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iClicker: Is $E = G$?
A: yes, they both have the same element
B: no, G has two elements and E has only one element

Important Sets: **N**, **Z**, **Q**, **R**

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These will be used throughout CS250, **Please Memorize!**

Set-Builder Notation

$$S = \{t \in T \mid P(t)\}$$

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iClicker: How many elements does the set H have?
A: 3, B: 4, C: 5

Subsets

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iClicker: Which of the above containments are proper?

A: all of them

B: all except the first

C: all except the second

Cartesian Product

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- What is $|\mathbf{N}|$?

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Answers to Quiz R2

True or False wrt the following sets:

$$\begin{aligned} A &= \mathbf{Z}^{\text{nonneg}} \\ B &= \left\{ n \in \mathbf{Z} \;\middle|\; -5 \leq n \leq 5 \right\} \\ C &= \{2, 4, 6, 8, 10, ...\} \\ E &= \left\{ z \in A \;\middle|\; z \textbf{ mod } 2 = 0, \text{ i.e, } z \text{ is even} \right\} \end{aligned}$$

1. $B \subseteq A$: False: $-5 \in B - A$

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1. $B \subseteq A$: False: $-5 \in B - A$
2. $E \subseteq A$: True: E contains exactly the even elements of A.

Answers to Quiz R2

True or False wrt the following sets:

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3. C is a proper subset of E: True: $C = E - \{0\}$

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4. There exists $x \in E$ s.t. $x \in B$: True: $\{0, 2, 4\} \subseteq B \cap E$

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12. How many elements are in the set $\{a, \{a\}, \{\{a\}\}\}$?: 3: the listed elements are all distinct.

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16. For $A = \{a, b\}$ and $S = \{0, 1\}$, which of the following is **not** an element of $A \times S$? $(1, b), (a, 0), (a, 1), (b, 1)$:
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17. For $A = \{a, b\}$ and $S = \{0, 1\}$, is $A \times S = S \times A$? No, for example $(a, 0) \in A \times S \ - \ S \times A$.