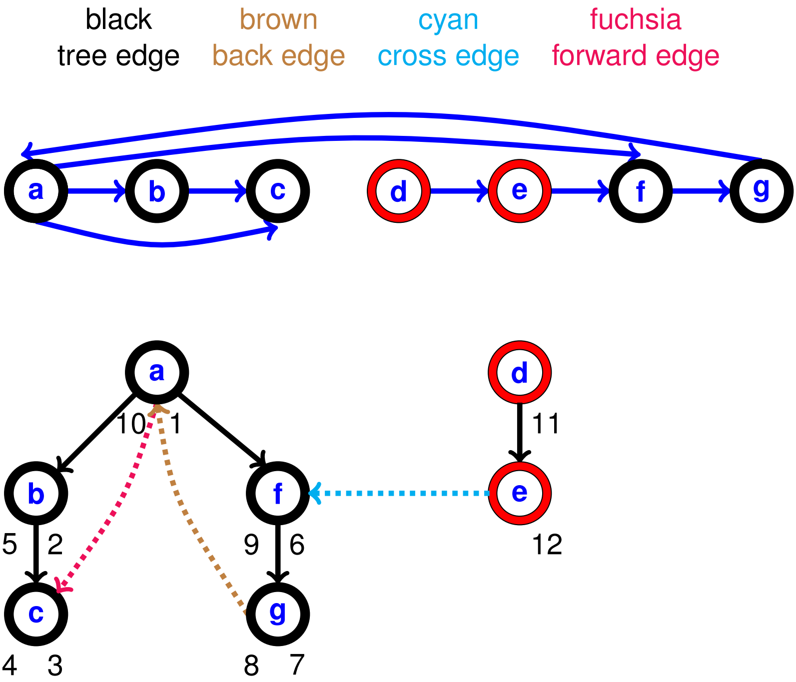
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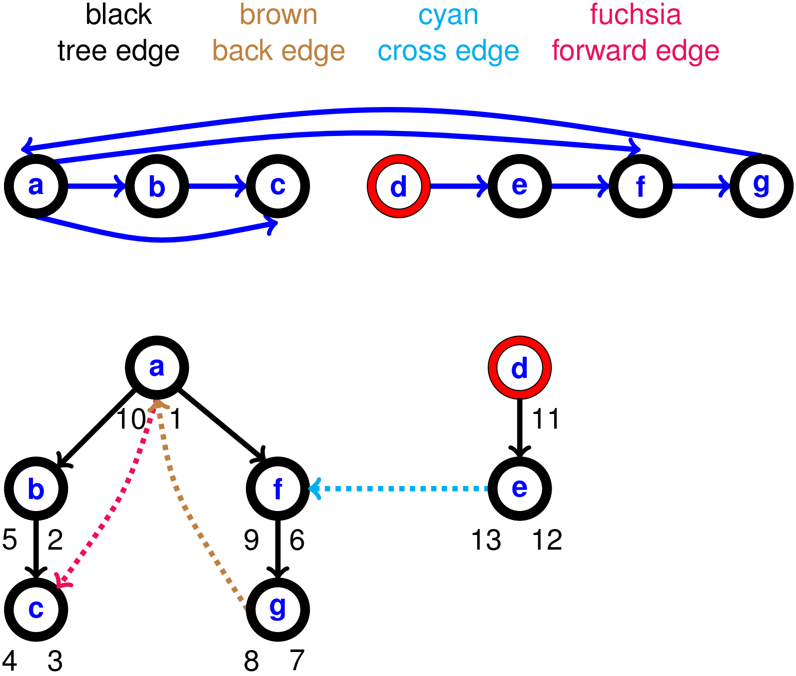
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cyan
cross edge
fuchsia
forward edge
a
b
c
d
e
f
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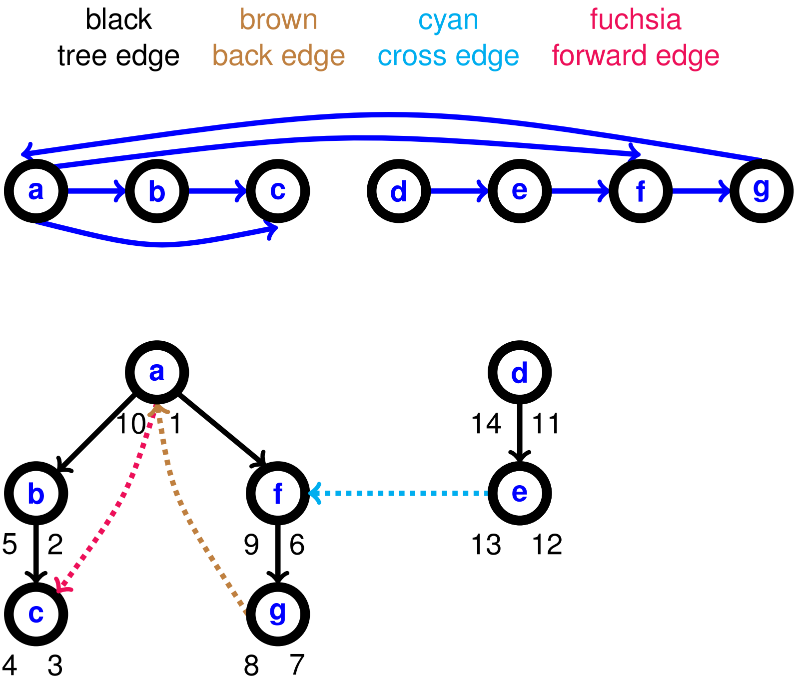
black
tree edge
brown
back edge
cyan
cross edge
fuchsia
forward edge
a
b
c
d
e
f
g
a
10 1
b
5 2
c
4 3
f
9 6
g
8 7
d
14 11
e
13 12

Depth First Search

```
DFSmain(G)
 for each u in V :
   color[u] = white
   parent[u] = NULL
 time=0
 for each u in V :
   if (color[u] == white):
     DFSVisit(u)
```

```
DFSVisit(u)
 color[u] = red                  // in process
 d[u] = ++time                   // discover
 for each v in Adj(u) :
   if (color[v] == white) :      // unseen
     parent[v] = u               // tree edge
     DFSVisit(v)
 color[u] = black                // done
 f[u] = ++time                   // finish
```

Thm. (Properties of DFS on directed Graphs)

Let G be an directed graph with n vertices and m edges. Then:

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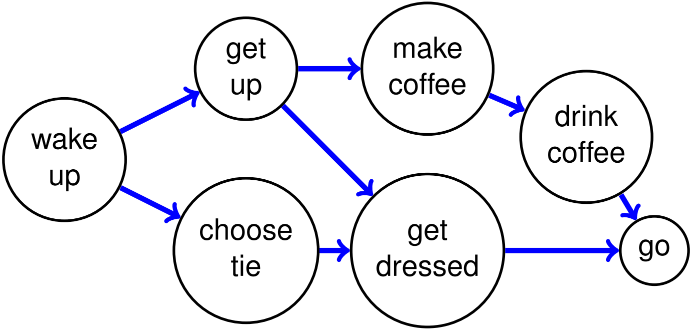
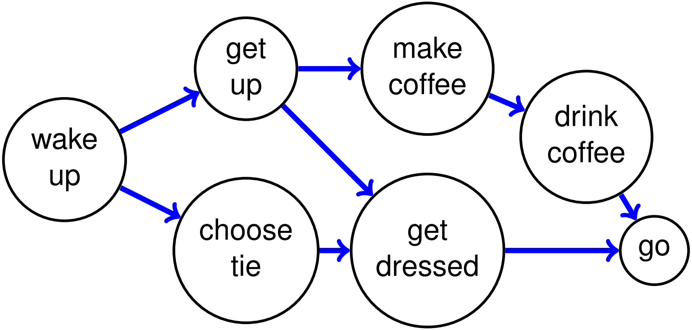
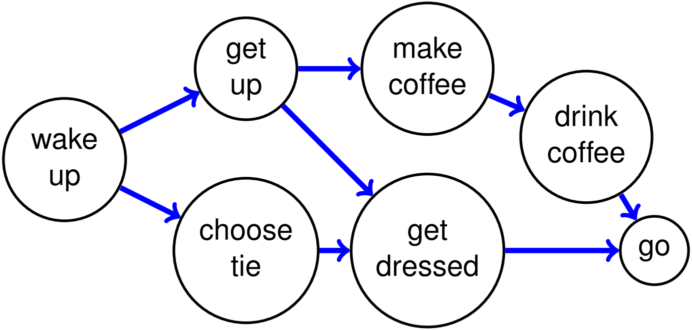
Proof: 1. 2., and 3. are similar as for undirected graphs. ✓


wake up
get up
choose tie
make coffee
get dressed
drink coffee
go



Def. A **topological ordering** of a DAG G, is an ordering of $V^G = \{v_1, \ldots, v_n\}$ such that for all $(v_i, v_j) \in E^G$, $i < j$.



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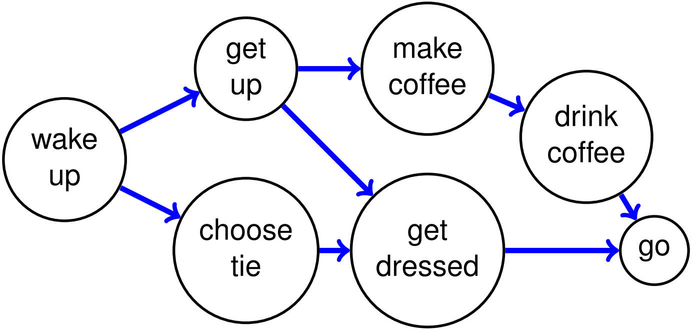

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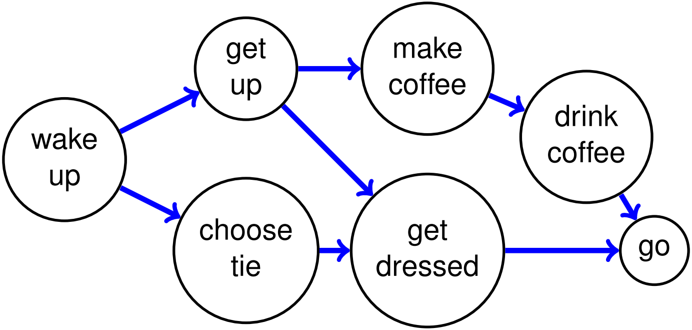

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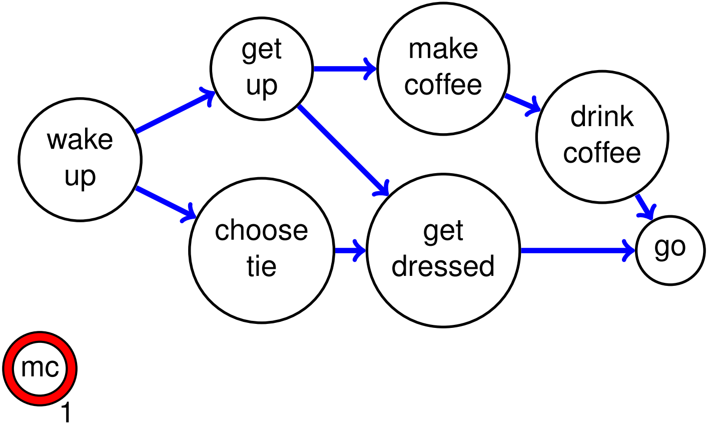
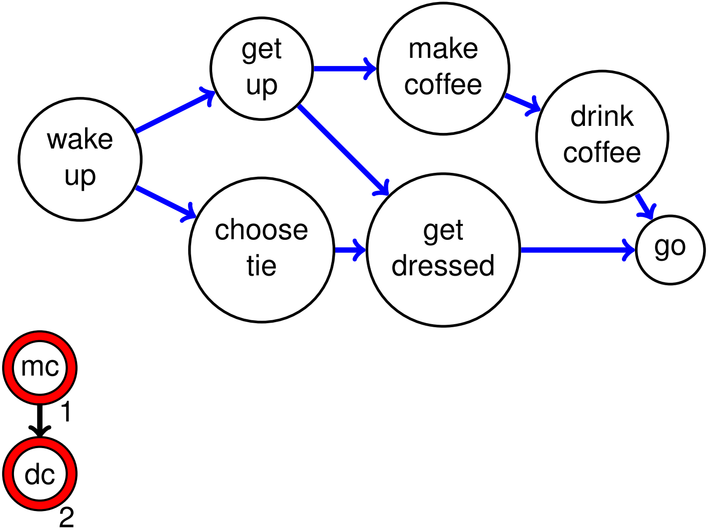
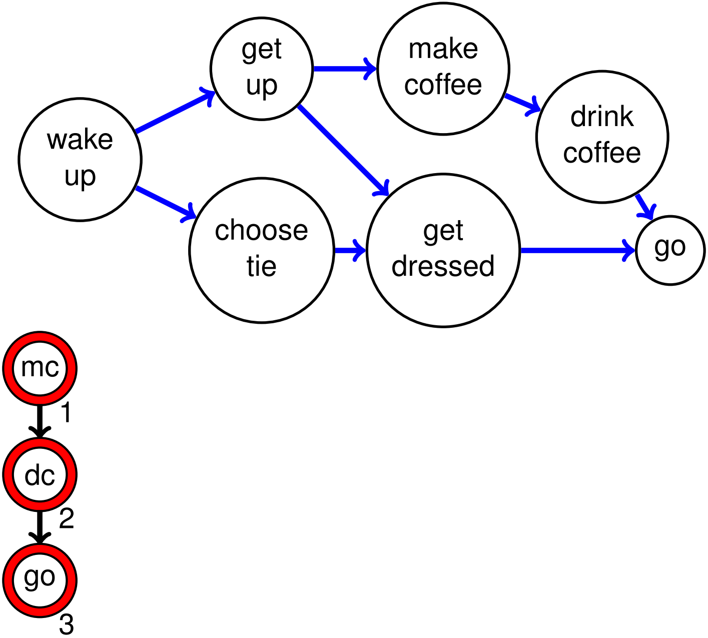

wake up
get up
make coffee
drink coffee
choose tie
get dressed
go
mc
1


wake up
get up
make coffee
drink coffee
choose tie
get dressed
go
mc
1
dc
2


wake up
get up
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go
mc
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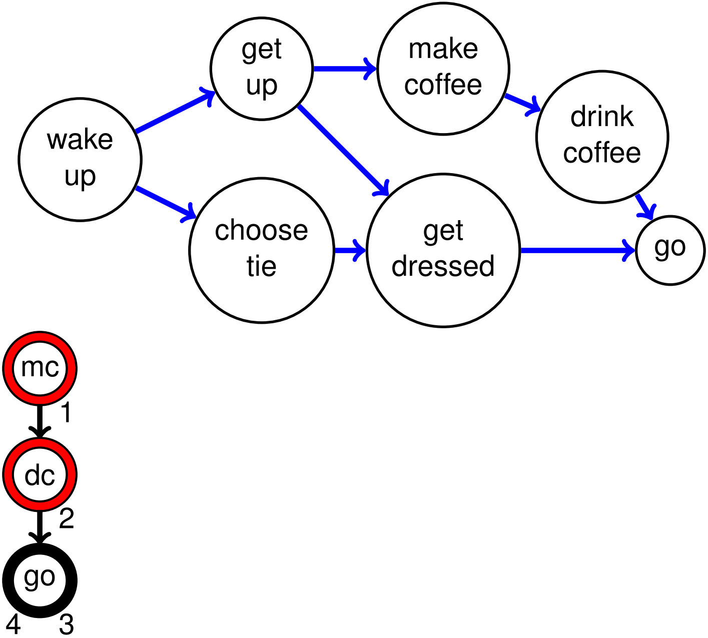
dc
2
go
3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mc
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dc
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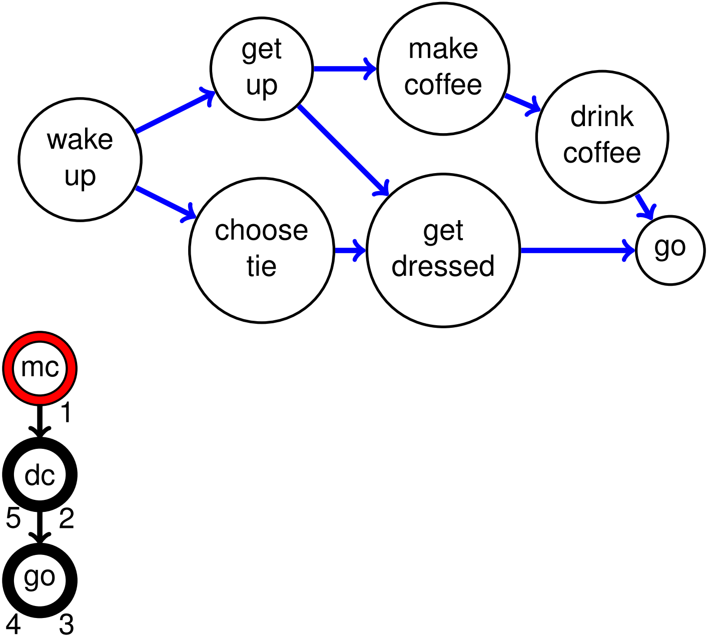
go
4
3


wake up
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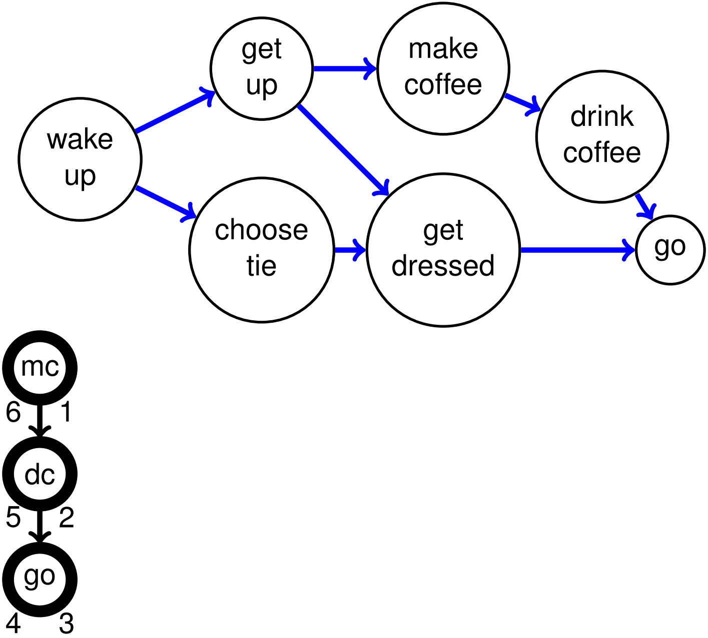
4
3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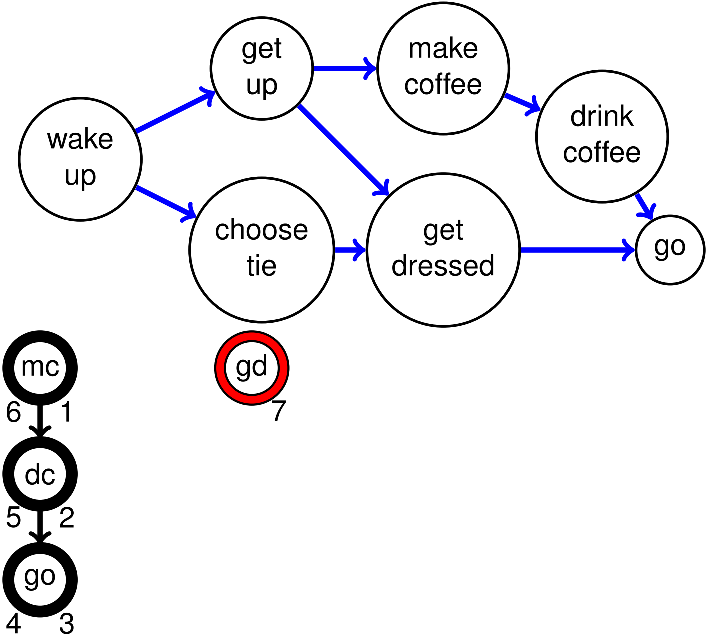
wake up
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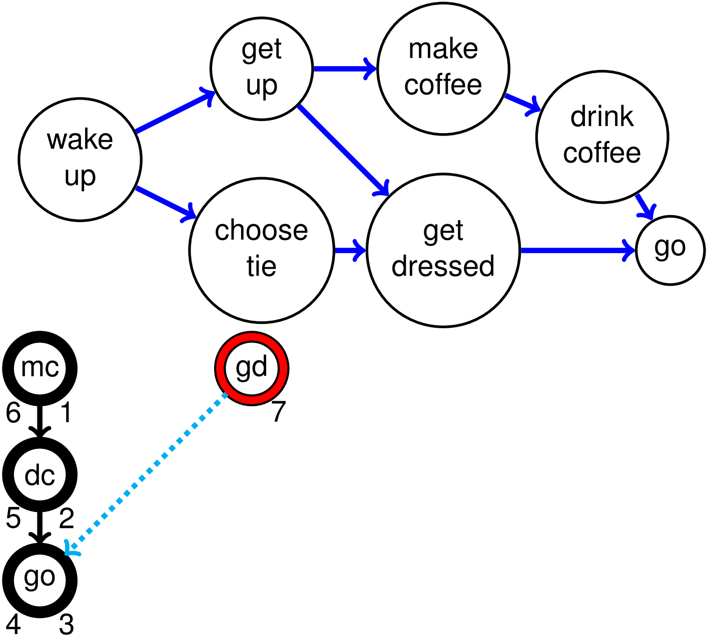

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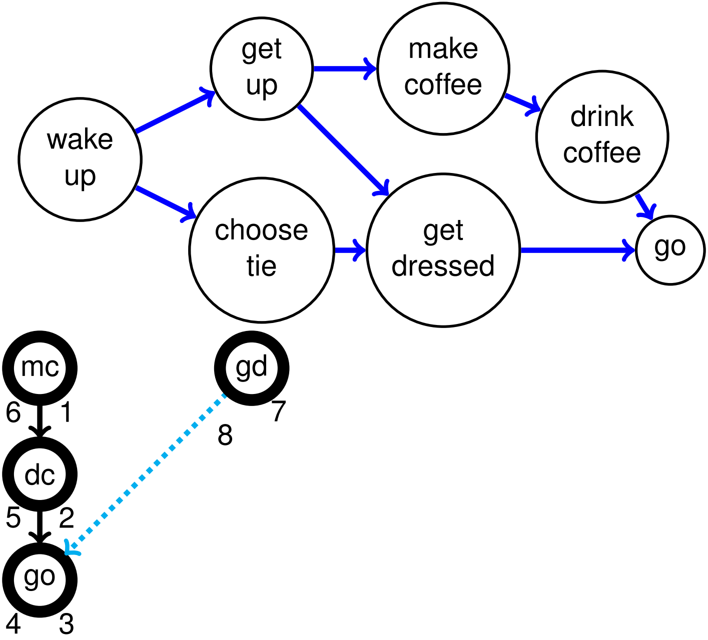

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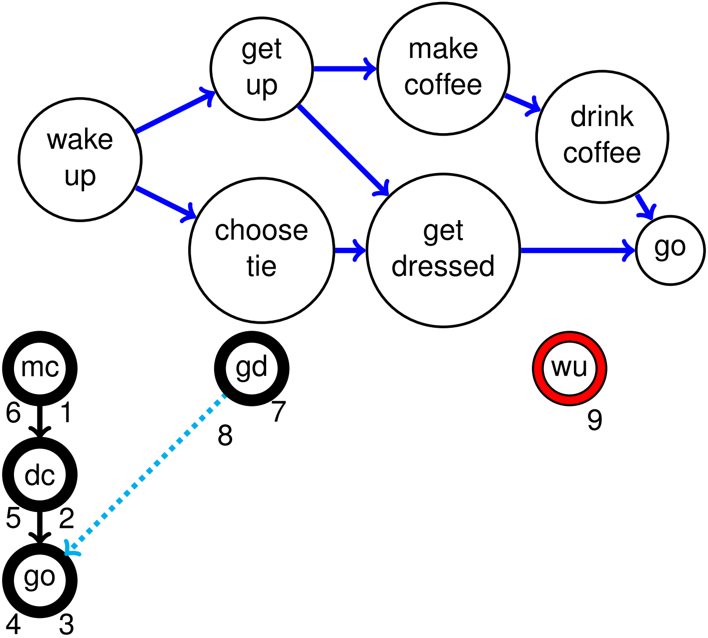

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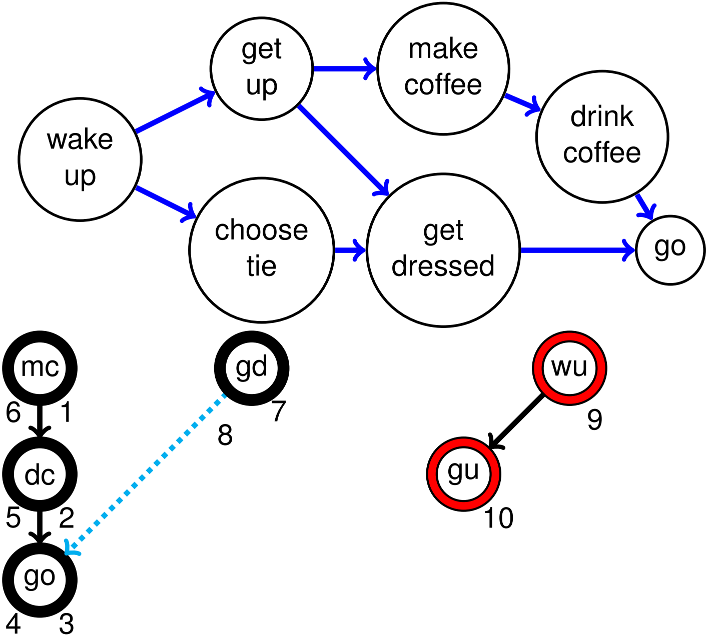

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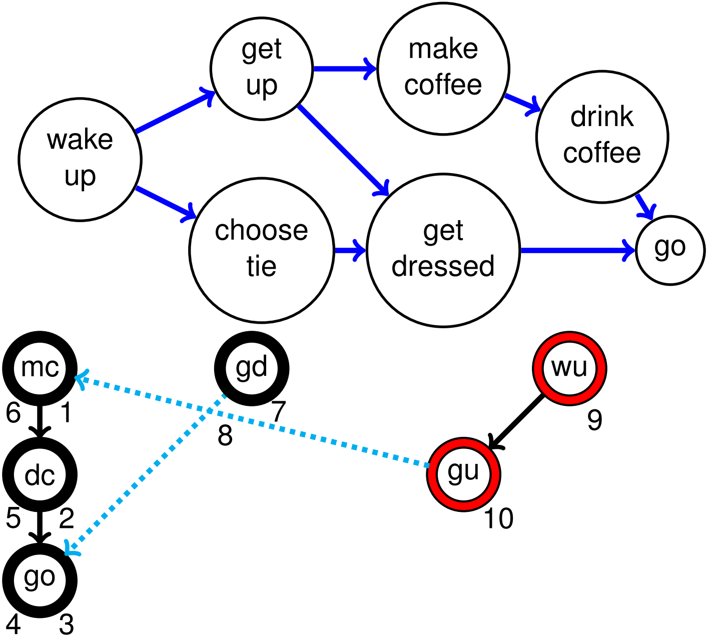
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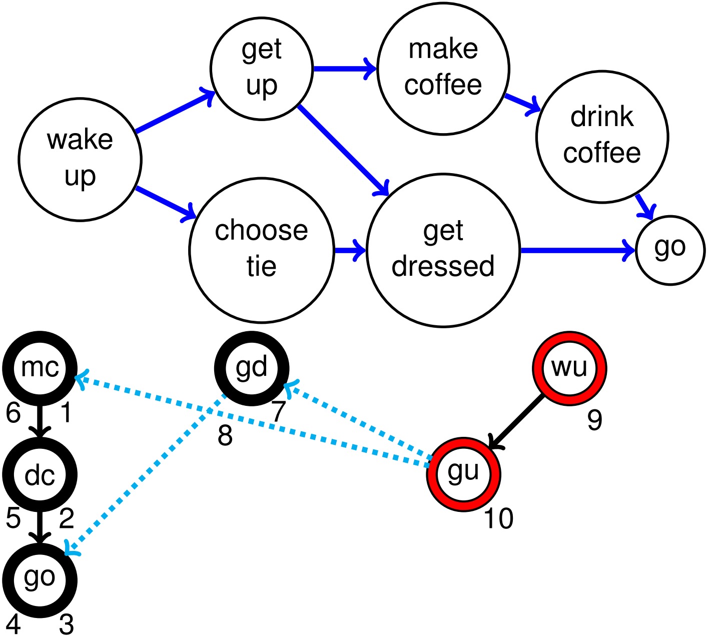

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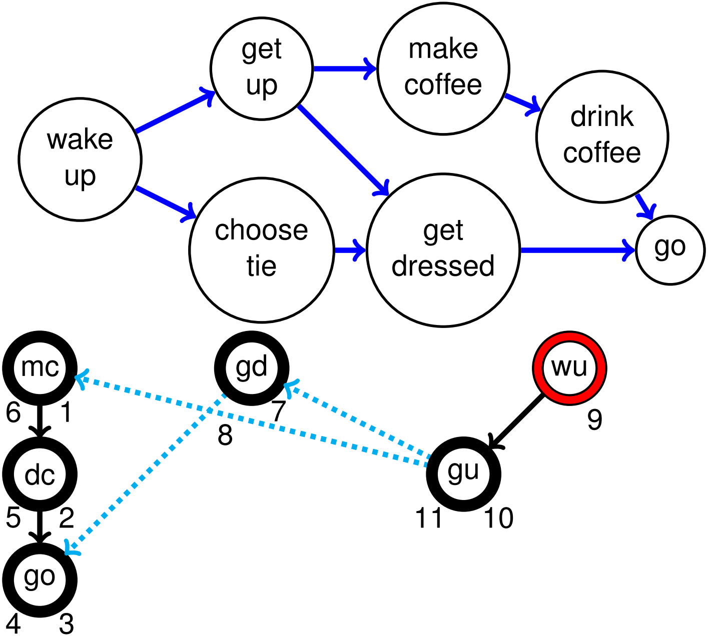
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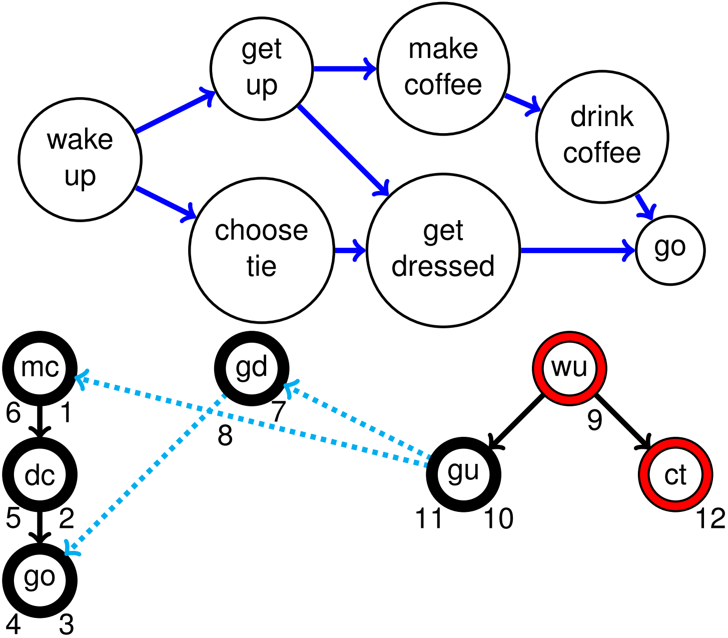

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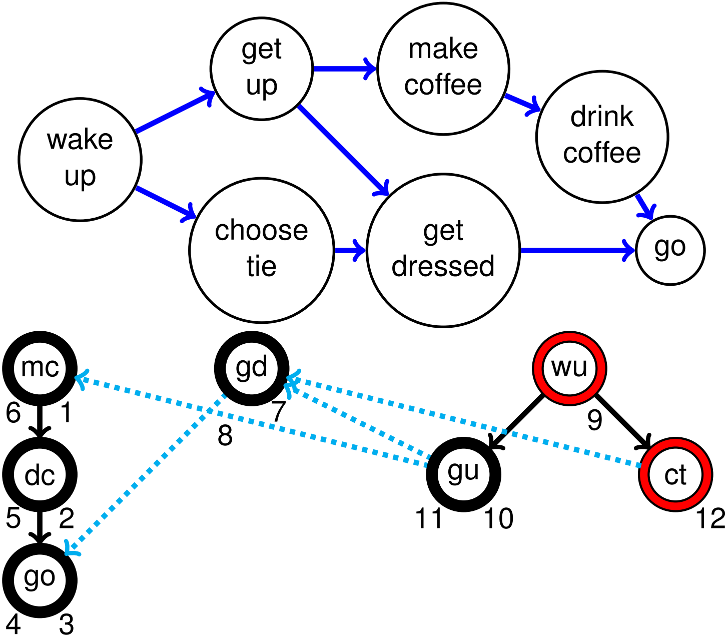

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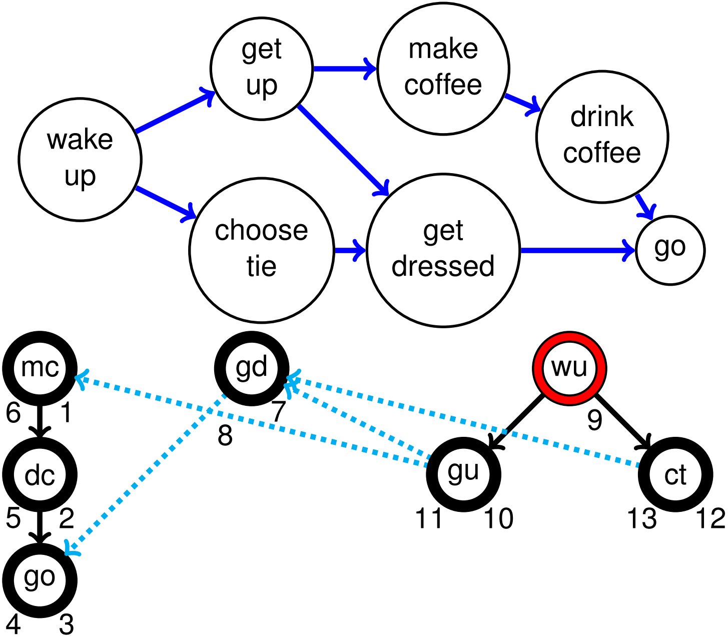

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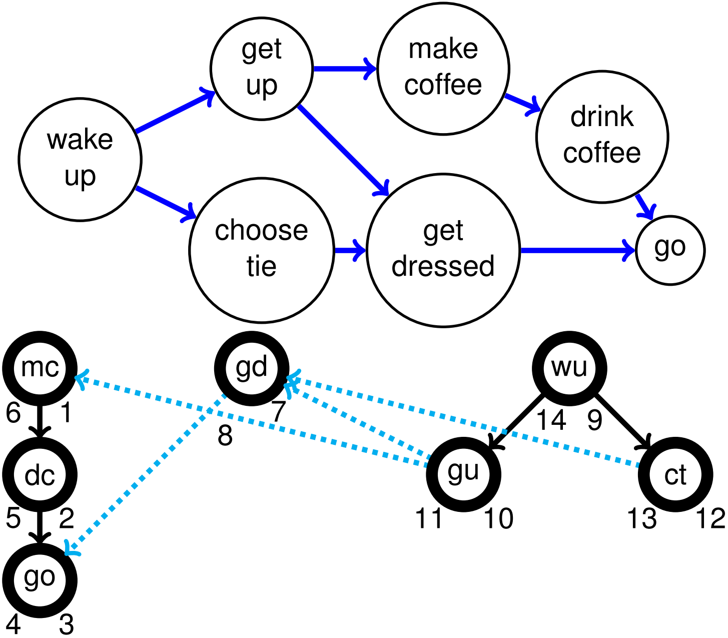

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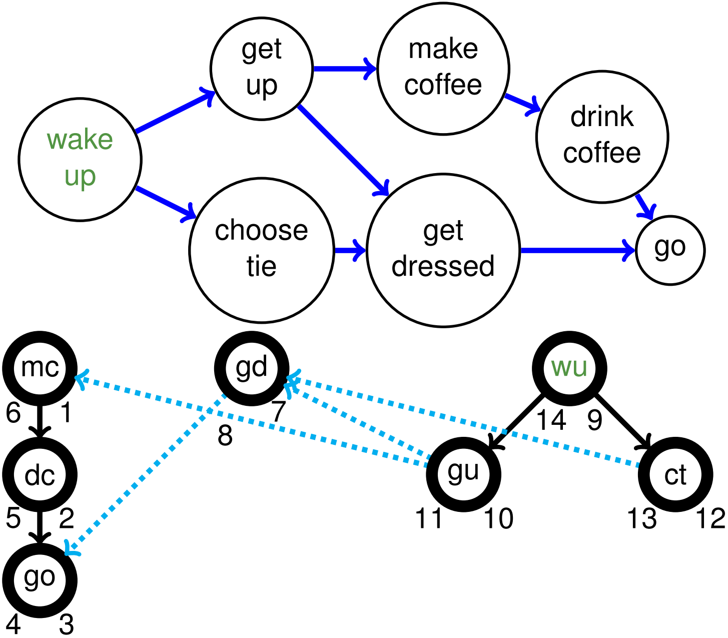

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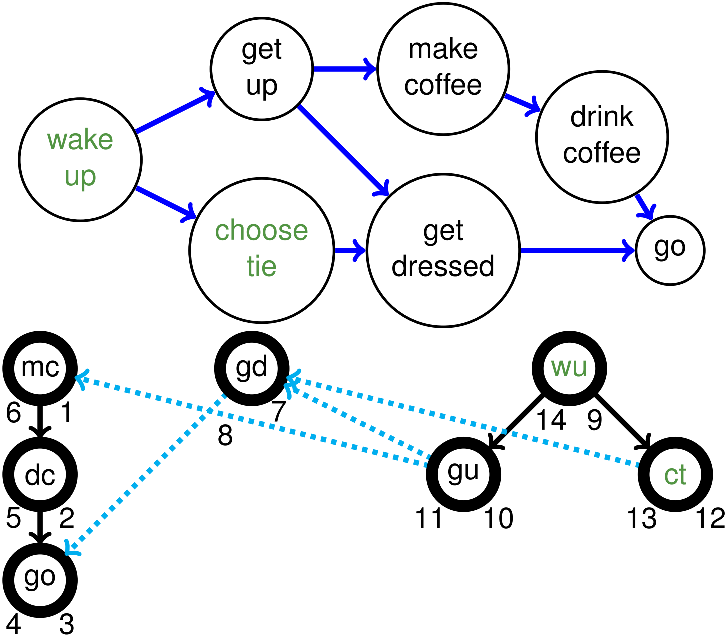

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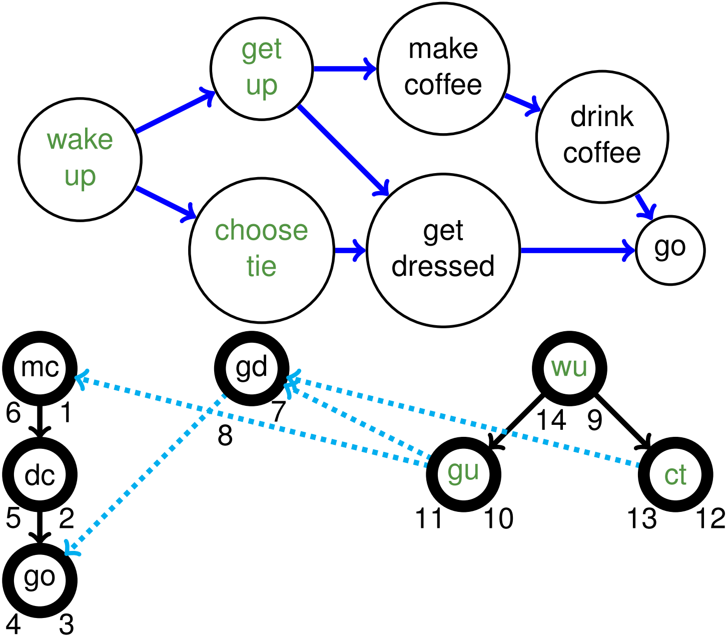
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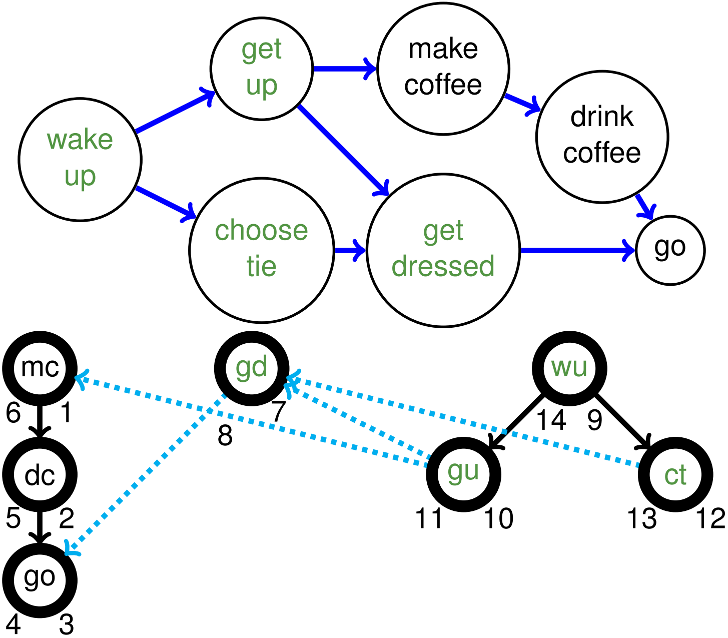

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