Neural Language Models

CS 685, Spring 2025

Advanced Natural Language Processing

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many slides from Richard Socher and Matt Peters and Mohit Iyyer

Deadlines

- **2/17: Quiz 1 due**
- **2/14**: HW 0 due
- **2/14**: Final project group assignments due
- **3/7**: Project proposals due
- **5/9**: Final project reports due
- **5/9**: Last day to submit extra credit
  - This Wednesday, we will have another talk (https://nlp.cs.umass.edu/seminar/)

Project Rubric

- Rubric
  - Acceptable for main NLP conferences: 100
  - Acceptable for NLP workshops: 97
  - Having one of the noticeable great attributes among Effort, Novelty, Usefulness, and Implication: 95
  - Is a complete NLP scientific report: 90
    - Have some findings but also contain some major weaknesses: 85
    - Can see that students spend some effort on the project: 80
- Policy of one project for multiple courses
  - The instructor of the other course needs to agree
  - All your group members need to agree
  - You need to indicate that in the final report for all courses
    - Your effort needs to be proportional to the number of credits

Details are NOT Important

The key is generalization ability

👍 SFT

👍 Pretraining (Small LM) SFT

👍 Pretraining (Large LM) SFT

👍 Test Time Training

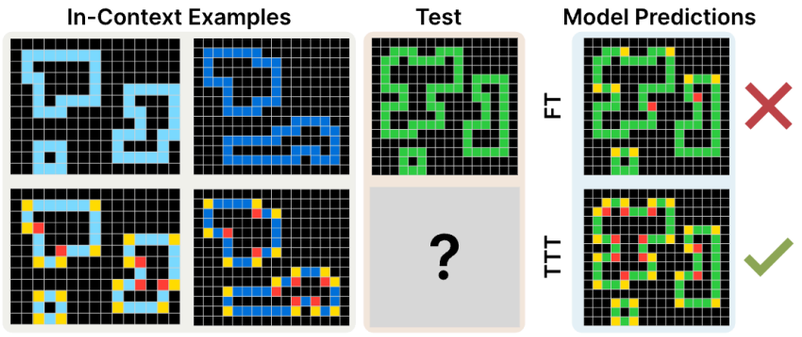
👎 Training on Testing data

Large

Small

Distance between training and testing

Akyürek, Ekin, et al. "The surprising effectiveness of test-time training for abstract reasoning." *arXiv preprint arXiv:2411.07279* (2024).(https://arxiv.org/abs/2411.07279)

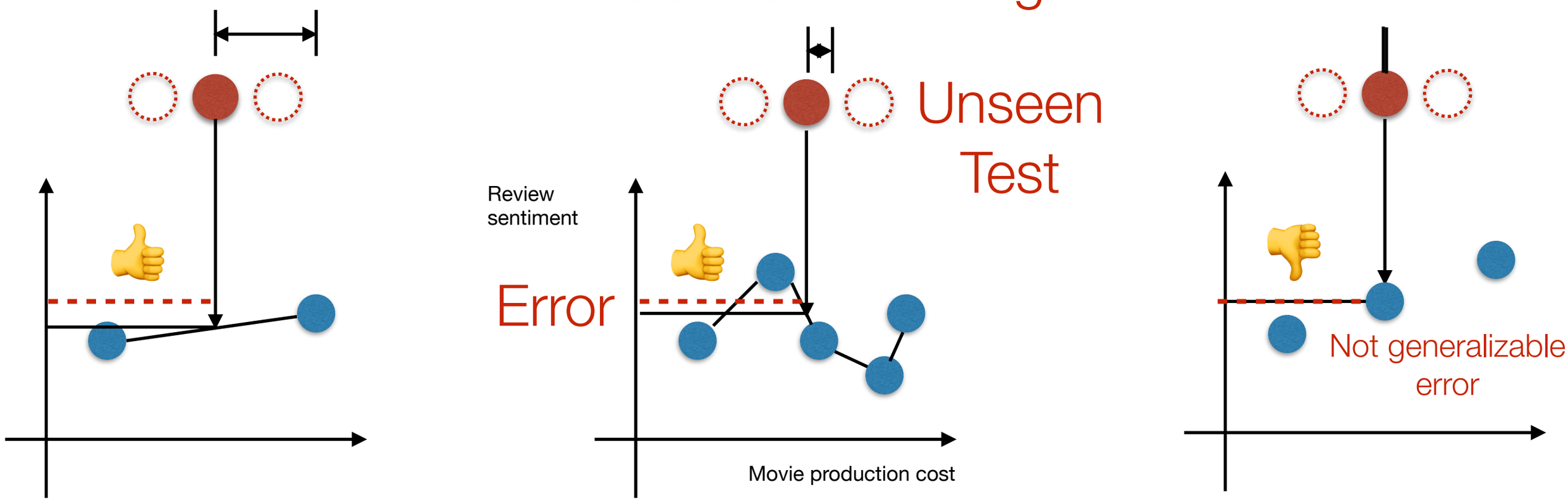


Data is the King for Maximizing Task Performances

FT

Pretraining
(Large LM)
FT

Training on
Testing data



Bitter Lesson (http://www.incompleteideas.net/IncIdeas/BitterLesson.html)

One of the most important concepts in this course, but More Data -> AGI? I personally disagree.

Multiple Token Prediction

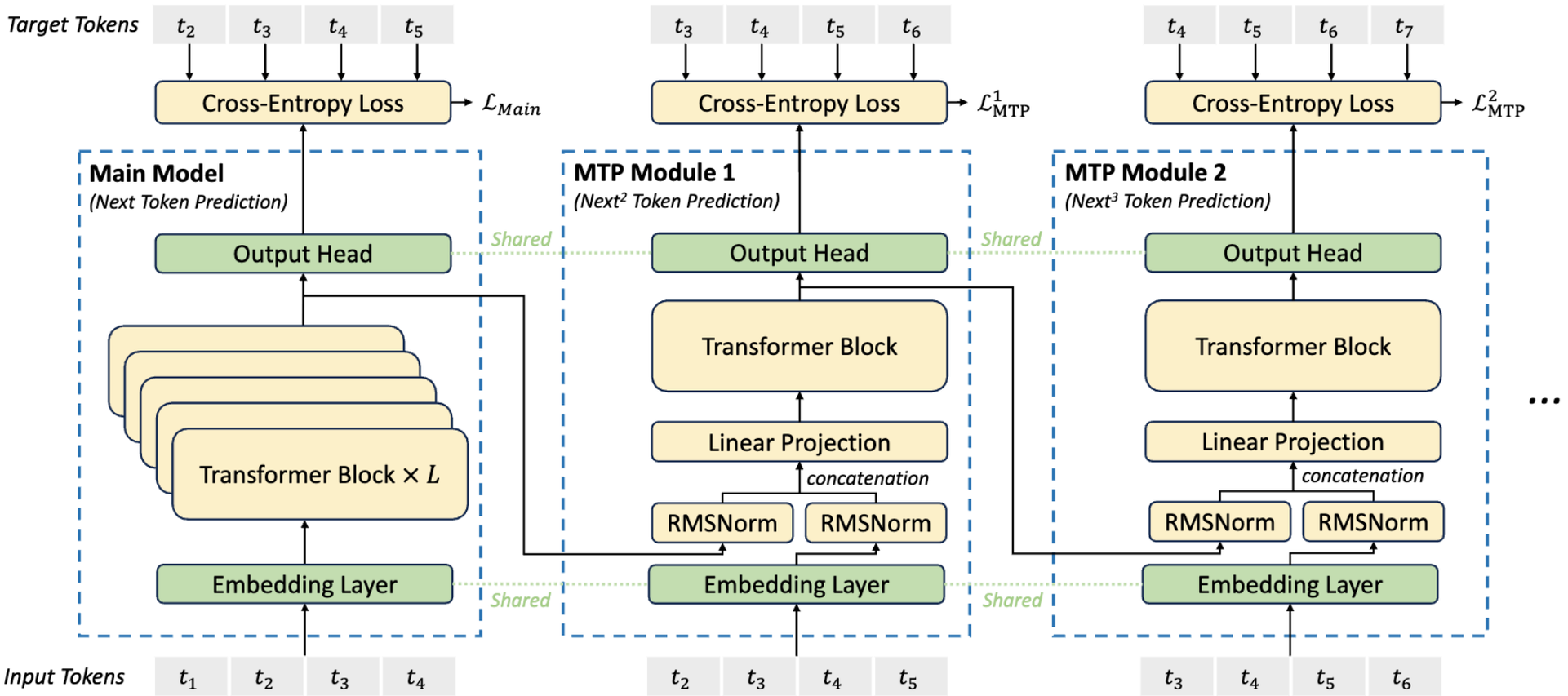


Figure 3 | Illustration of our Multi-Token Prediction (MTP) implementation. We keep the complete causal chain for the prediction of each token at each depth.

- DeepSeek V3 (https://arxiv.org/pdf/2412.19437v1)

Please Ask Questions if You don’t Understand

language model review

- Goal: compute the probability of a sentence or sequence of words:

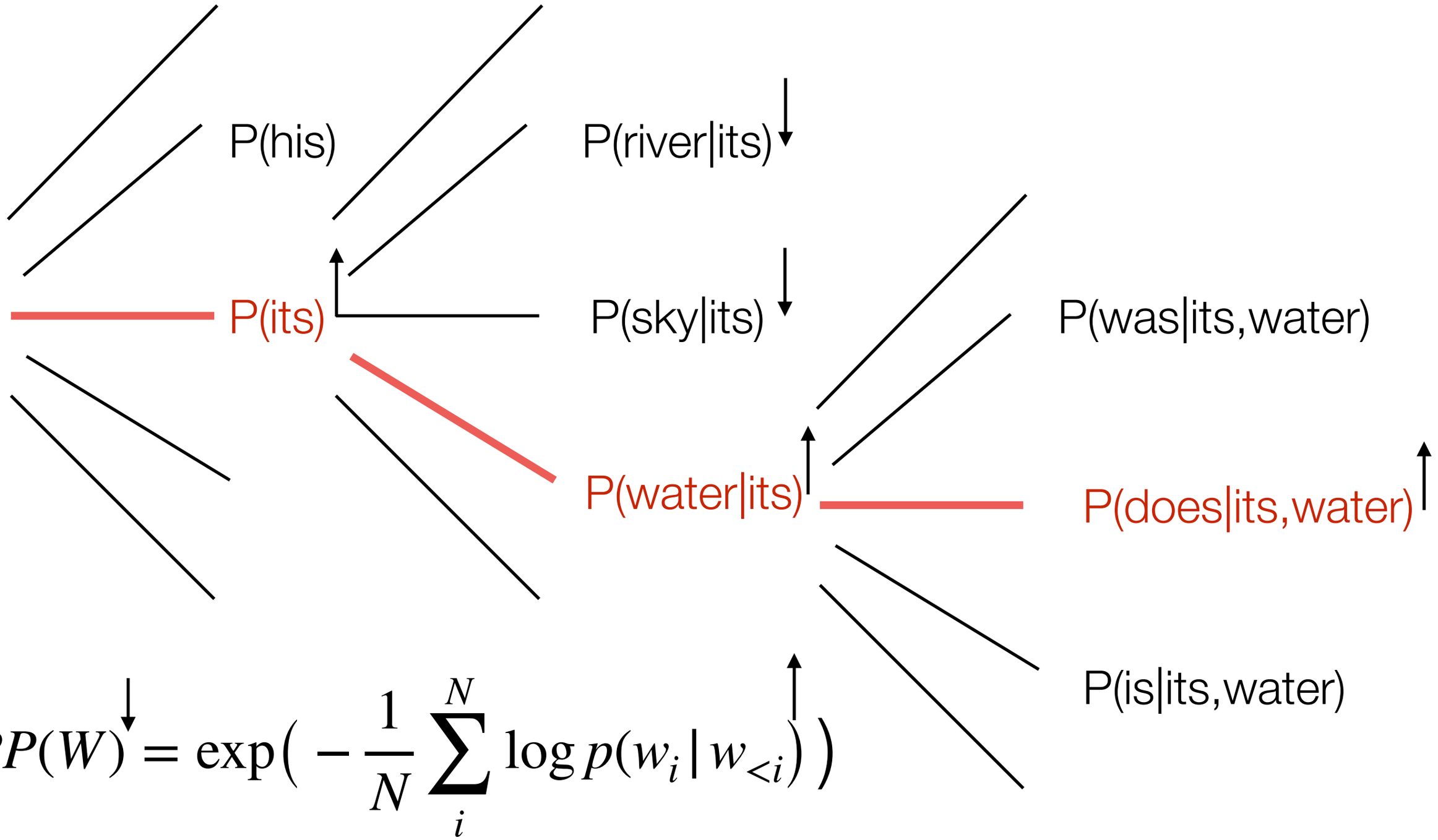
$$P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$$

- Related task: probability of an upcoming word:

$$P(w_5|w_1,w_2,w_3,w_4)$$

- A model that computes either of these:

$P(W)$ or $P(w_n|w_1,w_2 \ldots w_{n-1})$ is called a **language model** or **LM**



Testing data: **its water does**

n-gram models

$$p(w_j \mid \text{students opened their}) = \frac{\text{count(students opened their } w_j)}{\text{count(students opened their)}}$$

Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their* $\boldsymbol{w}_j$*"* never occurred in data? Then $\boldsymbol{w}_j$ has probability 0!

$$p(w_j \mid \text{students opened their}) = \frac{\text{count(students opened their } w_j)}{\text{count(students opened their)}}$$

Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their* w_j*"* never occurred in data? Then w_j has probability 0!

(Partial) Solution: Add small δ to count for every $w_j \in V$. This is called *smoothing*.

$$p(w_j \mid \text{students opened their}) = \frac{\text{count(students opened their } w_j)}{\text{count(students opened their)}}$$

Problems with n-gram Language Models

Storage: Need to store count for all possible *n*-grams. So model size is O(exp(*n*)).

$$P(\boldsymbol{w}_j|\text{students opened their}) = \frac{\text{count(students opened their } \boldsymbol{w}_j\text{)}}{\text{count(students opened their)}}$$

Increasing *n* makes model size huge!

another issue:

- We treat all words / prefixes independently of each other!

students opened their ___

pupils opened their ___

scholars opened their ___

undergraduates opened their ___

students turned the pages of their ___

students attentively perused their ___

…

Shouldn't we *share information* across these semantically-similar prefixes?

one-hot vectors

- n-gram models rely on the “bag-of-words” assumption
- represent each word/n-gram as a vector of zeros with a single 1 identifying its index in the vocabulary

vocabulary
i
hate
love
the
movie
film

movie = <0, 0, 0, 0, 1, 0>

film = <0, 0, 0, 0, 0, 1>

what are the issues of representing a word this way?

all words are equally (dis)similar!

movie = <0, 0, 0, 0, 1, 0>

film = <0, 0, 0, 0, 0, 1>

dot product is zero!

these vectors are **orthogonal**

What we want is a representation space in which words, phrases, sentences etc. that are semantically similar also have similar representations!

Task -> Loss -> Model -> Optimization

- Task:
  - Predict the next token
  - Prior to 2018, only a few NLP researchers studied LMs
  - Looking for ELMo's friends: Sentence-Level Pretraining Beyond Language Modeling (https://openreview.net/forum?id=Bkl87h09FX)
- Loss:
  - Maximal Likelihood / Cross-entropy
- Model:
  - Tables -> **Neural Network** -> Transformer
- Optimization:
  - Counting -> Gradient Descent

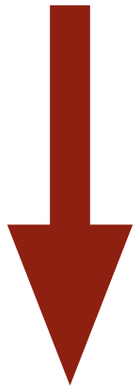
Enter neural networks!

Students opened their

books

Enter neural networks!

Students opened their



neural language model

books

This lecture: the *forward pass*, or how we compute a prediction of the next word given an existing neural language model

Enter neural networks!

Students opened their

neural language model

books

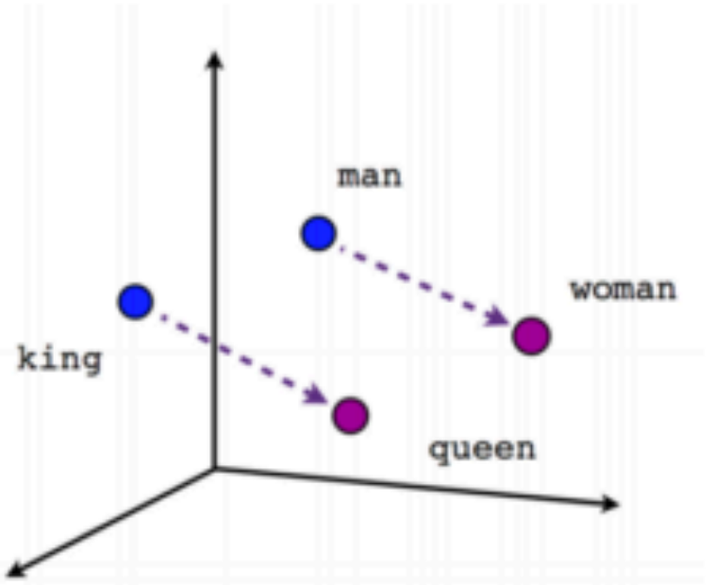
This lecture: the *forward pass*, or how we compute a prediction of the next word given an existing neural language model

Next lecture: the *backward pass*, or how we train a neural language model on a training dataset using the backpropagation algorithm

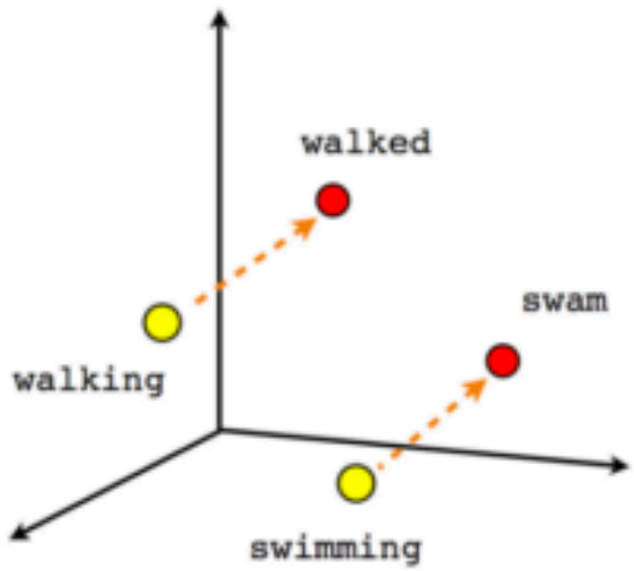
words as basic building blocks

- represent words with low-dimensional vectors called **embeddings** (Mikolov et al., NIPS 2013)

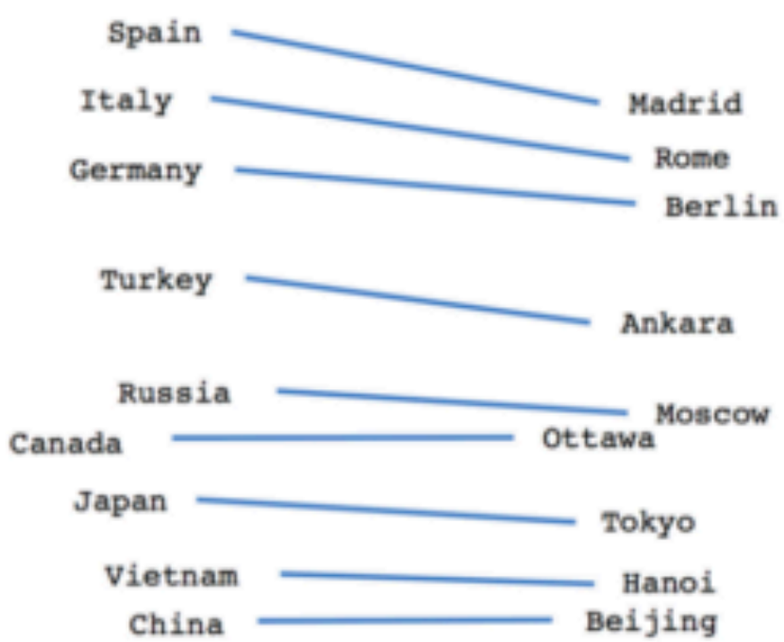
king =
[0.23, 1.3, -0.3, 0.43]



Male-Female



Verb tense



Country-Capital

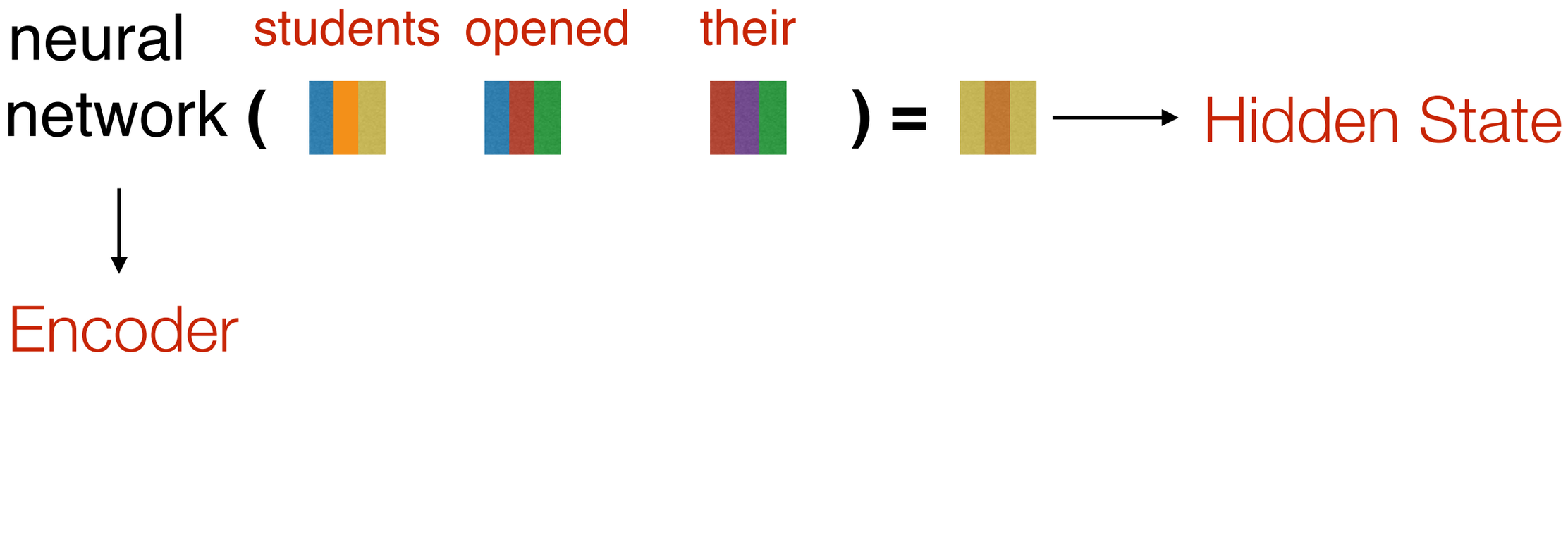
```
model['nuclear']
```

```
array([ 0.58108 ,  0.66825 ,  1.0771  ,  0.34879 , -0.34613 ,  0.20463 ,
        0.78436 ,  0.11287 ,  0.77594 ,  0.43579 ,  0.18566 , -0.20375 ,
       -0.53369 ,  0.55578 , -0.099609,  1.1739  ,  0.83277 ,  1.2848  ,
       -0.19772 ,  0.41573 ,  1.1255  , -0.31634 ,  0.22493 , -1.0348  ,
        0.28462 , -2.7709  ,  0.80654 ,  0.24704 ,  0.64272 ,  0.41439 ,
        2.4058  , -1.1552  , -1.3758  , -0.90799 ,  0.20109 , -0.29947 ,
        0.10769 ,  0.29975 , -0.94256 ,  0.26281 , -0.17048 , -1.1831  ,
        0.99454 , -0.50074 ,  1.0424  ,  0.8123  , -0.20606 ,  1.9433  ,
       -1.2817  , -0.49774 ])
```

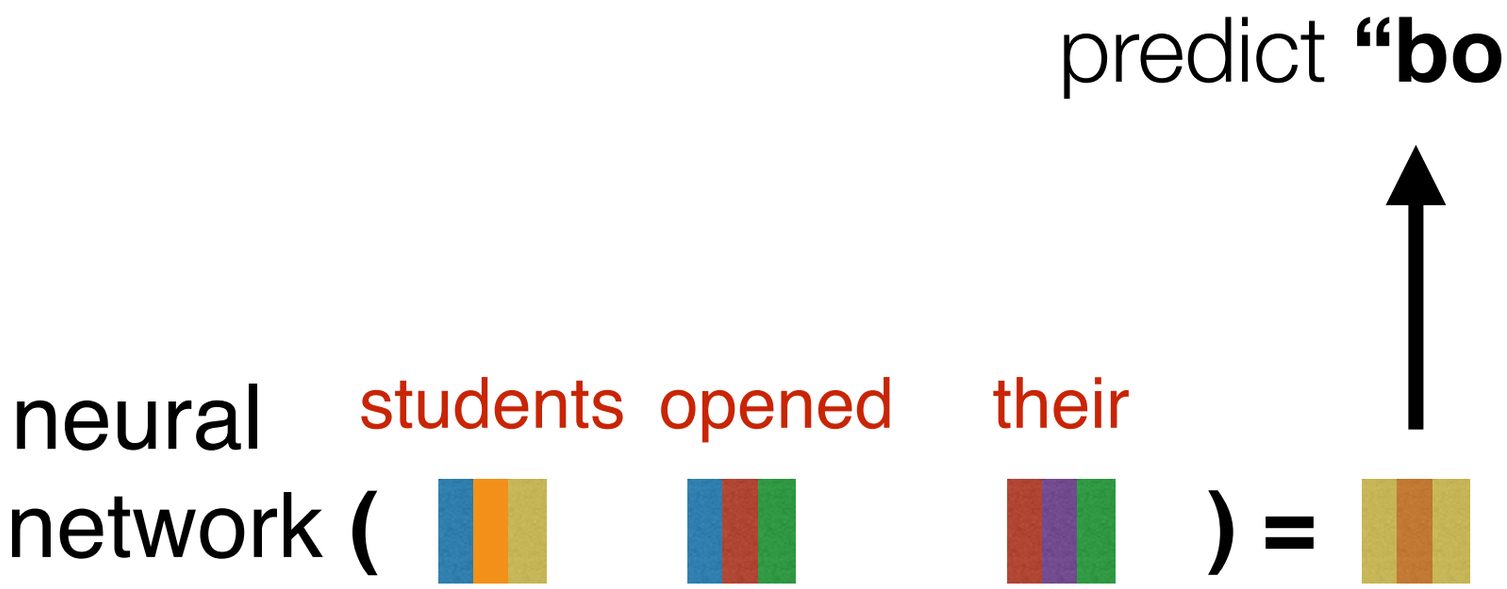
- Each dimension is not an attribute
  - If you force non-negative embeddings during training, it will have meaning
  - **Linear combination of dimensions could reveal attributes**

composing embeddings

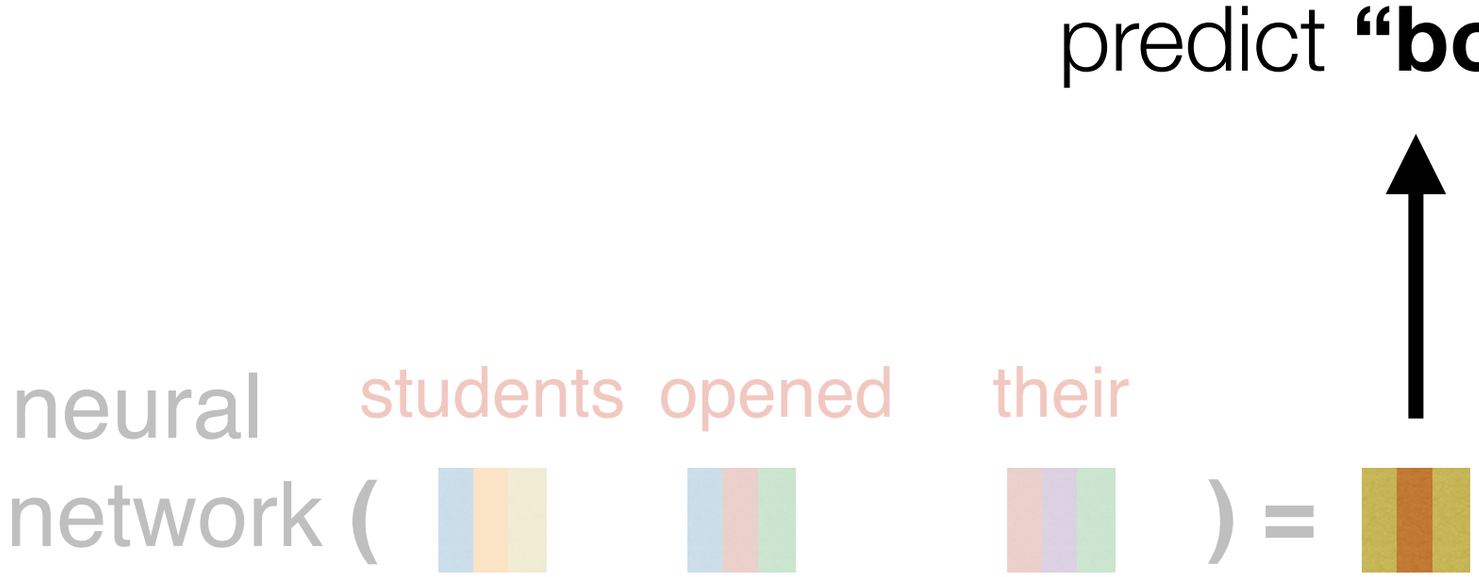
- neural networks **compose** word embeddings into vectors for phrases, sentences, and documents



Predict the next word from composed prefix representation



How does this happen? Let's work our way backwards, starting with the prediction of the next word

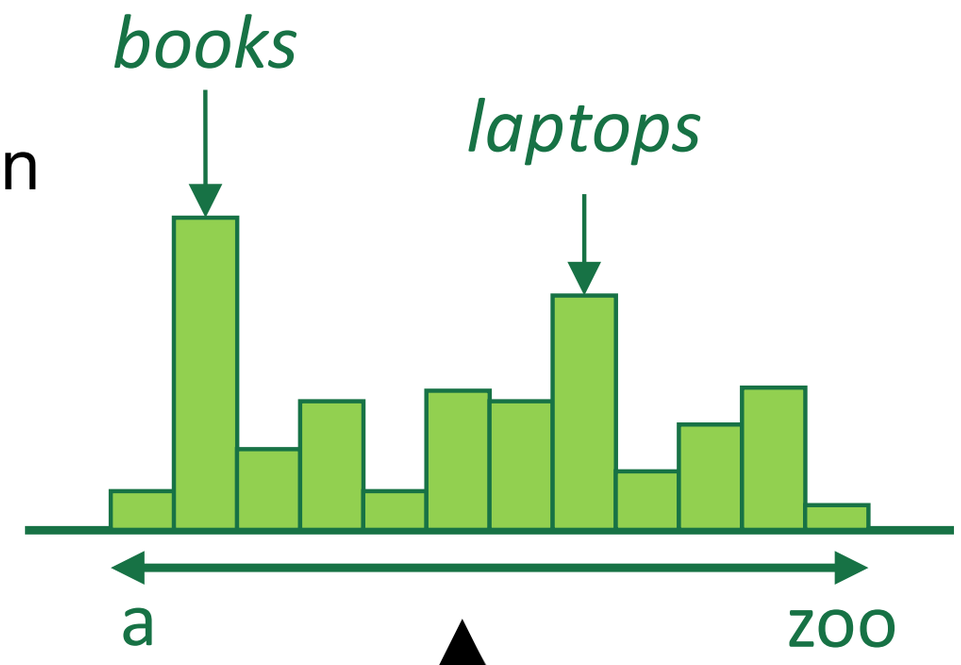


How does this happen? Let's work our way backwards, starting with the prediction of the next word

predict **"books"**

Softmax layer:
convert a vector representation into a probability distribution over the entire vocabulary

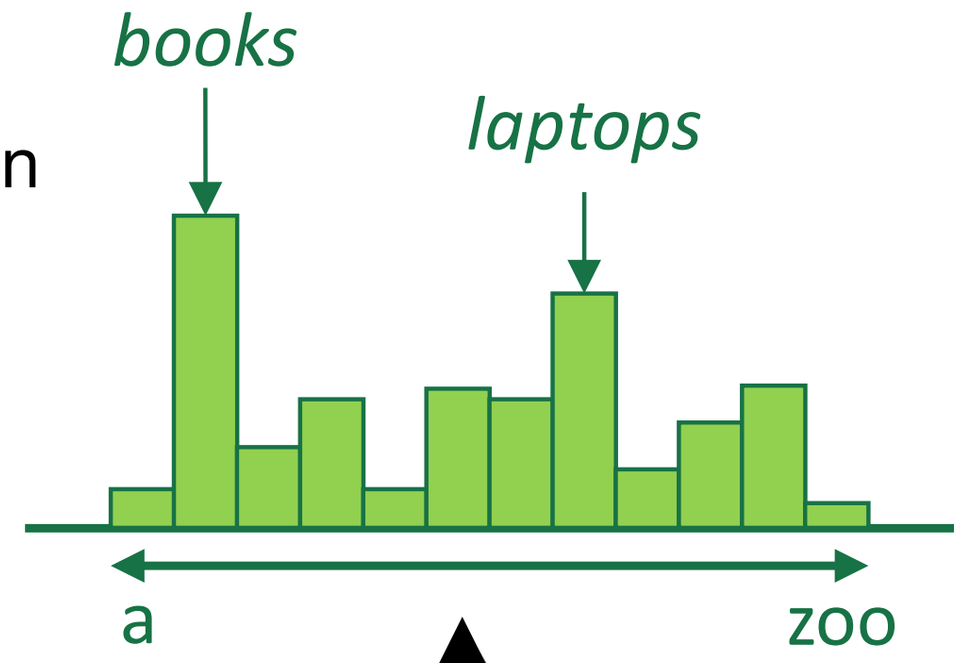
Probability distribution over the entire vocabulary



Low-dimensional representation of “students opened their”

$P(w_i \mid \text{vector for "students opened their"})$

Probability distribution over the entire vocabulary



Low-dimensional representation of "students opened their"

Let’s say our output vocabulary consists of just four words: “books”, “houses”, “lamps”, and “stamps”.

Low-dimensional representation of “students opened their”

Let's say our output vocabulary consists of just four words: "books", "houses", "lamps", and "stamps".

books houses lamps stamps

<0.6, 0.2, 0.1, 0.1>

We want to get a probability distribution over these four words

Low-dimensional representation of "students opened their"

$\boldsymbol{x}$ = <-2.3, 0.9, 5.4>

Here's an example 3-d prefix vector

W is a *weight matrix*. It contains *parameters* that we can *update* to control the final probability distribution of the next word

$$\mathbf{W} = \begin{Bmatrix} 1.2, & -0.3, & 0.9 \\ 0.2, & 0.4, & -2.2 \\ 8.9, & -1.9, & 6.5 \\ 4.5, & 2.2, & -0.1 \end{Bmatrix}$$

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first, we'll project our 3-d prefix representation to 4-d with a matrix-vector product

$$\boldsymbol{x} = \langle -2.3, 0.9, 5.4 \rangle$$

Here's an example 3-d prefix vector

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$\boldsymbol{x}$ = <-2.3, 0.9, 5.4>

intuition: each dimension of $\boldsymbol{x}$ corresponds to a *feature* of the prefix

intuition: each row of **W** contains *feature weights* for a corresponding word in the vocabulary

$$\mathbf{W} = \begin{Bmatrix} 1.2, & -0.3, & 0.9 \\ 0.2, & 0.4, & -2.2 \\ 8.9, & -1.9, & 6.5 \\ 4.5, & 2.2, & -0.1 \end{Bmatrix}$$

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CAUTION: we can't easily *interpret* these features! For example, the second dimension of ***x*** likely does not correspond to any linguistic property

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$$\boldsymbol{x} = \langle -2.3, 0.9, 5.4 \rangle$$

intuition: each dimension of ***x*** corresponds to a *feature* of the prefix

$$\mathbf{W}\boldsymbol{x} = \langle 1.8,\ -11.9,\ 12.9,\ -8.9 \rangle$$

How did we compute this? It's just the dot product of each row of **W** with ***x***!

$$\mathbf{W} = \begin{Bmatrix} 1.2, & -0.3, & 0.9 \\ 0.2, & 0.4, & -2.2 \\ 8.9, & -1.9, & 6.5 \\ 4.5, & 2.2, & -0.1 \end{Bmatrix}$$

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W*x* = <1.8, -11.9, 12.9, -8.9>

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1.2 * -2.3
+ -0.3 * 0.9
+ 0.9 * 5.4

x = <-2.3, 0.9, 5.4>

Some LMs uses Wx + b,
but it doesn't matter. Why?

Okay, so how do we go from this 4-d vector to a probability distribution?

$\mathbf{W}\boldsymbol{x}$ = <1.8, -11.9, 12.9, -8.9>

We'll use the **softmax** function!

$$\text{softmax}(Wx) = \frac{e^{Wx}}{\sum_j e^{[Wx]_j}}$$

PyTorch softmax function is more numerically stable

- Wx is a vector
- $[Wx]_j$ is dimension *j* of Wx
- each dimension *j* of the softmaxed output represents the probability of class *j*

W*x* = <1.8, -1.9, 2.9, -0.9> ⟶ Logit

softmax(W*x*) = <0.24, 0.006, 0.73, 0.02>

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books houses lamps stamps

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softmax(W*x*) = <0.24, 0.006, 0.73, 0.02>

We'll see the softmax function over and over again this semester, so be sure to understand it!

so to sum up…

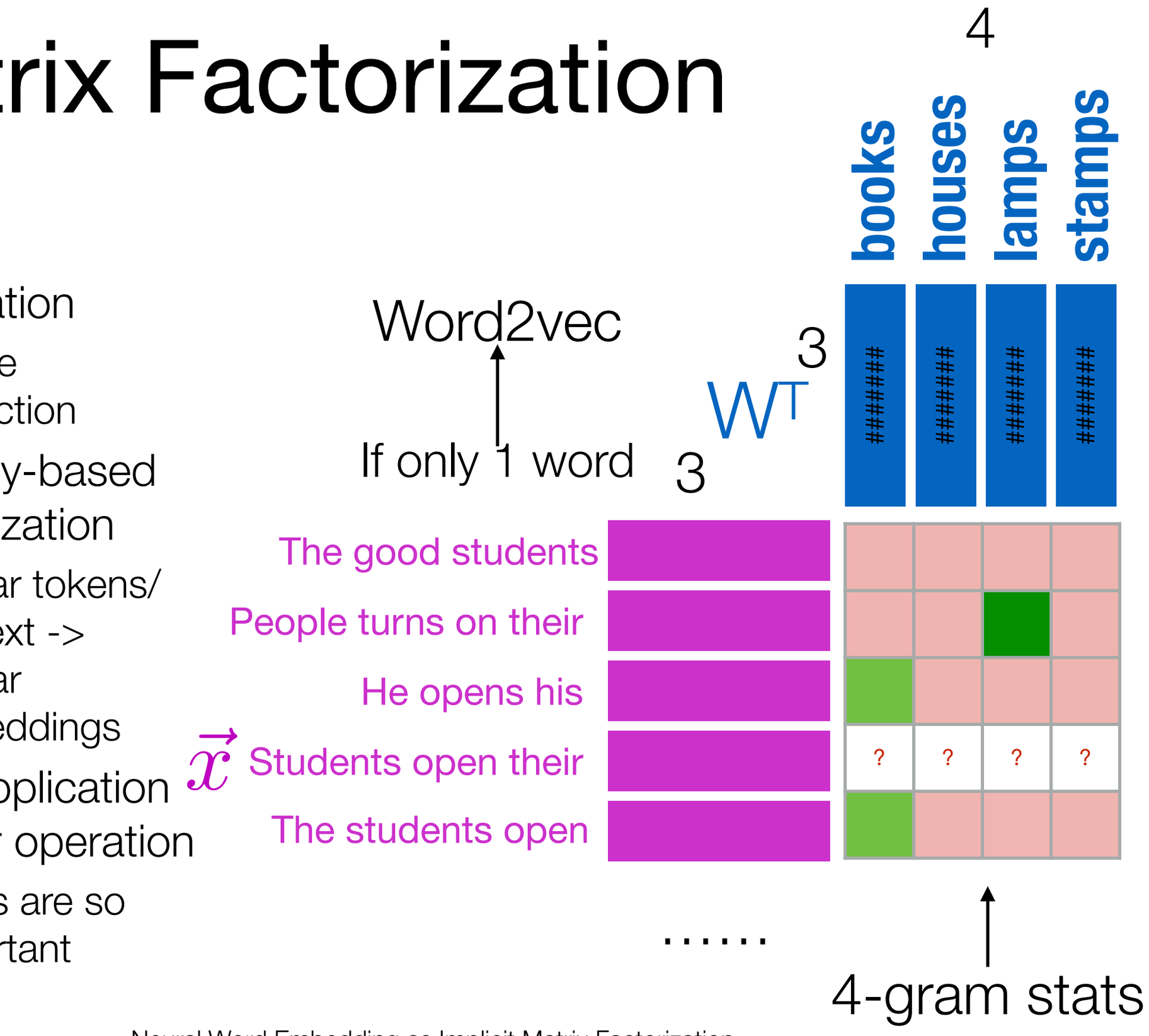
- Given a *d*-dimensional vector representation ***x*** of a prefix, we do the following to predict the next word:

1. Project it to a *V*-dimensional vector using a matrix-vector product (a.k.a. a “linear layer”, or a “feedforward layer”), where *V* is the size of the vocabulary

2. Apply the softmax function to transform the resulting vector into a probability distribution

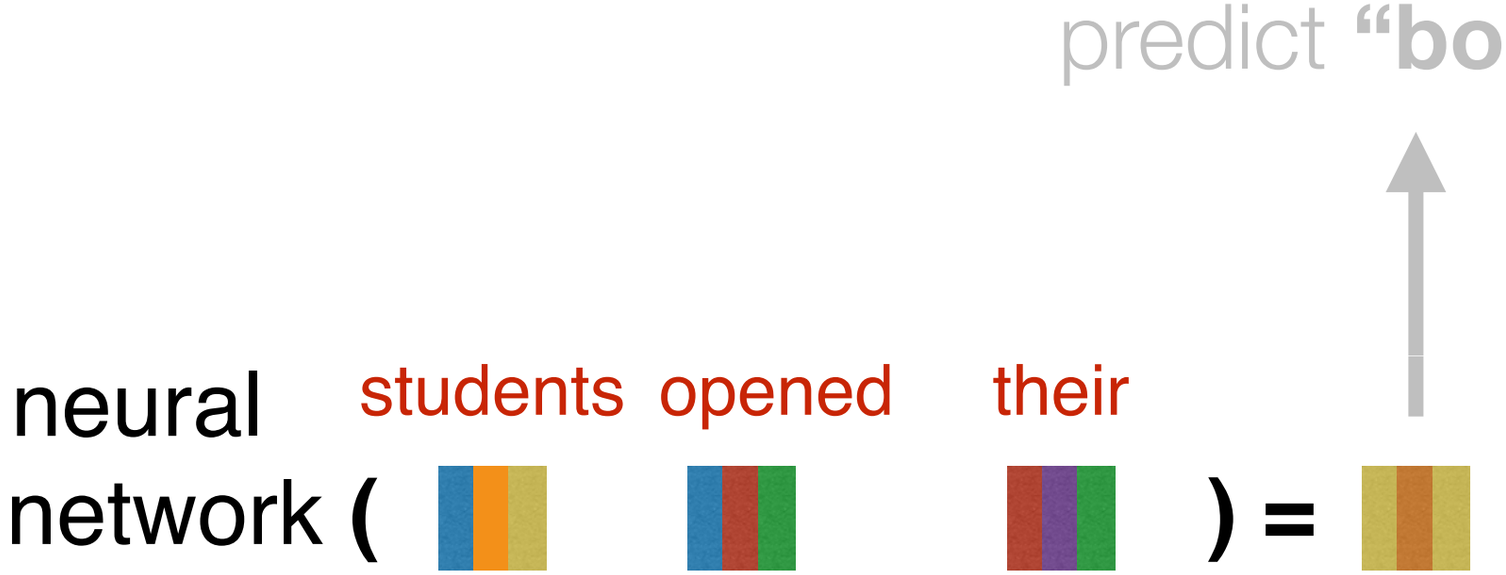
Matrix Factorization

- Sparse observation
  - Dense prediction
- Similarity-based generalization
  - Similar tokens/ context -> Similar embeddings
- Wide application of linear operation
  - GPUs are so important



Neural Word Embedding as Implicit Matrix Factorization
(https://proceedings.neurips.cc/paper_files/paper/2014/file/feab05aa91085b7a8012516bc3533958-Paper.pdf)

Now that we know how to predict **“books”**, let’s focus on how to compute the prefix representation ***x*** in the first place!



Composition functions

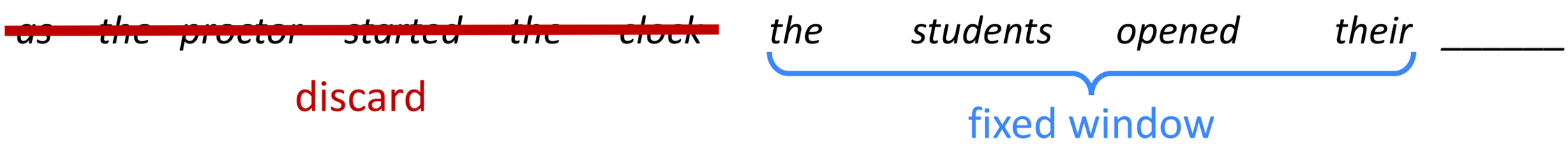
input: sequence of word embeddings corresponding to the tokens of a given prefix

output: single vector

- Element-wise functions
  - e.g., just sum up all of the word embeddings!
- Concatenation
- Feed-forward neural networks
- Convolutional neural networks
- Recurrent neural networks
- Transformers (our focus this semester)

Let’s look first at *concatenation* + *feedforward NN*, an easy to understand but limited composition function

A fixed-window neural Language Model



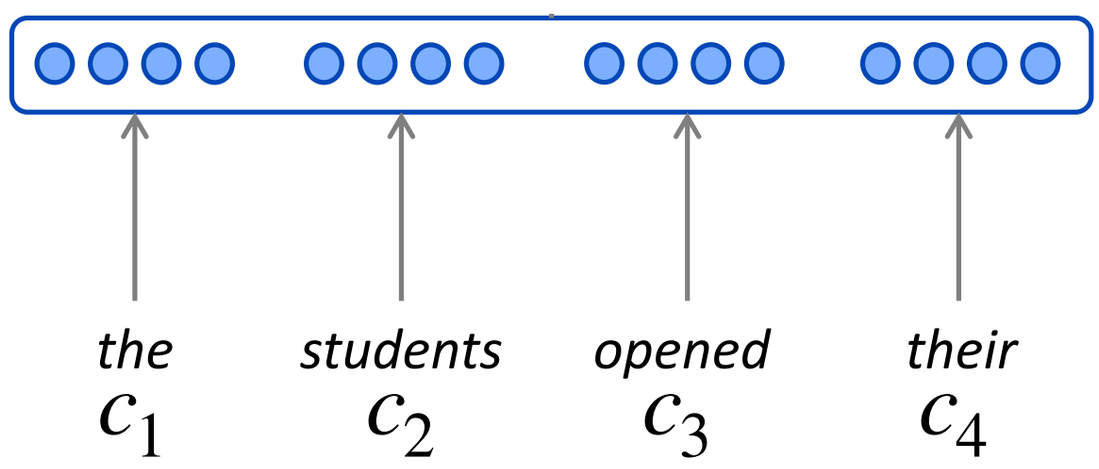
A fixed-window neural Language Model

concatenated word embeddings

$$x = [c_1; c_2; c_3; c_4]$$

words / one-hot vectors

$$c_1, c_2, c_3, c_4$$



A fixed-window neural Language Model

hidden layer

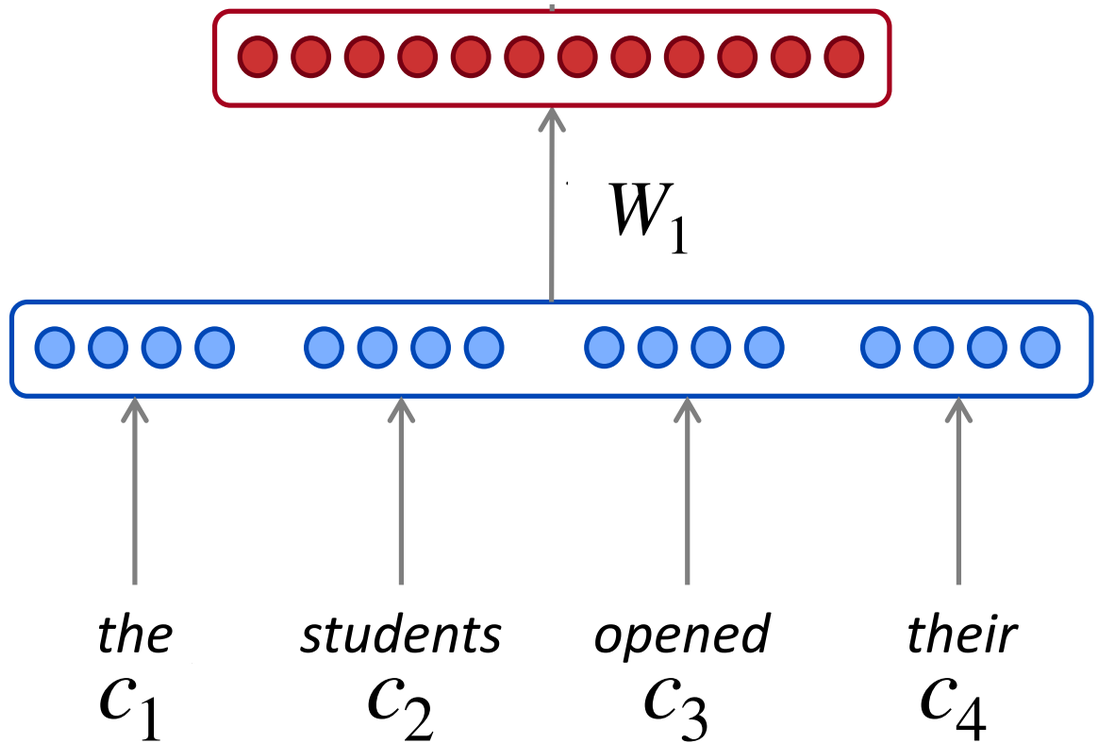
$$h = f(W_1 x)$$

concatenated word embeddings

$$x = [c_1; c_2; c_3; c_4]$$

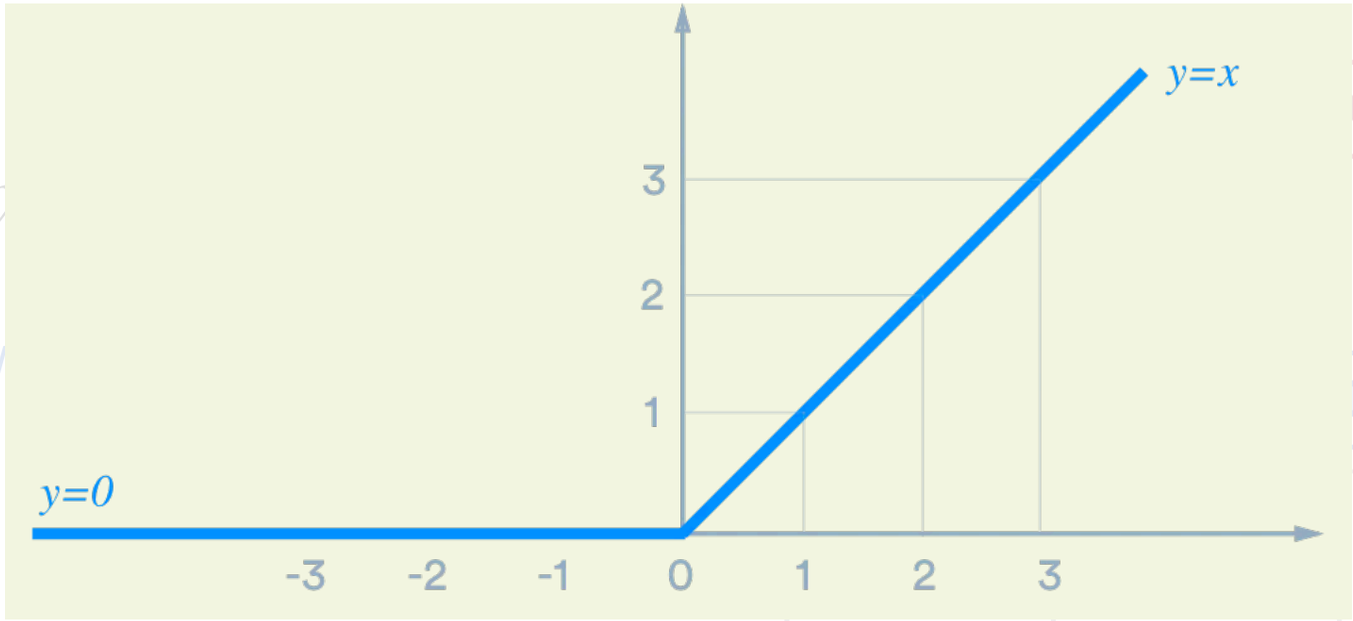
words / one-hot vectors

$$c_1, c_2, c_3, c_4$$



A fixed-window neural Language Model

f is a *nonlinearity*, or an element-wise nonlinear function.
The most commonly-used choice today is the rectified linear unit (**ReLu**), which is just **ReLu**(x) = max(0, x).
Other choices include **tanh** and **sigmoid**.



hidden layer

$h = f(W_1$

concatenated v

$x = [c_1;$

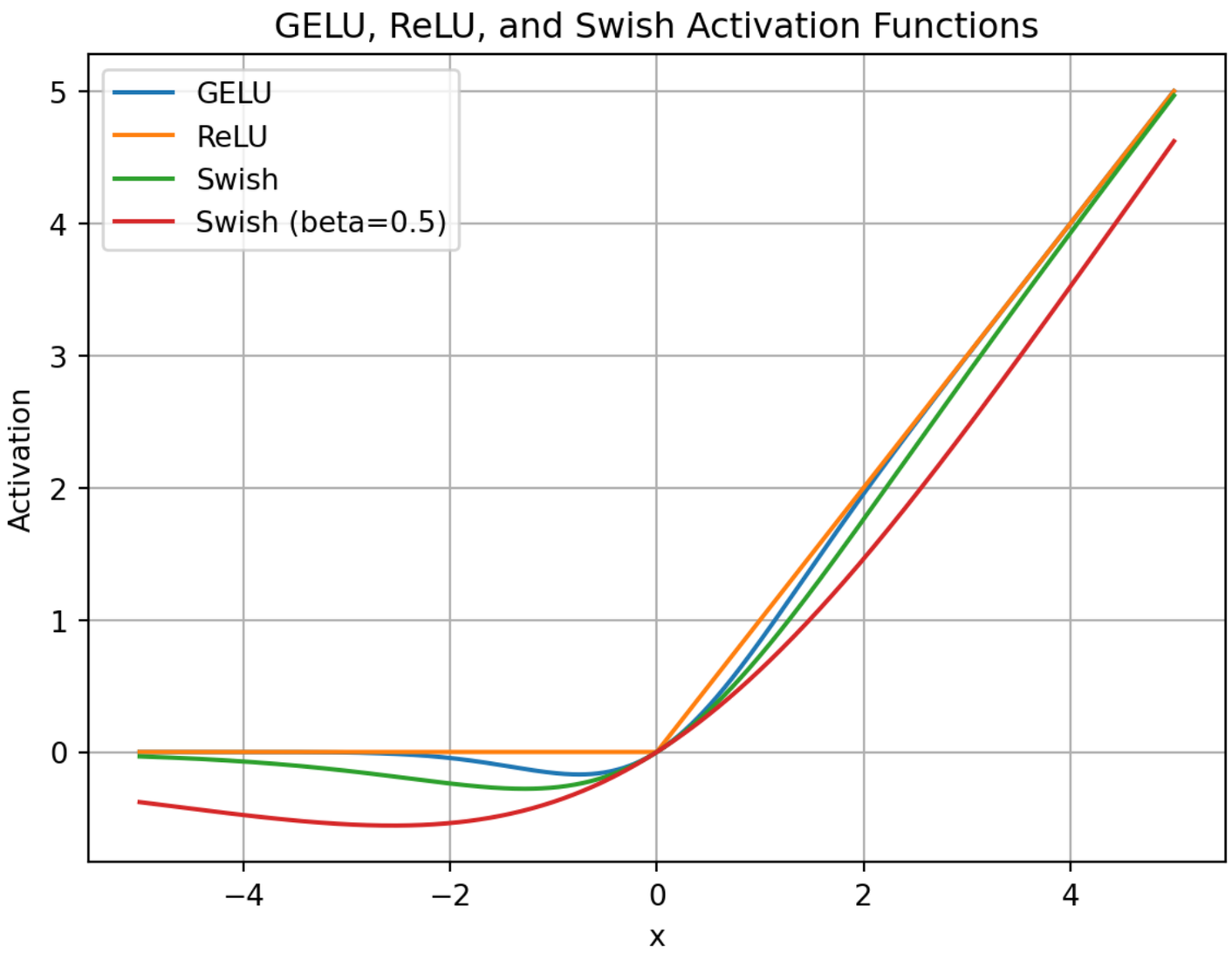
words / one-hot vectors

c_1, c_2, c_3, c_4

the *students* *opened* *their*

c_1 c_2 c_3 c_4

Activation Function Matter



Training Steps	65,536	524,288
$\text{FFN}_{\text{ReLU}}(baseline)$	1.997 (0.005)	1.677
FFN_{GELU}	1.983 (0.005)	1.679
$\text{FFN}_{\text{Swish}}$	1.994 (0.003)	1.683
FFN_{GLU}	1.982 (0.006)	1.663
$\text{FFN}_{\text{Bilinear}}$	1.960 (0.005)	1.648
$\text{FFN}_{\text{GEGLU}}$	**1.942** (0.004)	**1.633**
$\text{FFN}_{\text{SwiGLU}}$	**1.944** (0.010)	**1.636**
$\text{FFN}_{\text{ReGLU}}$	1.953 (0.003)	1.645

https://arxiv.org/pdf/2002.05202v1

A fixed-window neural Language Model

output distribution

$$\hat{y} = \text{softmax}(W_2 h)$$

hidden layer

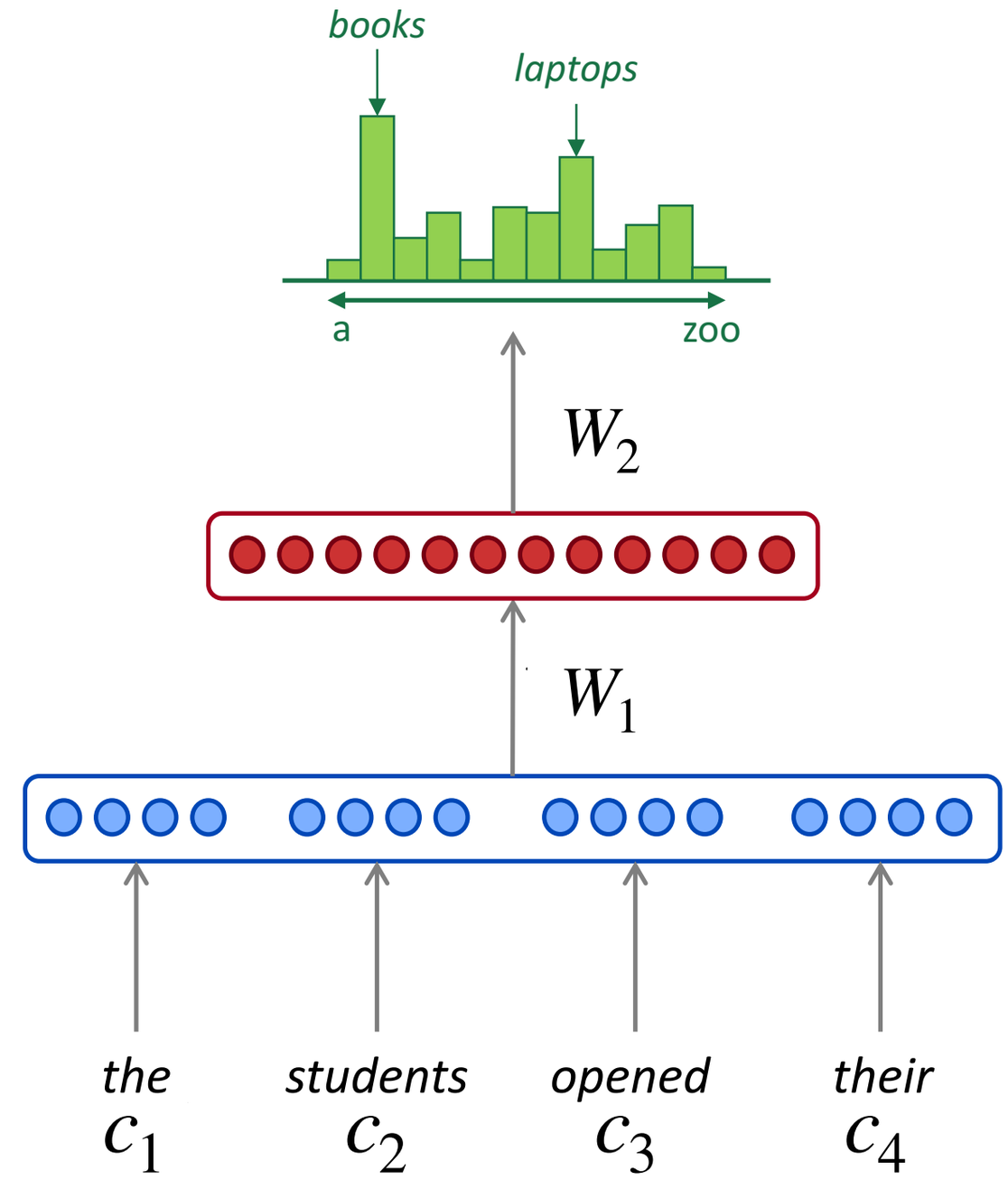
$$h = f(W_1 x)$$

concatenated word embeddings

$$x = [c_1; c_2; c_3; c_4]$$

words / one-hot vectors

c_1, c_2, c_3, c_4



This is also called Feedforward NN , MLP (multilayer perceptron), or fully connected network

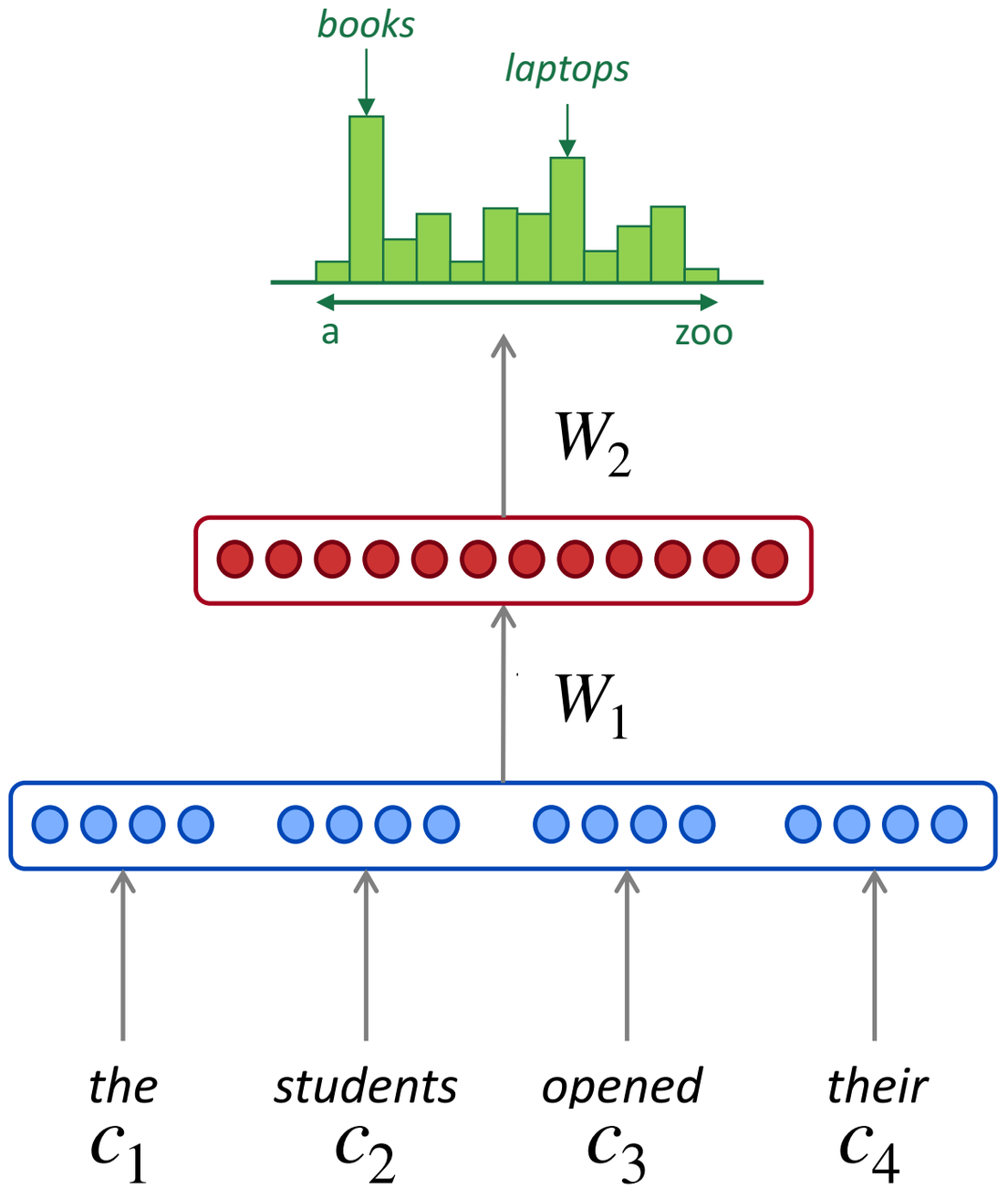
how does this compare to a normal n-gram model?

Improvements over *n*-gram LM:

- No sparsity problem
- Model size is O(*n*) not O(exp(*n*))

Remaining **problems**:

- Fixed window is too small
- Enlarging window enlarges $\boldsymbol{W}$
- Window can never be large enough!
- Each c_i uses different rows of $\boldsymbol{W}$. We don't share weights across the window.

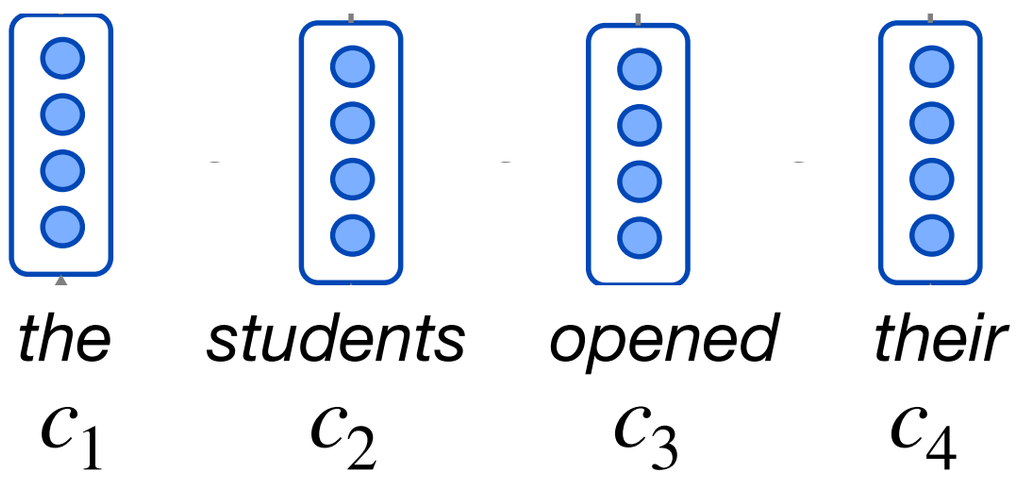


Recurrent Neural Networks!

A RNN Language Model

word embeddings

$$c_1, c_2, c_3, c_4$$



A RNN Language Model

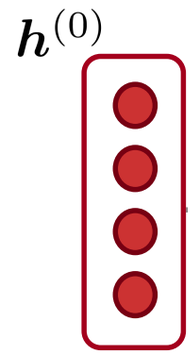
hidden states

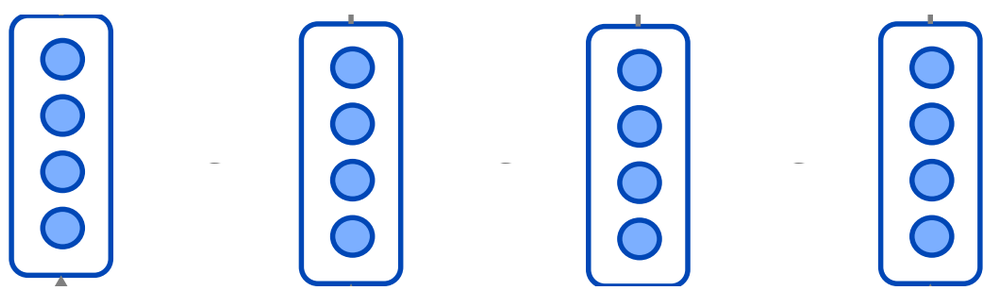
$$h^{(t)} = f(W_h h^{(t-1)} + W_e c_t)$$

h(0) is initial hidden state!

word embeddings

$$c_1, c_2, c_3, c_4$$





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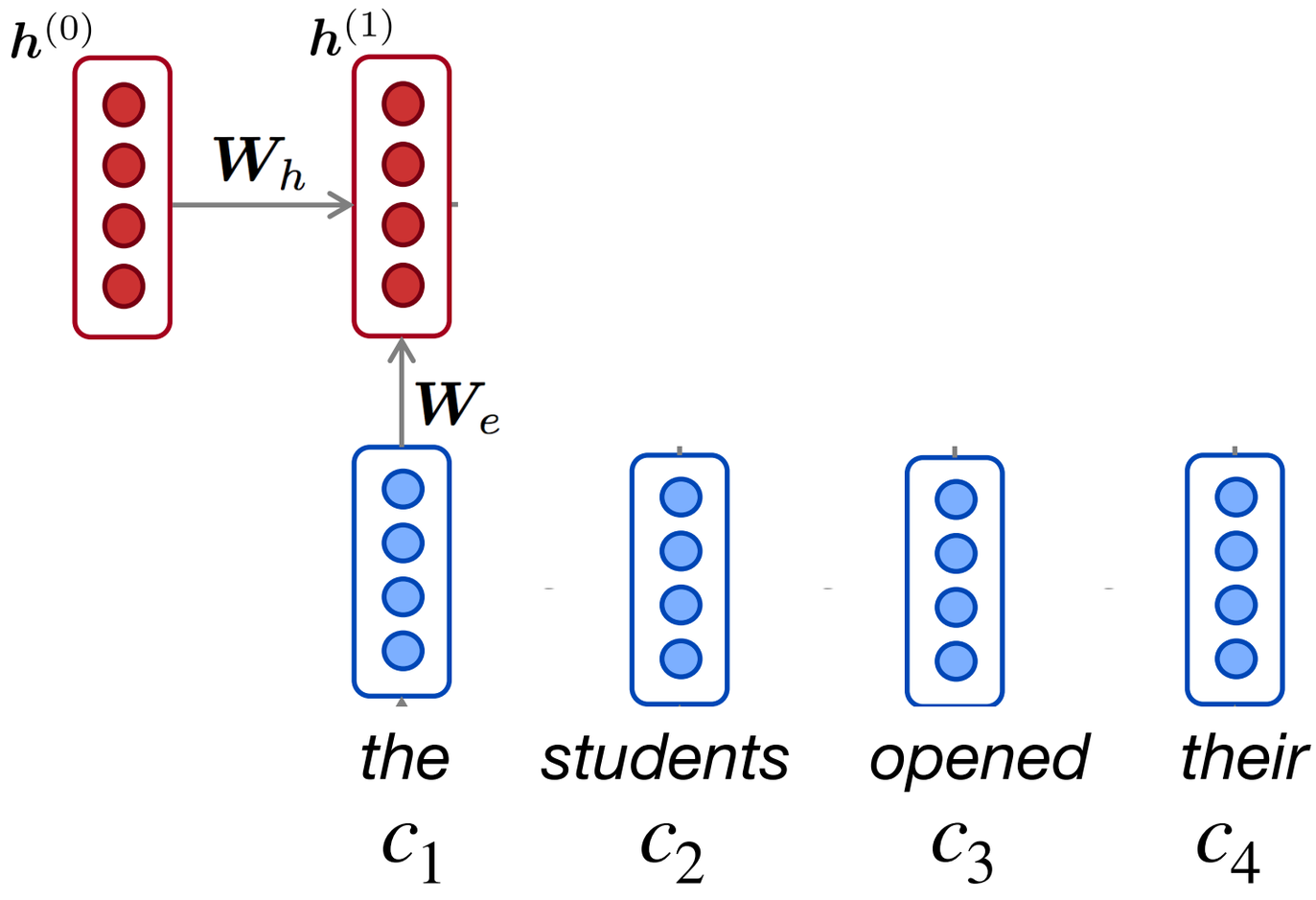

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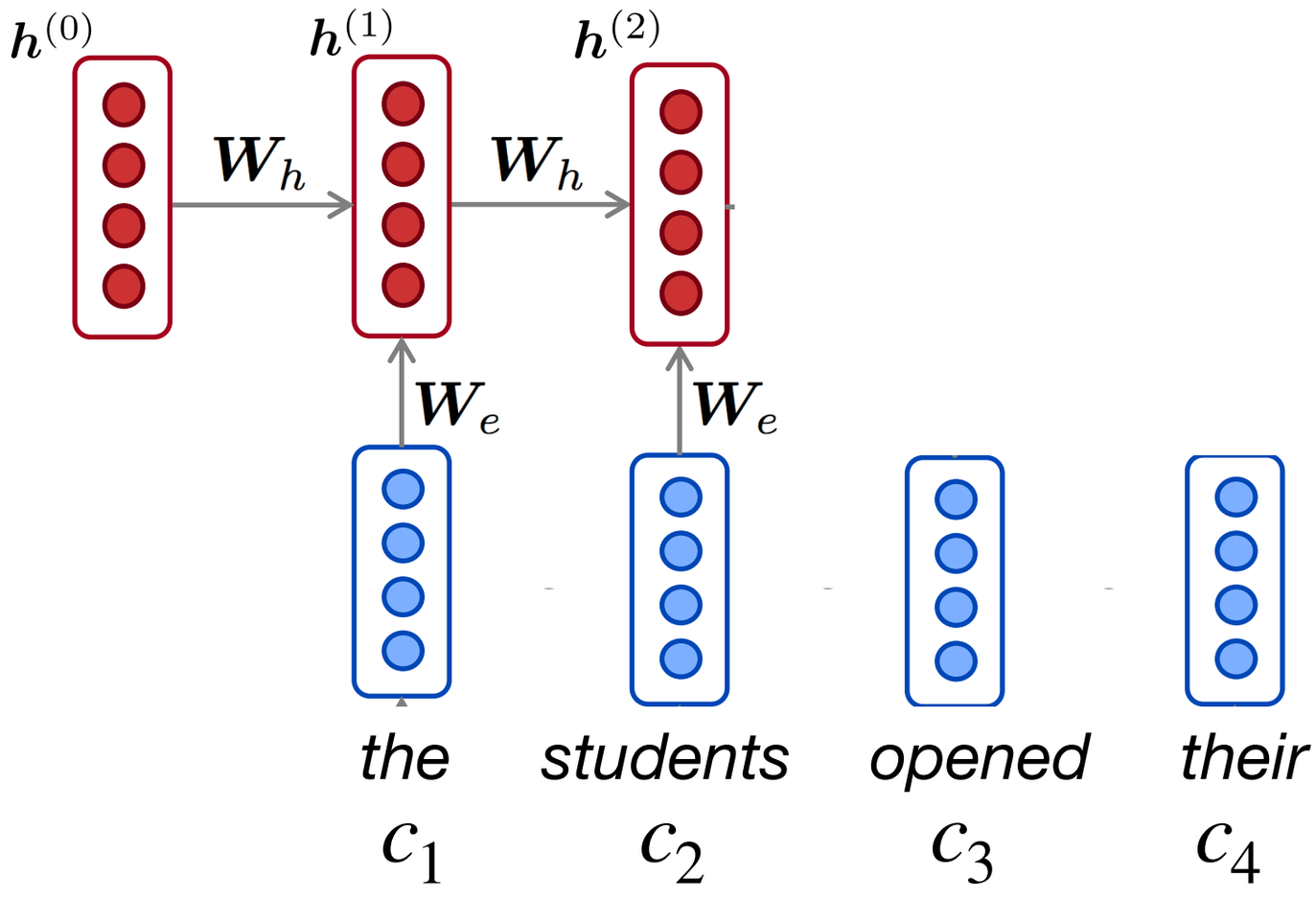

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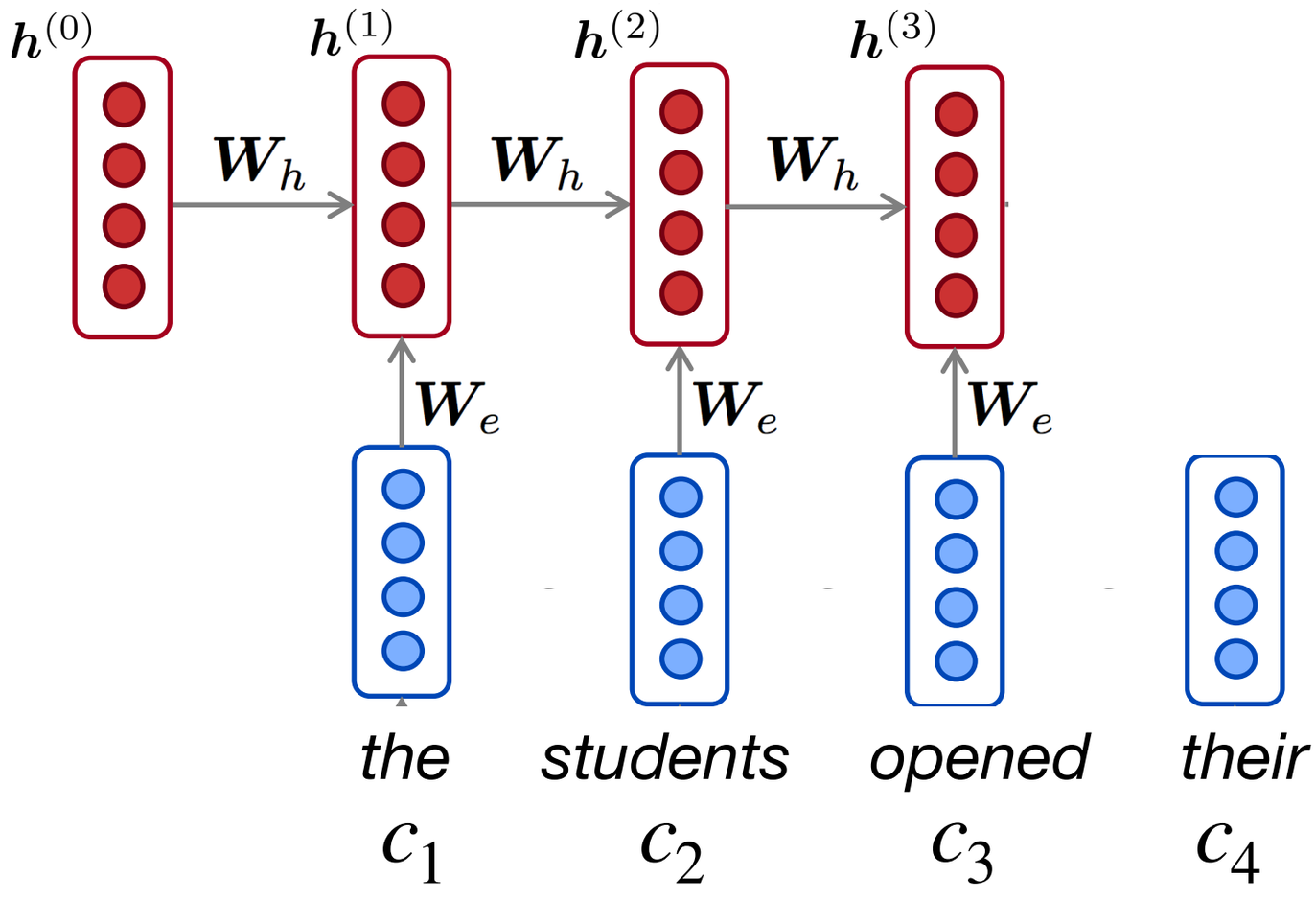

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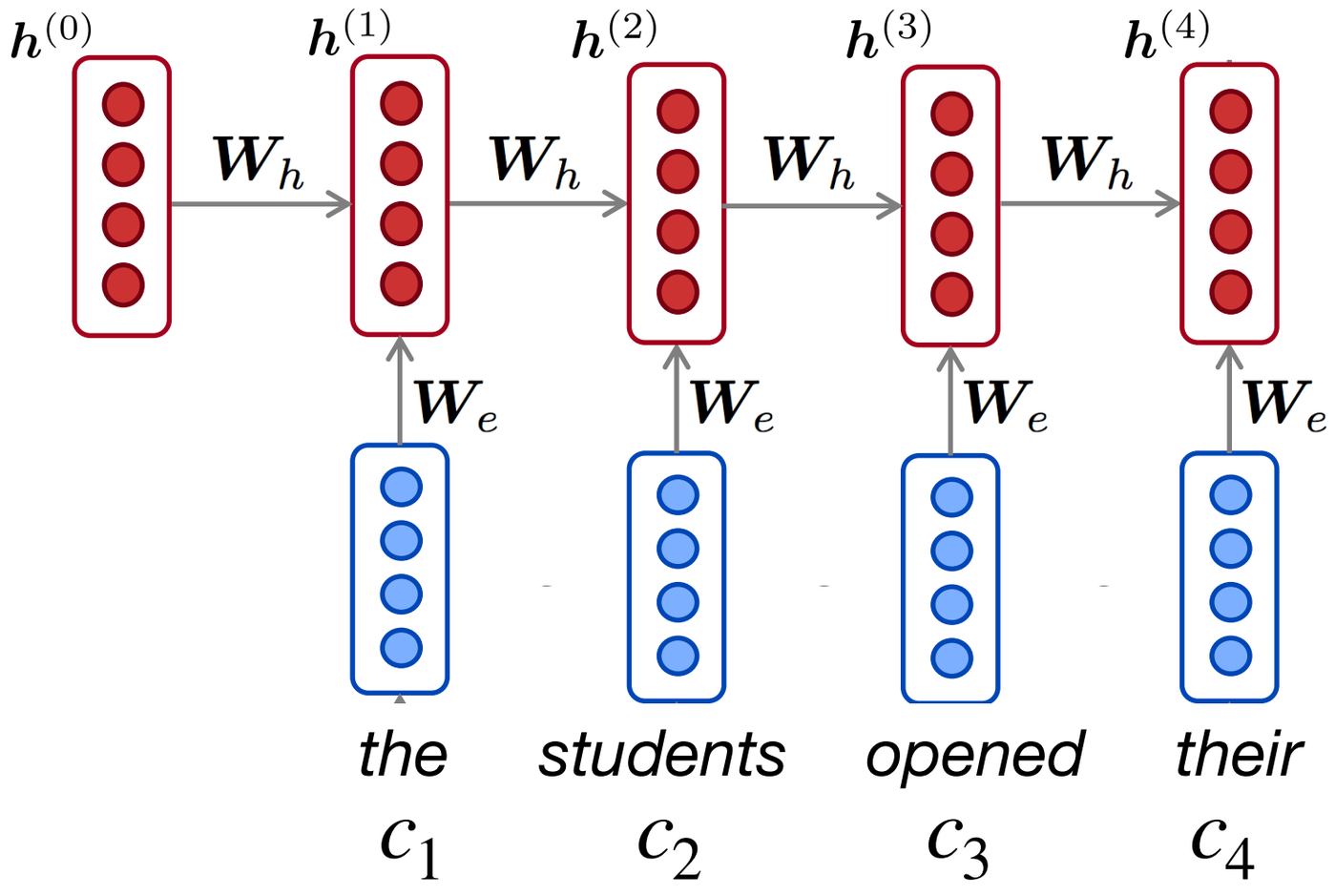


A RNN Language Model

output distribution

$$\hat{y} = \text{softmax}(W_2 h^{(t)})$$

hidden states

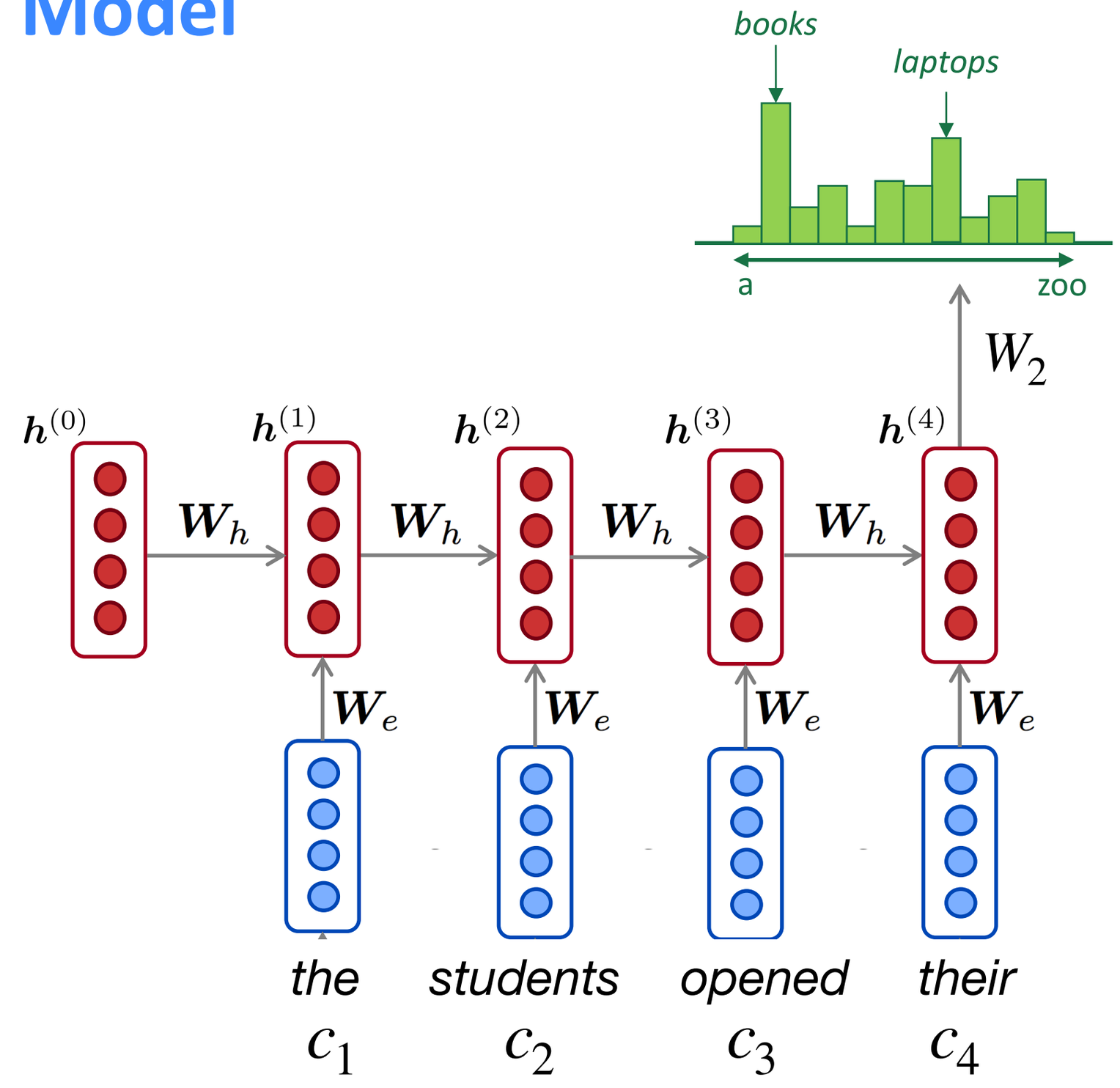
$$h^{(t)} = f(W_h h^{(t-1)} + W_e c_t)$$

$h^{(0)}$ is initial hidden state!

word embeddings

$$c_1, c_2, c_3, c_4$$

$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$



why is this good?

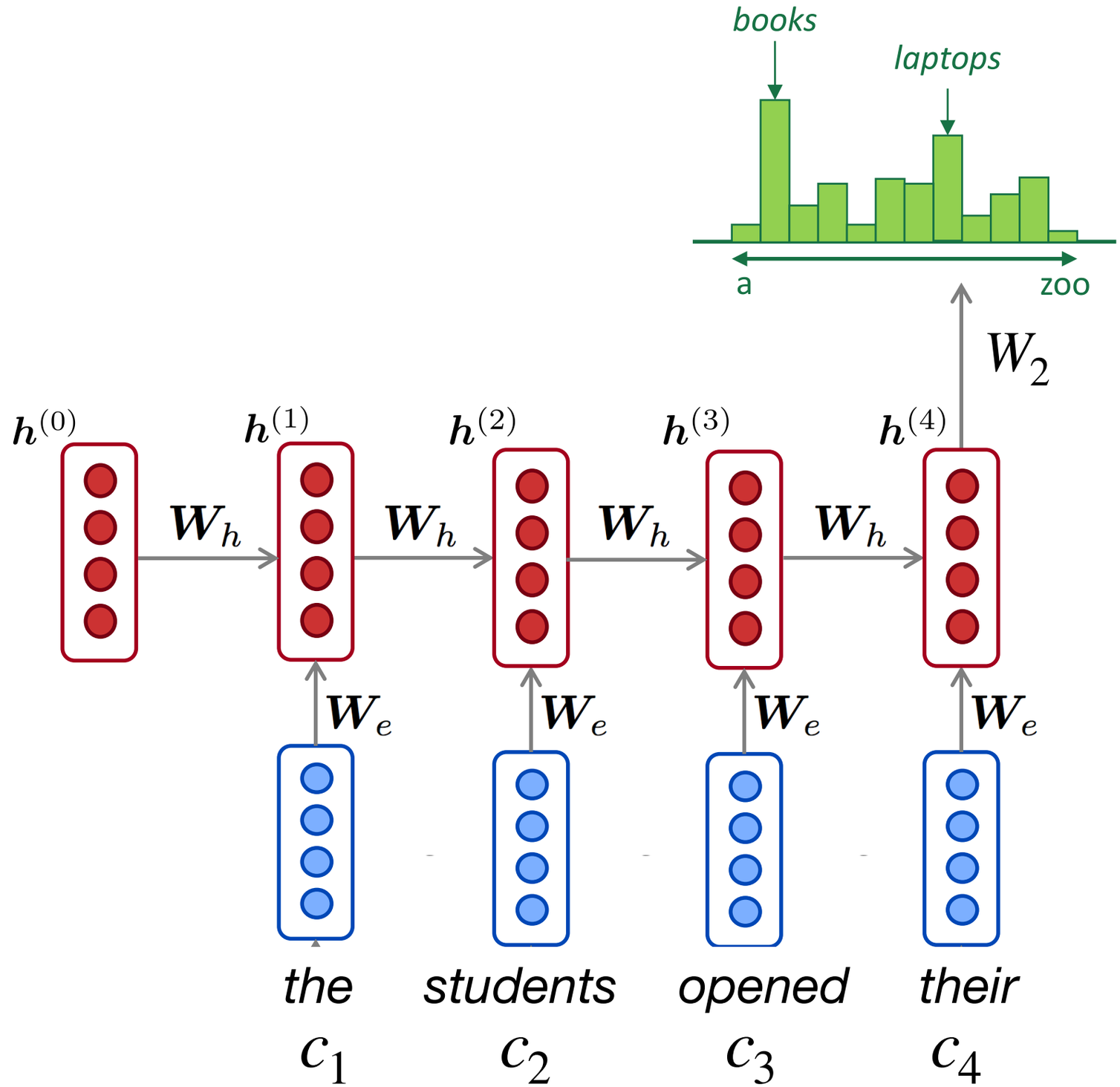
RNN **Advantages**:

- Can process any length input
- Model size doesn't increase for longer input
- Computation for step *t* can (in theory) use information from many steps back
- Weights are shared across timesteps → representations are shared

RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back

$$\hat{\boldsymbol{y}}^{(4)} = P(\boldsymbol{x}^{(5)}|\text{the students opened their})$$



Sequence Processing

Feedforward NN

RNN

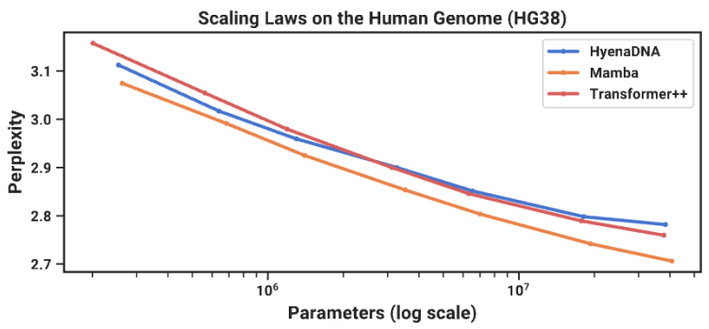
CNN

Transformer

Computation Cost — More →

Long Dependency — More →

Position Information — Less →



Mamba: Linear-Time Sequence Modeling with Selective State Spaces (https://arxiv.org/pdf/2312.00752)

Be on the lookout for…

- Next lecture on **backpropagation**, which allows us to actually train these networks to make reasonable predictions
- After that, we'll focus on **attention mechanisms** and build our way to the **Transformer** architecture, which is the most popular composition function used today