CS 520

Theory and Practice of Software Engineering
Fall 2021

Automated theorem proving

November 23, 2021

Programs are known to be error-prone

- Capture complex aspects such as:
  - Threads and synchronization (e.g., Java locks)
  - Dynamically heap allocated structured data types (e.g., Java classes)
  - Dynamically stack allocated procedures (e.g., Java methods)
  - Non-determinism (e.g., Java HashSet, Java locks)
  - Many input/output pairs (e.g., Numbers)

- Challenging to reason about all possible behaviors of these programs

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  - Threads and synchronization (e.g., Java locks)
  - Dynamically heap allocated structured data types (e.g., Java classes)
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- Challenging to reason about all possible behaviors of these programs

Overview of theorem provers

Key idea: Constraint satisfaction problem

Take as input:

- a **program** modeled in first-order logic (i.e. a set of boolean formulae)
- a **question** about that program also modeled in first-order logic (i.e. additional boolean formulae)

Overview of theorem provers

Use **formal reasoning (e.g., decision procedures)** to produce as output one of the following:

- **satisfiable**: For some input/output pairs (i.e. variable assignments), the program does satisfy the question
- **unsatisfiable**: For all input/output pairs (i.e. variable assignment), the program does not satisfy the question

Possible uses of theorem provers

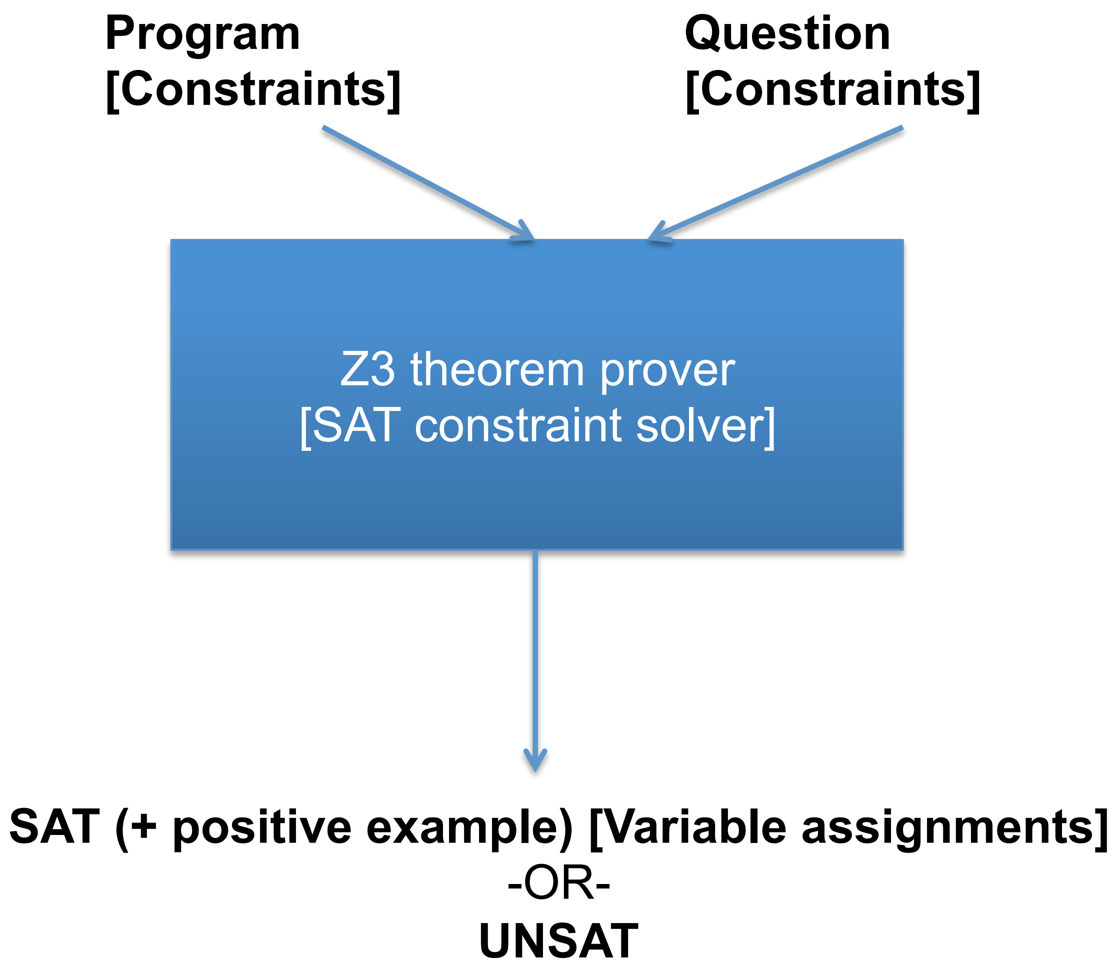
- Testing, e.g., detecting mutants
- Analysis
- Verification

Z3

- Online interfaces:
  - https://rise4fun.com/z3
  - https://compsys-tools.ens-lyon.fr/z3/index.php

- Download: https://github.com/Z3Prover/z3

Theorem prover architecture: Z3



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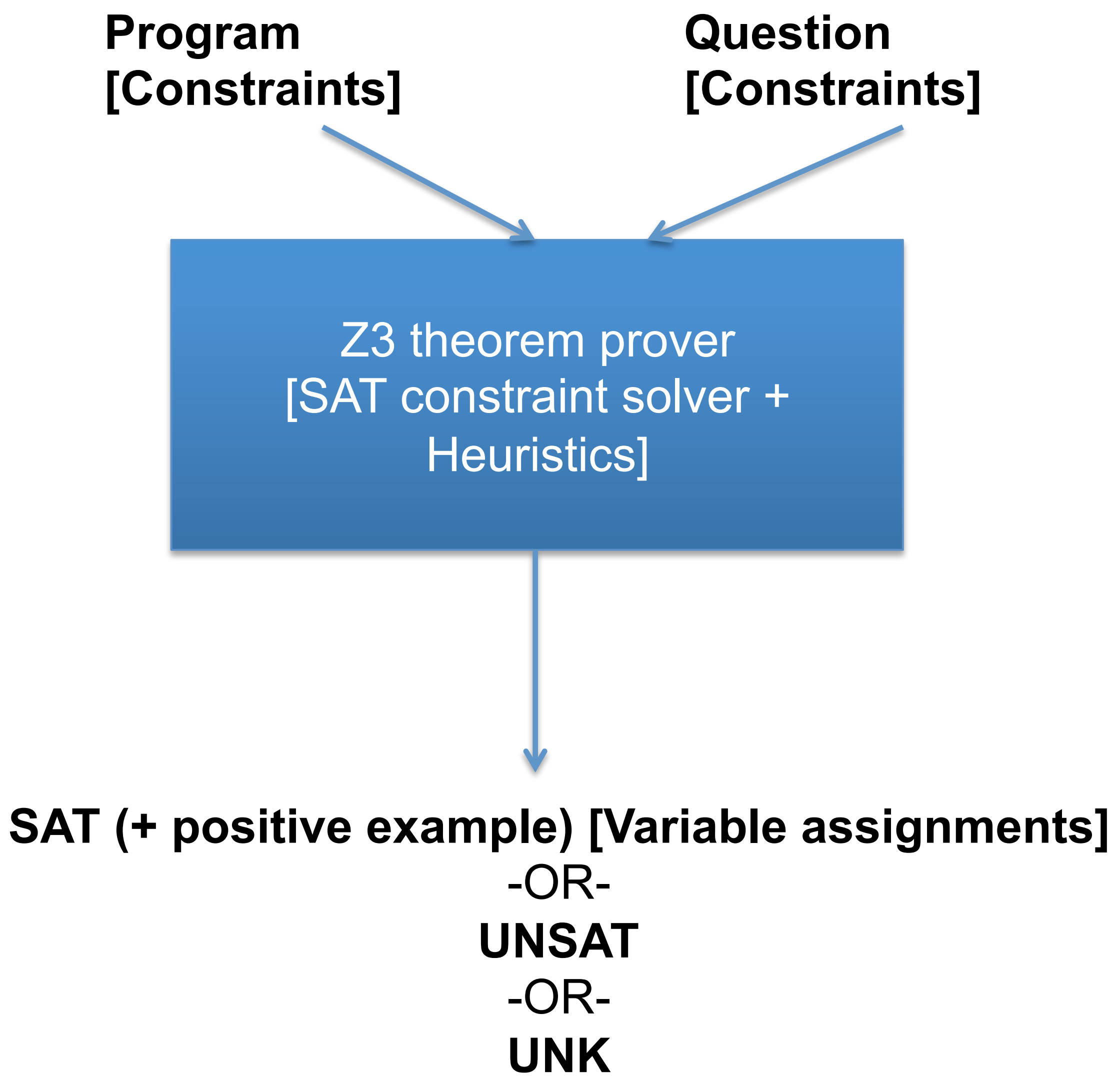

Programming language: SMT (Satisfiability Modulo Theories)

Supports the following:

- Variables, e.g., *(declare-const a Int)*
- Assertions, e.g., *(assert (> a 0))*
- Print statements, e.g., *(echo “Printing…”)*
- Comments, e.g., *;; This is a comment.*
- Functions, e.g.,

  (declare-fun compareTo (Int Int) Bool)
- *…*

http://smtlib.cs.uiowa.edu/

Example: Simple program

Java:

```
int sum (int a, int b) {
    return a + b;
}
```

Z3 input:

Example: Simple program

Java:

```
int sum (int a, int b) {
    return a + b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
```

Example: Simple program

Java:

```
int sum (int a, int b) {
    return a + b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
```

Example: Simple program

Java:

```
int sum (int a, int b) {
    return a + b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(assert (= (+ a b) r1))
```

Z3’s question types

- Basic boolean equations
- More complex boolean equations involving existential and universal quantification
- Certain math equations involving numbers, and linear arithmetic (addition, subtraction, multiplication, division, and ordering)

Z3's questions and possible answers

- **Can ask "Is this possible (i.e. satisfiable)?"**

(check-sat)

- **If satisfiable, can ask "What is an example (i.e. a satisfying variable assignment)?"**

(get-model)

- **If unsatisfiable (or unknown), cannot ask "What is an example?"**

Example: Simple program

Question: Can sum ever return 0?

```
(assert (= r1 0)) ;; We want r1 = a + b to be 0
(check-sat)       ;; Ask if this is possible
(get-model)       ;; It is, so let's get an example
```

SAT constraint solving

- **Satisfiability** is about finding a solution to a set of constraints (in our case formulae).

- A formula F is **satisfiable** if there is some assignment of appropriate values to its uninterpreted function and constant symbols under which F evaluates to true.

From https://rise4fun.com/Z3/tutorial/guide

SAT constraint solving

- **Validity** is about finding a proof of a statement (in our case a formula F).

- A formula F is **valid** if F always evaluates to true for any assignment of appropriate values to its uninterpreted function and constant symbols.

SAT constraint solving

- F is **satisfiable** if and only if not F is not **valid** (is **invalid**).
  - Report that there exists a satisfying assignment

- F is **valid** precisely when not F is not **satisfiable** (is **unsatisfiable**).
  - Report that none of the assignments are satisfying

Example: Simple program

Z3 run:

z3 Z3code.simple.smt2

Z3 output:

```
sat
(model
  (define-fun a () Int 0)
  (define-fun b () Int 0)
  (define-fun r1 () Int 0)
)
```

Example: Simple program

Z3 run:

z3 Z3code.simple.smt2

Z3 output:

```
sat
(model
  (define-fun a () Int 0)
  (define-fun b () Int 0)
  (define-fun r1 () Int 0)
)
```

Here is the expected result.
The sum is 0 when
both a and b are 0.

Detect mutants using Z3

1. Given an original program and a mutant, use Z3 to show that mutant is either detectable or undetectable

2. If the mutant is detectable, use Z3's output to create a JUnit test to kill it

Show a mutant is either detectable or undetectable

- If two functions are behaviorally equivalent (i.e. undetectable mutants), for all inputs, they act the same (in our case produce the same outputs)

- We can ask if two functions are **NOT** behaviorally equivalent (i.e. detectable mutants), does there exist an input for which they act differently (in our case produce different outputs)

Z3

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- Download: https://github.com/Z3Prover/z3

- Examples: https://people.cs.umass.edu/~hconboy/class/2021Fall/CS520/lectures/20211123automatedTheoremProving-programs.zip

Example: Pair 0

Java:

```
int normal_sum (int a, int b) {
    return a + b;
}

int mutant_sum (int a, int b) {
    return a * b;
}
```

Z3 input:

Example: Pair 0

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int normal_sum (int a, int b) {
    return a + b;
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int mutant_sum (int a, int b) {
    return a * b;
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```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
```

Example: Pair 0

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```
int normal_sum (int a, int b) {
   return a + b;
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Z3 input:

```
(declare-const a Int)
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```

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```
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    return a + b;
}

int mutant_sum (int a, int b) {
    return a * b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(declare-const mutated_r1 Int)
```

Example: Pair 0

Java:

```
int normal_sum (int a, int b) {
    return a + b;
}

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    return a * b;
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Z3 input:

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(declare-const a Int)
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```

Example: Pair 0

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```
int normal_sum (int a, int b) {
    return a + b;
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int mutant_sum (int a, int b) {
    return a * b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(declare-const mutated_r1 Int)

(assert (= (+ a b) r1))
(assert (= (* a b) mutated_r1))
```

Example: Pair 0

Java:

```
int normal_sum (int a, int b) {
    return a + b;
}

int mutant_sum (int a, int b) {
    return a * b;
}
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Z3 input:

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(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(declare-const mutated_r1 Int)

(assert (= (+ a b) r1))
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Example: Pair 0

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int normal_sum (int a, int b) {
   return a + b;
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int mutant_sum (int a, int b) {
   return a * b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(declare-const mutated_r1 Int)

(assert (= (+ a b) r1))
(assert (= (* a b) mutated_r1))

(assert (not (= r1 mutated_r1)))
```

Example: Pair 0

Java:

```
int normal_sum (int a, int b) {
    return a + b;
}

int mutant_sum (int a, int b) {
    return a * b;
}
```

Z3 input:

```
(declare-const a Int)
(declare-const b Int)
(declare-const r1 Int)
(declare-const mutated_r1 Int)

(assert (= (+ a b) r1))
(assert (= (* a b) mutated_r1))

(assert (not (= r1 mutated_r1)))

(check-sat)
;;(get-model)
```

Example: Pair 0 (cont.)

Z3 output:

```
sat

(get-model)
(model
  (define-fun mutated_r1 () Int (- 8))
  (define-fun r1 () Int 2)
  (define-fun b () Int 4)
  (define-fun a () Int (- 2))
)
```

Example: Pair 0 (cont.)

Z3 output:

sat

(model
  (define-fun mutated_r1 () Int (- 8))
  (define-fun r1 () Int 2)
  (define-fun b () Int 4)
  (define-fun a () Int (- 2))
)

JUnit test case:

@Test
public killSimpleMutant {

}

Example: Pair 0 (cont.)

Z3 output:

```
sat

(model
  (define-fun mutated_r1 () Int (- 8))
  (define-fun r1 () Int 2)
  (define-fun b () Int 4)
  (define-fun a () Int (- 2))
)
```

JUnit test case:

```
@Test
public killSimpleMutant {
    int a = -2;
    int b = 4;
    assertEquals(
        2,
        sum(a,b));
}
```

Example: Pair 0 (cont.)

Z3 output:

```
sat

(model
  (define-fun mutated_r1 () Int (- 8))
  (define-fun r1 () Int 2)
  (define-fun b () Int 4)
  (define-fun a () Int (- 2))
)
```

JUnit test case:

```
@Test
public killSimpleMutant {
    int a = -2;
    int b = 4;
    assertEquals(
        2,              // Expected: 2
        sum(a,b)); // Actual: -8
}
```

Example: Pair 1

Example: Pair 1

Z3 input:

```
;;;;;;;;;;;;;;;;; START STUDENT CODE ;;;;;;;;;;;;;;;
( assert (= (+ x y) a1))
( assert (= (+ a1 z) a2))
( assert (= (- a1 z) mutated_a2))
( assert (not (= a2 mutated_a2)))
;;;;;;;;;;;;;;;;; END STUDENT CODE ;;;;;;;;;;;;;;;
```

Z3 output: SAT

Example: Pair 2

Example: Pair 2

Z3 input:

```
;;;;;;;;;;;;;;;;; START STUDENT CODE ;;;;;;;;;;;;;;;
(assert (= a-eq-b (= a b))) ;It is fine if these three lines are missing.
(assert (= a-eq-c (= a c))) ;Adding asserts to an unsat problem cannot make it sat
(assert (= b-eq-c (= b c))) ;so just the lines below are sufficient to prove unsat
(assert (= initial-condition (= trian 0)))
(assert (= mutated-condition (<= trian 0)))
;;;;;;;;;;;;;;;;; END STUDENT CODE ;;;;;;;;;;;;;;;
```

Z3 output: UNSAT

Example: Pair 3

Advantages and disadvantages of theorem proving

- **Automates reasoning** about **all program behaviors**
  - Considers all possible input/output pairs
- **Suffers from the state space explosion problem**
  - Incorporates heuristics that may lead to returning unknown
- **Requires expertise with modeling in first-order logic**
- **May be hard to update the proof after program changes**

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Repair/Synthesis of proofs

- Repair of proofs - https://github.com/uwplse/coq-change-analytics (Talia Ringer, **Alex Sanchez-Stern**, et al.)
- Synthesis of proofs - https://github.com/LASER-UMASS/TacTok

(**Emily First, Yuriy Brun**, Arjun Guha)

Homework 3: Questions?