Language modeling

CS 685, Spring 2025
Advanced Natural Language Processing

Haw-Shiuan Chang
College of Information and Computer Sciences
University of Massachusetts Amherst

Impending deadlines

- **2/14**: HW 0 due
- **2/14**: Final project group assignments due
  - Google Form for project teams to be posted tonight
  - https://forms.gle/PKvJRxZkUMgFrkVG8
- **3/7**: Project proposals due

NLP Model Evoluation

Pre-NN

Deep Learning	LM	LLM
SFT →	**Pretraining** SFT →	**Pretraining** SFT Preference / Reasoning
>2013	>2018	>2023

Let’s say I want to train a model for *sentiment analysis*

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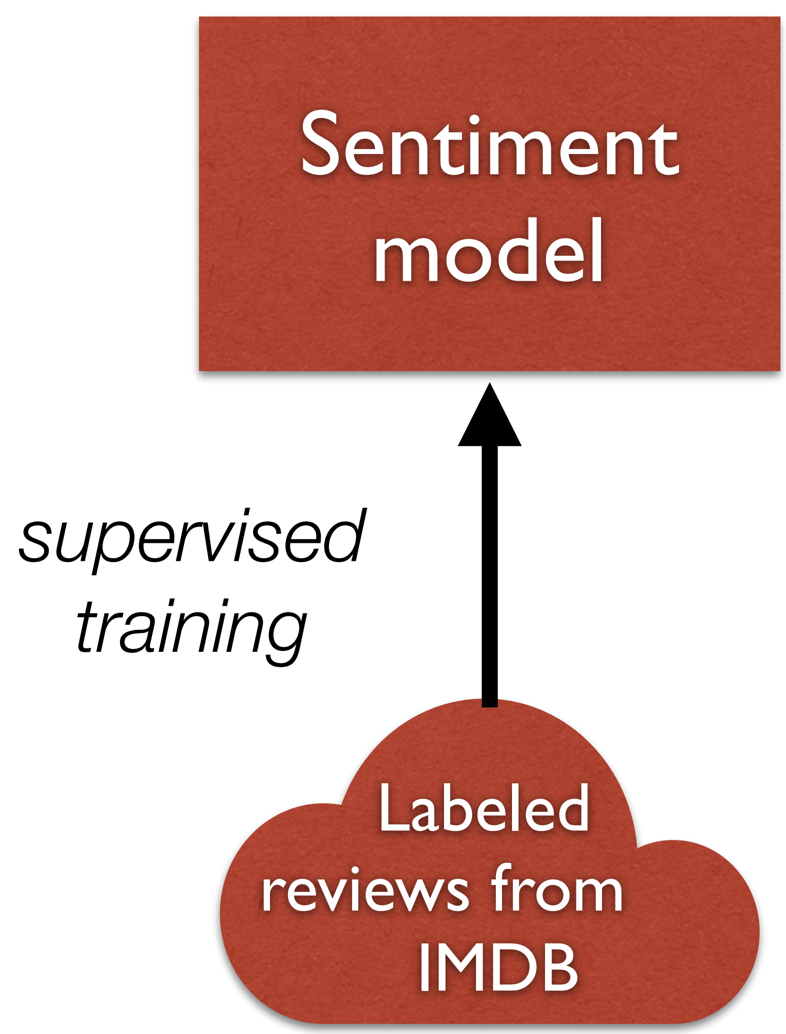
In the past, I would simply train a *supervised* model on labeled sentiment examples (i.e., review text / score pairs from IMDB)



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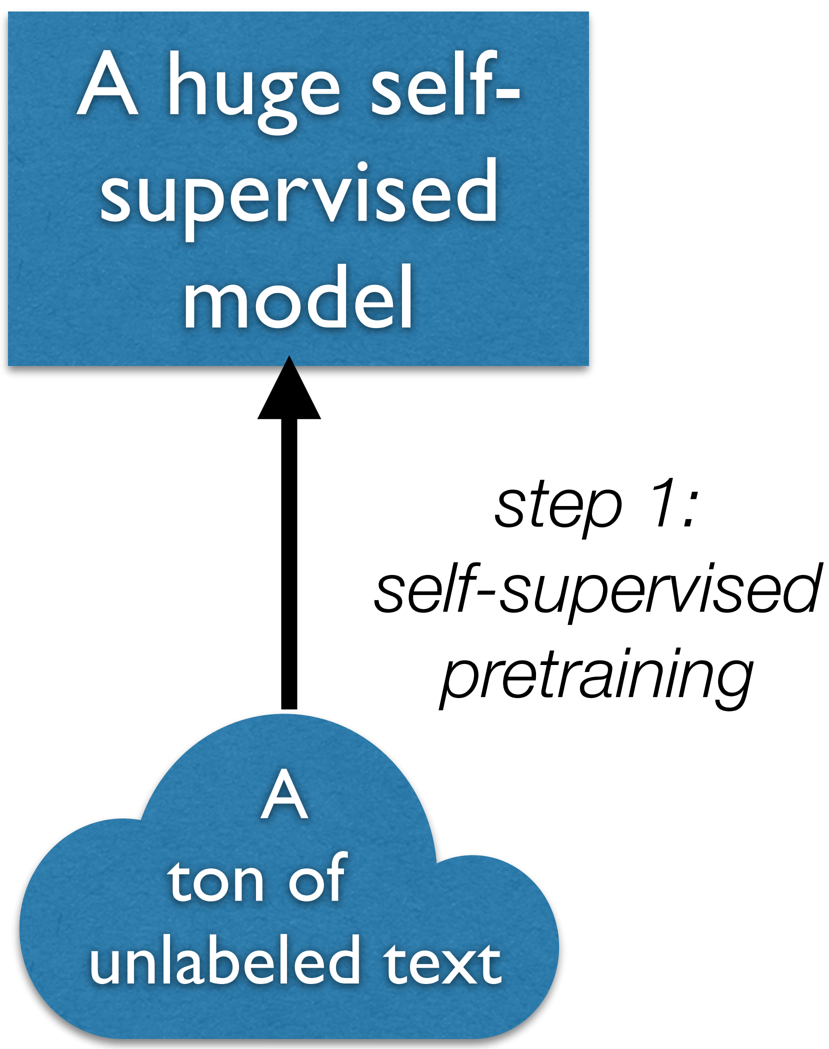
Nowadays, however, we take advantage of *transfer learning*:



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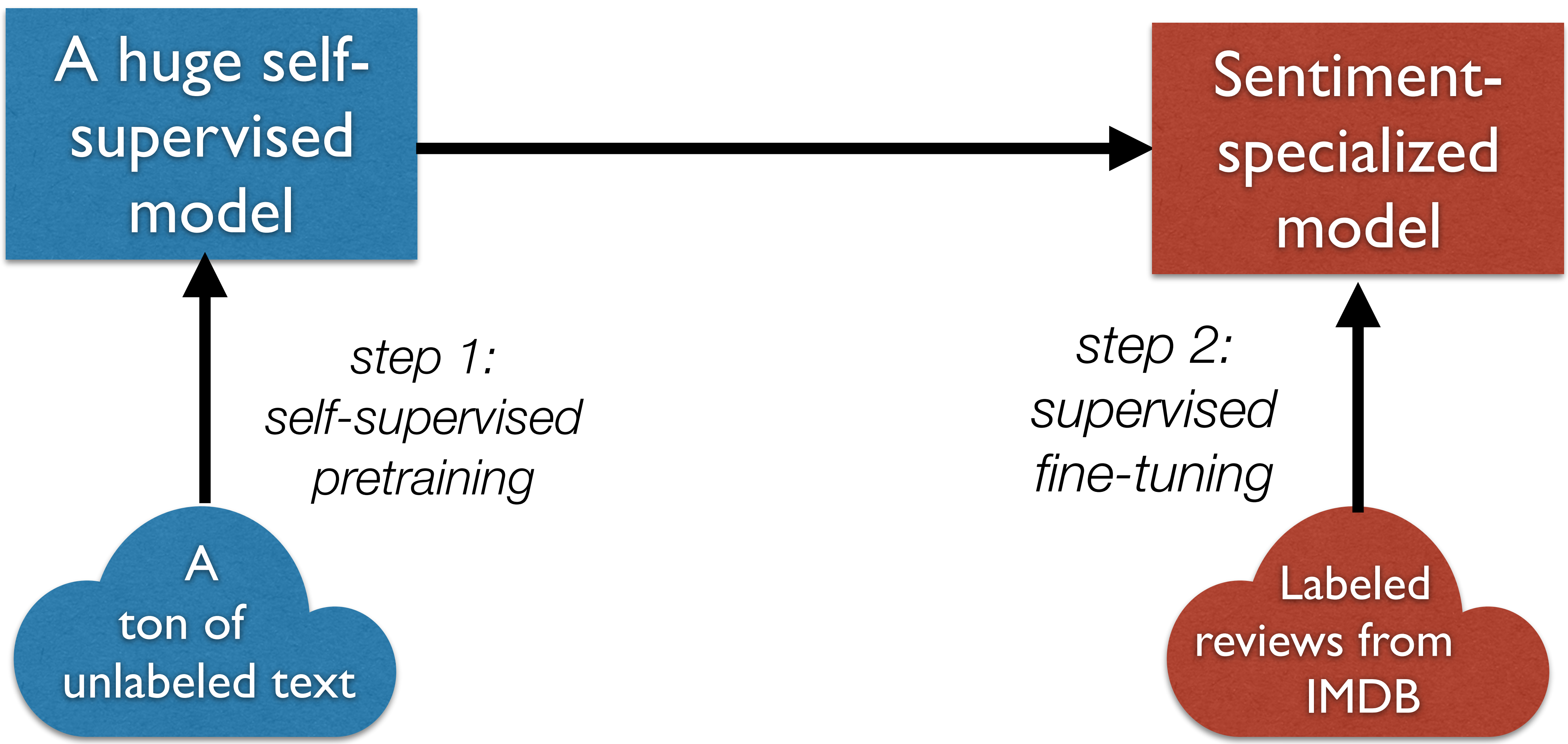
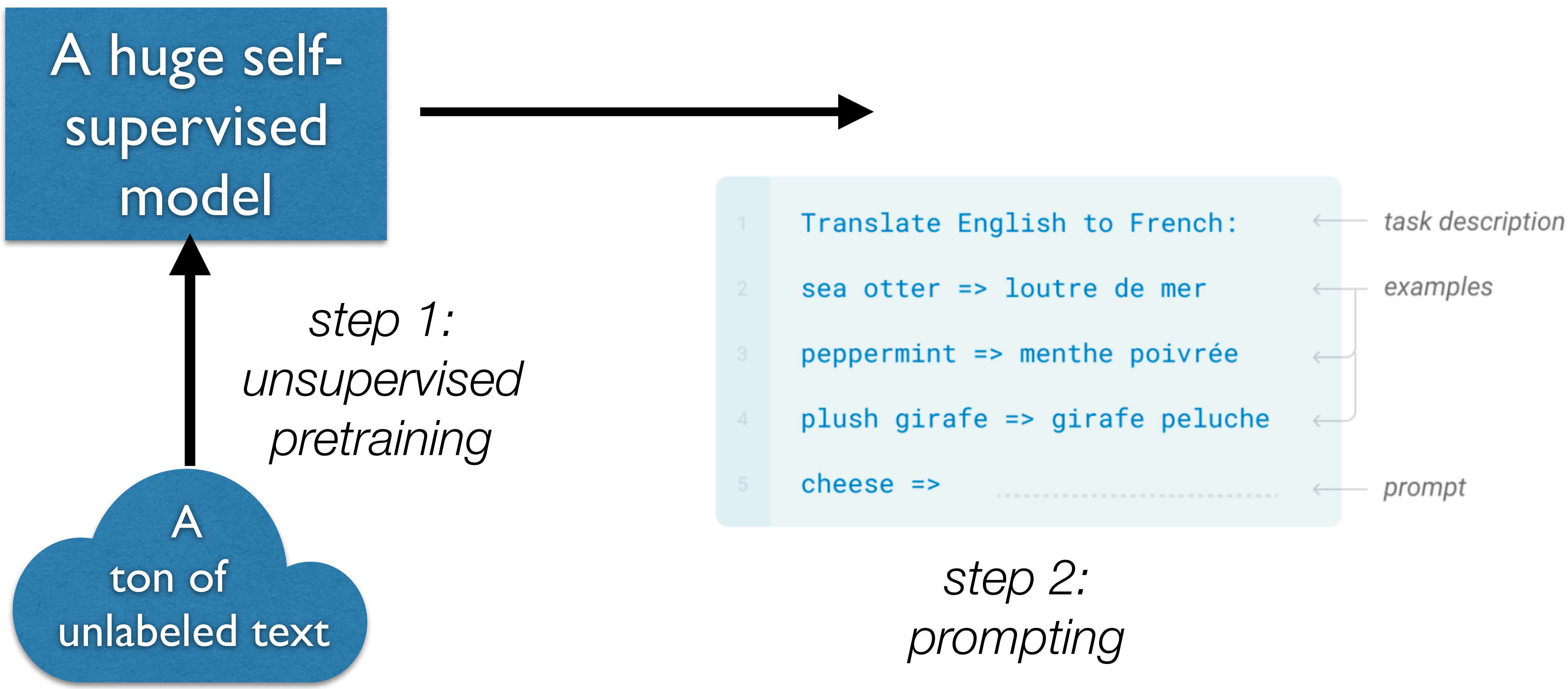
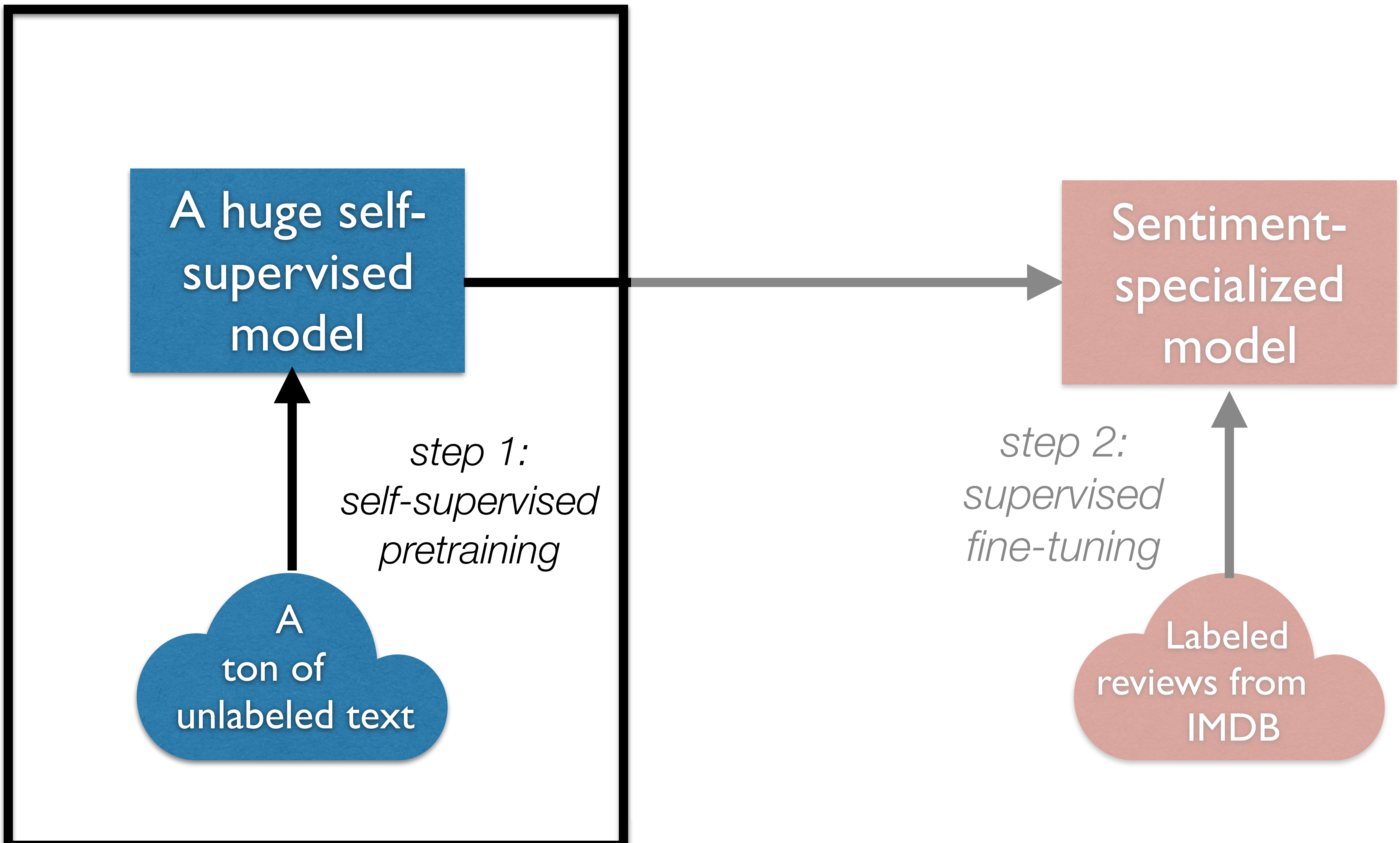


Let’s say I want to train a model for *sentiment analysis*

Or just rely entirely on the self-supervised model via prompting…



This lecture: **language modeling**, which forms the core of most self-supervised NLP approaches



Language models assign a probability to a piece of text

- why would we ever want to do this?
- translation:
  - P(i flew to the movies) **<<<<<** P(i went to the movies)
- speech recognition:
  - P(i saw a van) **>>>>>** P(eyes awe of an)

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

  $P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$

- Related task: probability of an upcoming word:

  $P(w_5|w_1,w_2,w_3,w_4)$

- A model that computes either of these:

  $P(W)$ or $P(w_n|w_1,w_2 \ldots w_{n-1})$ is called a **language model** or **LM**

How to compute P(W)

- How to compute this joint probability:
  - P(its, water, is, so, transparent, that)
- Intuition: let's rely on the Chain Rule of Probability

Reminder: The Chain Rule

- Recall the definition of conditional probabilities

  $\mathbf{P(B|A) = P(A,B)/P(A)}$ Rewriting: $\mathbf{P(A,B) = P(A)P(B|A)}$

- More variables:

  $P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$

- The Chain Rule in General

  $P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i \mid w_1 w_2 \dots w_{i-1})$$

P("its water is so transparent") =
P(its) × P(water|its) × P(is|its water)
× P(so|its water is) × P(transparent|its water is so)

The Chain Rule applied to compute joint probability of words in sent

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid \overbrace{w_1 w_2 \ldots w_{i-1}}^{\text{In HW0, we refer to this as a “prefix”}})$$

P(“its water is so transparent”) =

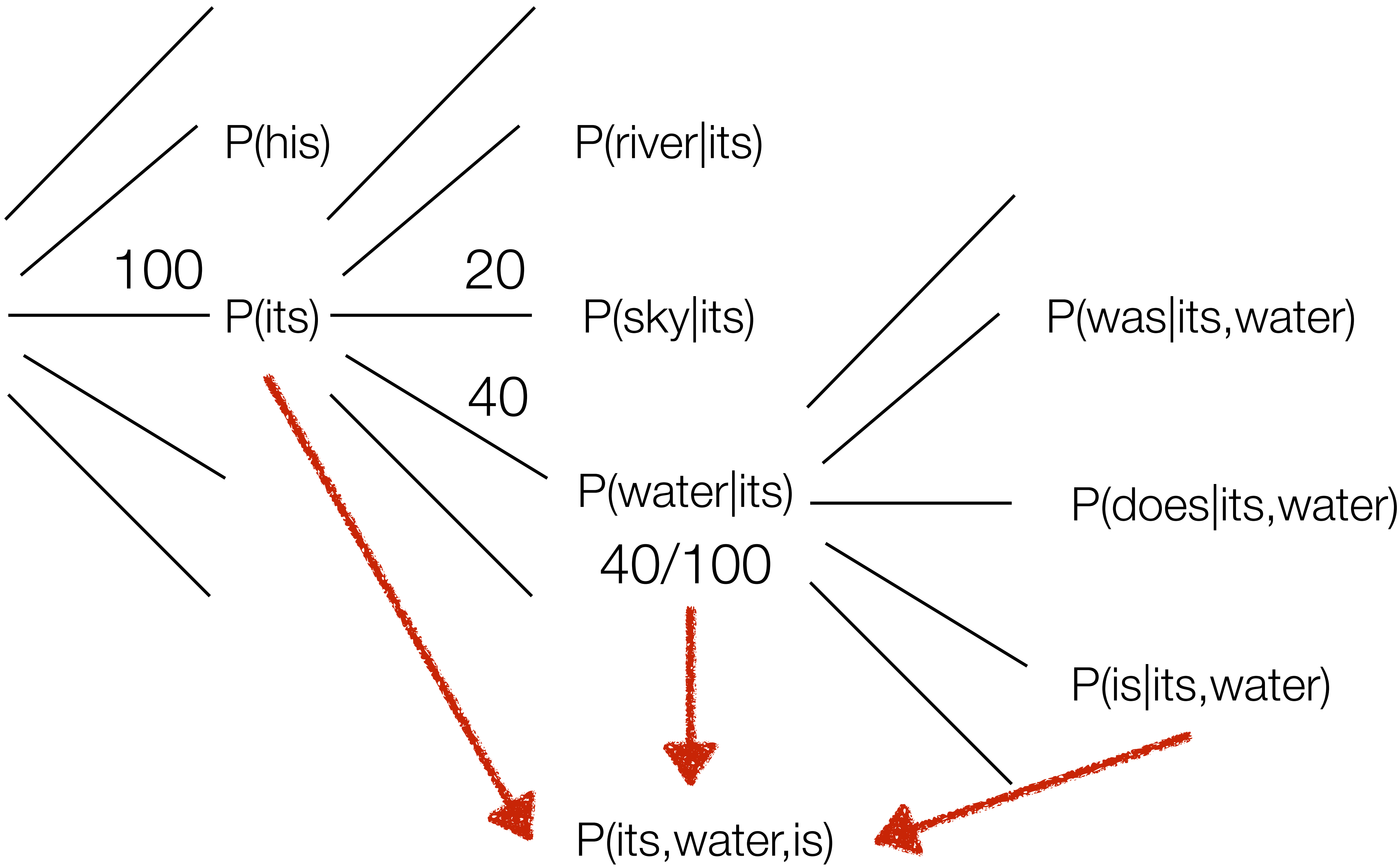
P(its) × P(water|its) × P(is|its water)

× P(so|its water is) × P(transparent|its water is so)

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$


P(his)
P(river|its)
100
P(its)
20
P(sky|its)
P(was|its,water)
40
P(water|its)
P(does|its,water)
40/100
P(is|its,water)
P(its,water,is)

Markov Assumption

Andrei Markov (1856~1922)

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

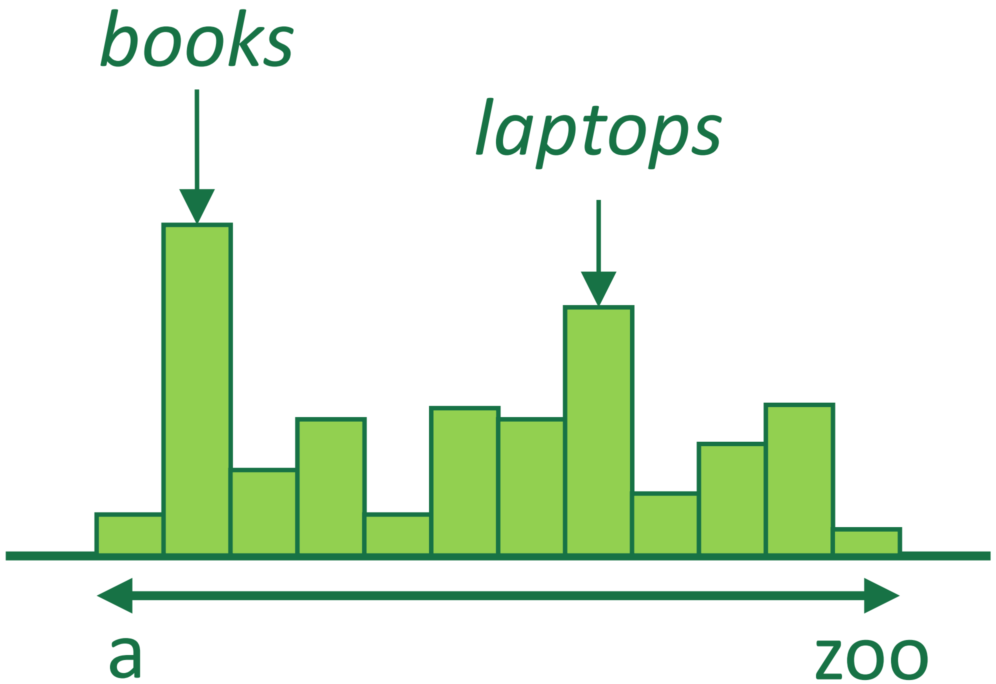
Some automatically generated sentences from a unigram model:

```
fifth, an, of, futures, the, an, incorporated, a, a,
the, inflation, most, dollars, quarter, in, is, mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

How can we generate text from a language model?

Decoding from an LM

Prefix: “students opened their”



Probability distribution over next word

Approximating Shakespeare

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; –It cannot be but so.

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

"The computer which I had just put into the machine room on the fifth floor crashed."

Estimating bigram probabilities

- The Maximum Likelihood Estimate (MLE)
  - relative frequency based on the empirical counts on a training set

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

c — count

An example

$$P(w_i \mid w_{i-1}) \overset{\text{MLE}}{=} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $\qquad P(\texttt{Sam} \mid \texttt{<s>}) = ???$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $\qquad P(\texttt{Sam} \mid \texttt{am}) = ???$

An example

$$P(w_i \mid w_{i-1}) \overset{\text{MLE}}{=} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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$$P(\texttt{I} \mid \texttt{<s>}) = \tfrac{2}{3} = .67 \qquad P(\texttt{Sam} \mid \texttt{<s>}) = \tfrac{1}{3} = .33 \qquad P(\texttt{am} \mid \texttt{I}) = \tfrac{2}{3} = .67$$

$$P(\texttt{</s>} \mid \texttt{Sam}) = \tfrac{1}{2} = 0.5 \qquad P(\texttt{Sam} \mid \texttt{am}) = \tfrac{1}{2} = .5 \qquad P(\texttt{do} \mid \texttt{I}) = \tfrac{1}{3} = .33$$

An example

Important terminology: a word **type** is a unique word in our vocabulary, while a **token** is an occurrence of a word type in a dataset.

$$P(w_i \mid w_{i-1}) \overset{\text{MLE}}{=} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

$$P(\texttt{I} \mid \texttt{<s>}) = \tfrac{2}{3} = .67 \qquad P(\texttt{Sam} \mid \texttt{<s>}) = \tfrac{1}{3} = .33 \qquad P(\texttt{am} \mid \texttt{I}) = \tfrac{2}{3} = .67$$

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A bigger example:
Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i'm looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i'm looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

note: this is only a subset of the (much bigger) bigram count table

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

$$P(w_i \mid w_{i-1}) \overset{\text{MLE}}{=} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Normalize by unigrams:
- Result:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

these probabilities get super tiny when we have longer inputs w/ more infrequent words… how can we get around this?

logs to avoid underflow

$$\log \prod p(w_i \mid w_{i-1}) = \sum \log p(w_i \mid w_{i-1})$$

Example with unigram model on a sentiment dataset:

sentence: I love love love love love the movie

logs to avoid underflow

$$\log \prod p(w_i \mid w_{i-1}) = \sum \log p(w_i \mid w_{i-1})$$

<u>Example with unigram model on a sentiment dataset:</u>

sentence: I love love love love love the movie

$p(\text{i}) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie})$ = 5.95374181e-7

$\log p(\text{i}) + 5 \log p(\text{love}) + \log p(\text{the}) + \log p(\text{movie})$

= -14.3340757538

What kinds of knowledge?

- P(english|want) = .0011 — about the world
- P(chinese|want) = .0065
- P(to|want) = .66 — grammar – infinitive verb
- P(eat | to) = .28
- P(food | to) = 0 — ???
- P(want | spend) = 0 — grammar
- P (i | <s>) = .25

Task -> Loss -> Model -> Optimization

- Task:
  - **Predict the next token**
  - (even humans do this)
  - (Recommending the next product is the same problem)
- Loss:
  - **Markov + Maximal Likelihood** / Cross-entropy
- Model:
  - **Tables** -> Neural Network / Transformer
- Optimization:
  - **Counting** -> Gradient Descent
- Loss + Model + Optimization -> Algorithm
- Why is decomposition useful? Tools
- Please try to report these in your project

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to “real” or “frequently observed” sentences
    - Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

Evaluation: How good is our model?

- The goal isn't to pound out fake sentences!
  - Obviously, generated sentences get "better" as we increase the model order
  - More precisely: using maximum likelihood estimators, higher order is always better likelihood on **training set,** but not **test set**

Example: I use a bunch of New York Times articles to build a bigram probability table

train →

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
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evaluate

Now I'm going to evaluate the probability of some *heldout* data using our bigram table

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spend	0.0036	0	0.0036	0	0	0	0	0

evaluate

A good language model should assign a high probability to heldout text!

Now I'm going to evaluate the probability of some *heldout* data using our bigram table

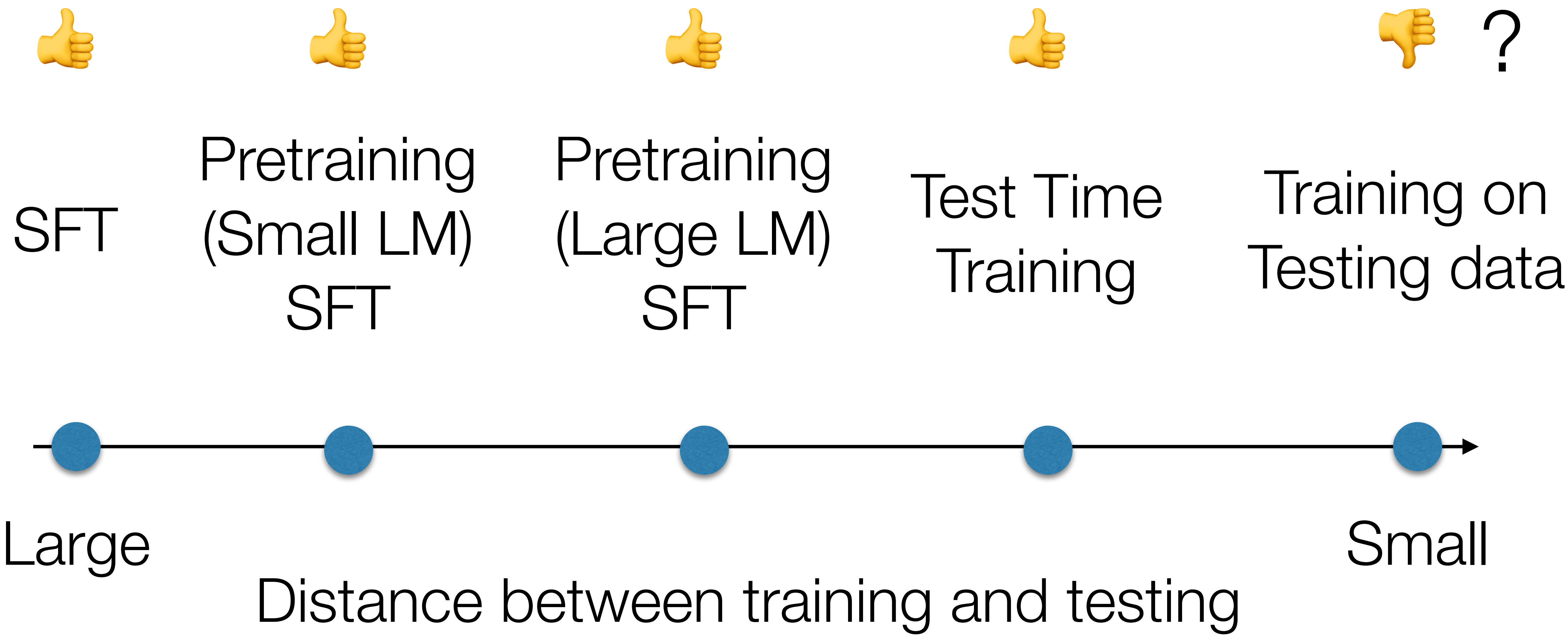
Training on the test set

- We can’t allow test sentences into the training set
- We will assign it an artificially high probability when we set it in the test set
- “Training on the test set”
- Bad science!

This advice is generally applicable to any downstream task! Do NOT do this in your final projects unless you want to lose a lot of points :)

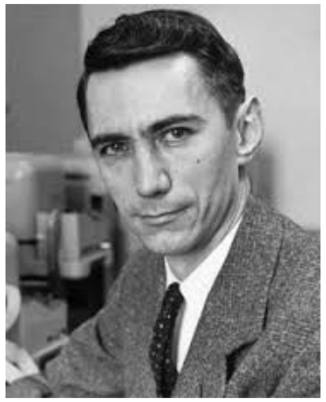
Example Midterm Question

The key is generalization ability



Akyürek, Ekin, et al. "The surprising effectiveness of test-time training for abstract reasoning." *arXiv preprint arXiv:2411.07279* (2024).(https://arxiv.org/abs/2411.07279)

Intuition of Perplexity



Claude Shannon
(1916~2001)

- The Shannon Game:
  - How well can we predict the next word?

    I always order pizza with cheese and ____

    The 33rd President of the US was ____

    I saw a ____

  - Unigrams are terrible at this game. (Why?)
- A better model of a text
  - is one which assigns a higher probability to the word that actually occurs

mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$PP(W) = P(w_1 w_2 ... w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 ... w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

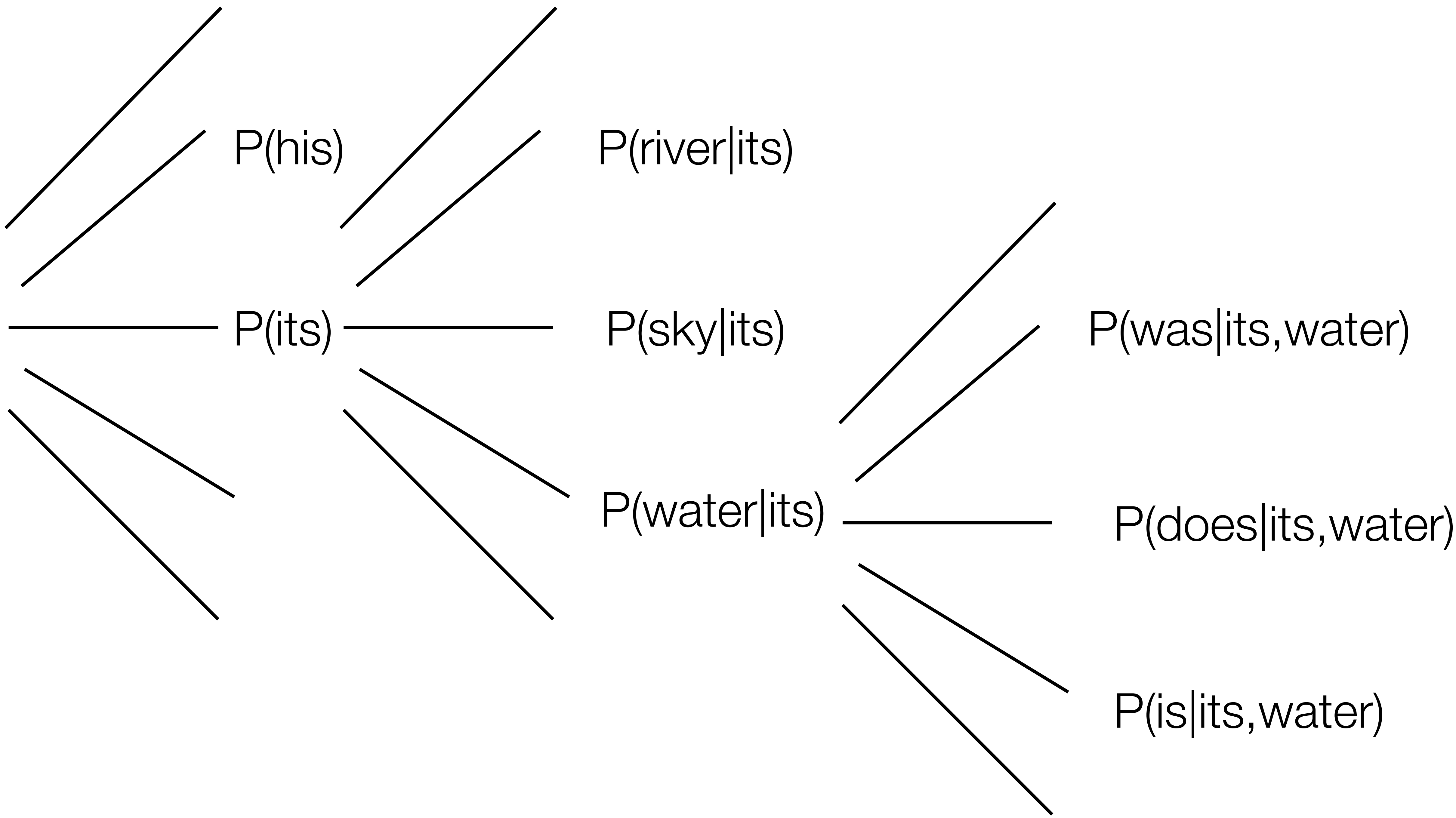
Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

Let's suppose a sentence consisting of random digits

What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= (\frac{1}{10}^N)^{-\frac{1}{N}} \\
&= \frac{1}{10}^{-1} \\
&= 10
\end{aligned}
$$


P(his)
P(its)
P(river|its)
P(sky|its)
P(water|its)
P(was|its,water)
P(does|its,water)
P(is|its,water)

In practice, we use log probs

$$PP(W) = \exp\Big(-\frac{1}{N}\sum_{i}^{N}\log p(w_i \mid w_{<i})\Big)$$

In practice, we use log probs

$$PP(W) = \exp\Big(-\frac{1}{N}\sum_{i}^{N} \log p(w_i \mid w_{<i}) \Big)$$

Perplexity is the exponentiated *token-level negative log-likelihood*

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, Wall Street Journal

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Shakespeare as corpus

- N=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844 million possible bigrams.
  - So 99.96% of the possible bigrams were never seen (have zero entries in the table)

Zeros

Training set:
… denied the allegations
… denied the reports
… denied the claims
… denied the request

P("offer" | denied the) = 0

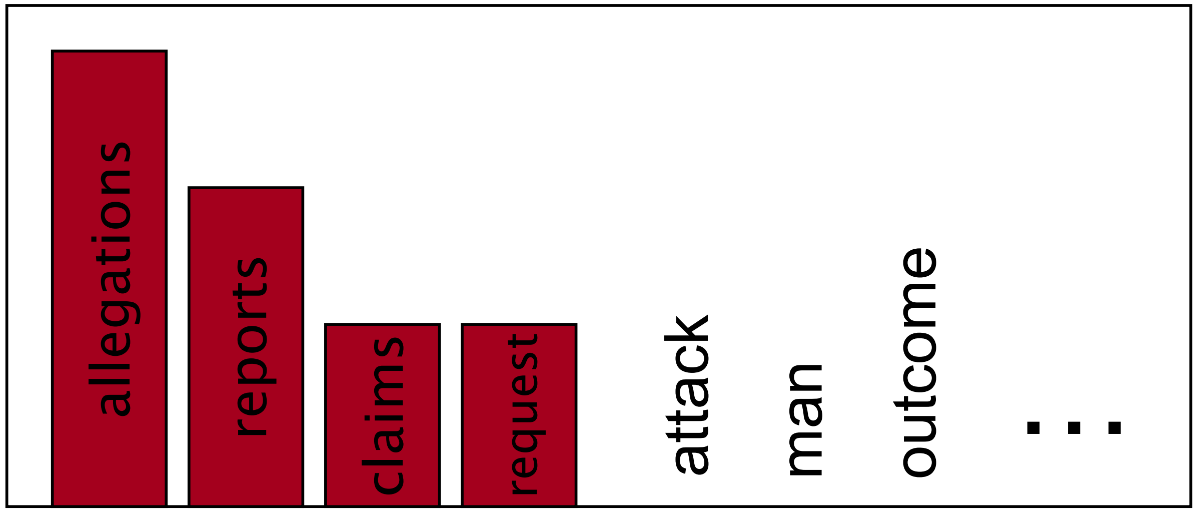
- Test set
  … denied the offer
  … denied the loan

The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

  P(w | denied the)
  3 allegations
  2 reports
  1 claims
  1 request

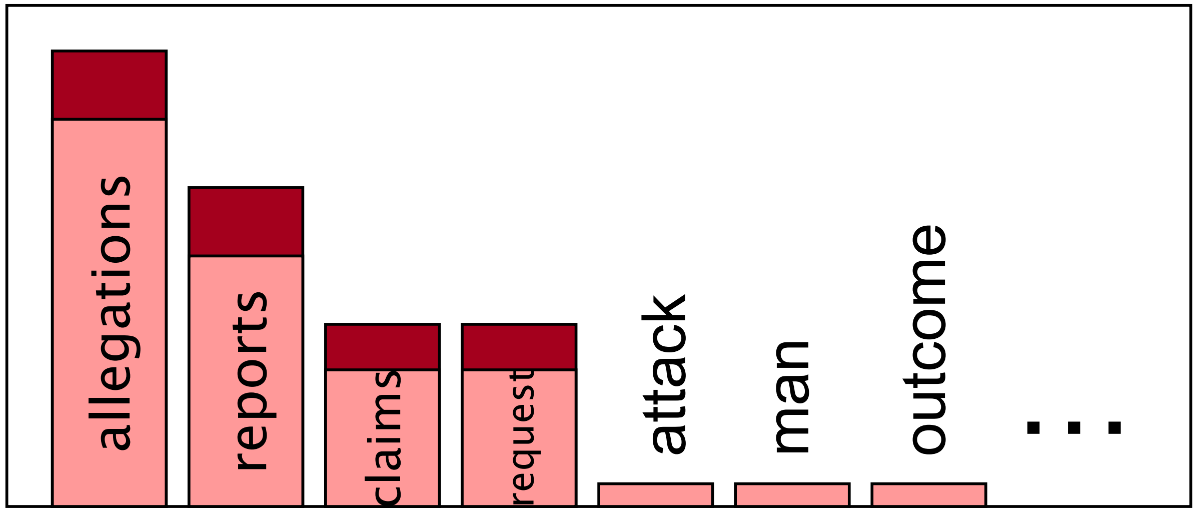
  7 total



- Steal probability mass to generalize better

  P(w | denied the)
  2.5 allegations
  1.5 reports
  0.5 claims
  0.5 request
  2 other

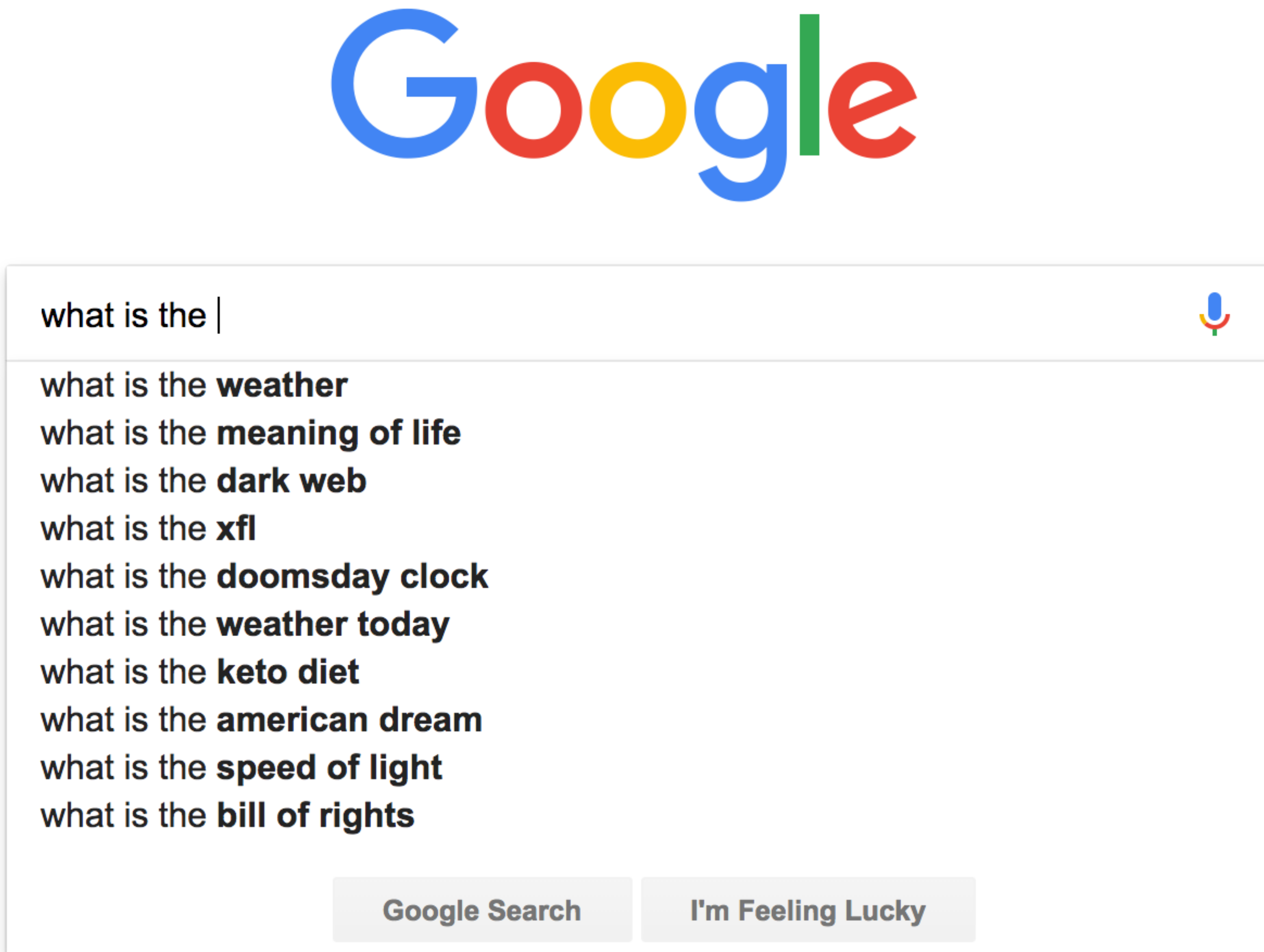
  7 total



Why n-gram LM still matters?

- Compared to neural LMs
  - No hallucination problem
    - Accurate when the n is small
    - Tracible sources
  - Fast and could work under low resources
    - auto-complete

You use Language Models every day!



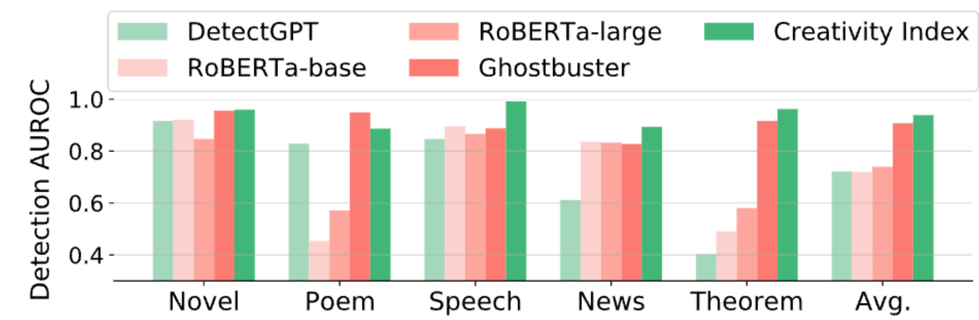
Infini-gram: a state of the art *n*-gram model on 1.4T tokens

https://arxiv.org/pdf/2401.17377.pdf

https://huggingface.co/spaces/liujch1998/infini-gram

Liu et al., 2024. "Infini-gram: Scaling Unbounded n-gram Language Models to a Trillion Tokens"

N-gram LM application



Great example of NLP!=LLM

Do not assert that something is outdated too quickly

https://arxiv.org/abs/2410.04265

Lu, Ximing, et al. "AI as Humanity's Salieri: Quantifying Linguistic Creativity of Language Models via Systematic Attribution of Machine Text against Web Text." *arXiv preprint arXiv:2410.04265* (2024).

Why do we predict the next word

- Why from left to right?
  - Will introduce masked language modeling
  - Some works try other orders, but hard to be significantly better
    - Arrows of Time for Large Language Models (https://arxiv.org/abs/2401.17505)
- Why not predict multiple tokens?
  - Dependency of words
    - Large Concept Models: Language Modeling in a Sentence Representation Space
    - (https://arxiv.org/abs/2412.08821)
    - Better & Faster Large Language Models via Multi-token Prediction (https://arxiv.org/pdf/2404.19737)
    - Predicting the next sentence (not word) in large language models: What model-brain alignment tells us about discourse comprehension (https://www.science.org/doi/10.1126/sciadv.adn7744)