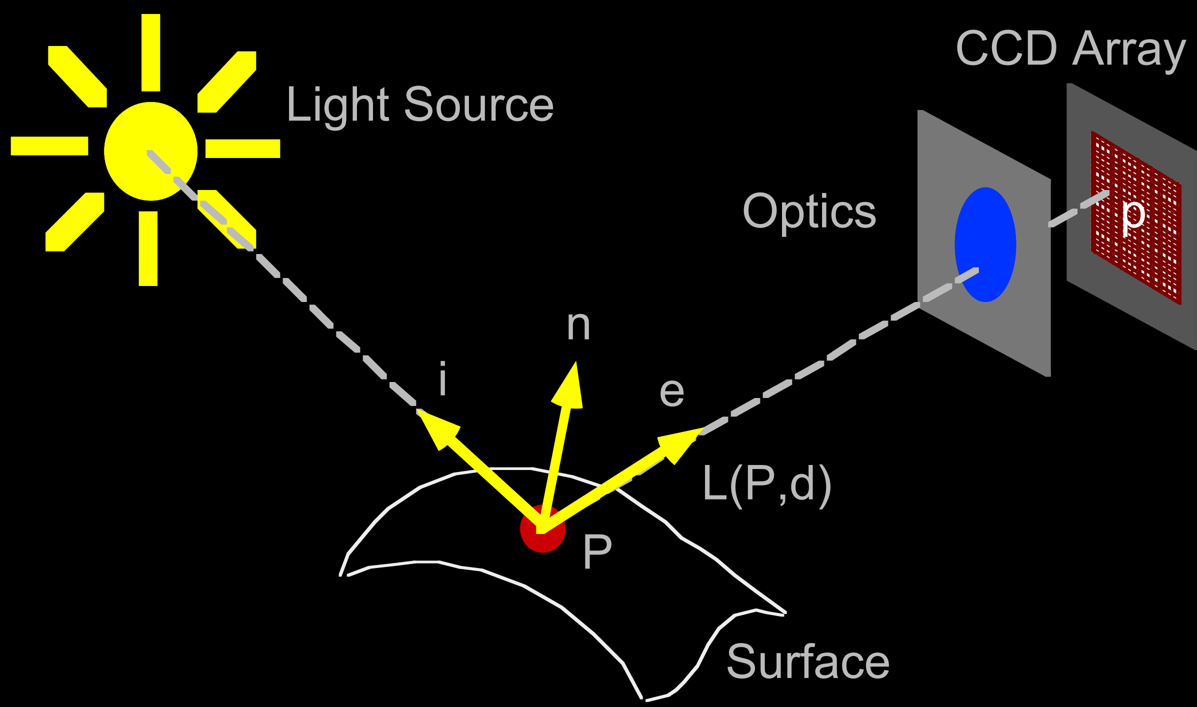
Understanding Variability

- Why so different?

- Light and Optics
  - Pinhole camera model
  - Perspective projection
  - Thin lens model
  - Fundamental equation
  - Distortion: spherical & chromatic aberration, radial distortion
  - Reflection and Illumination: color, Lambertian and specular surfaces, Phong, BRDF
- Sensing Light
- Conversion to Digital Images
- Sampling Theorem
- Other Sensors: frequency, type, ….

Basic Radiometry

- Radiometry is the part of image formation concerned with the relation among the amounts of light energy emitted from light sources, reflected from surfaces, and registered by sensors.



Lecture Assumptions

- Typical imaging scenario:
  - visible light
  - ideal lenses
  - standard sensor (e.g. TV camera)
  - opaque objects
- Goal

> To create 'digital' images which can be processed to recover some of the characteristics of the 3D world which was imaged.

Steps

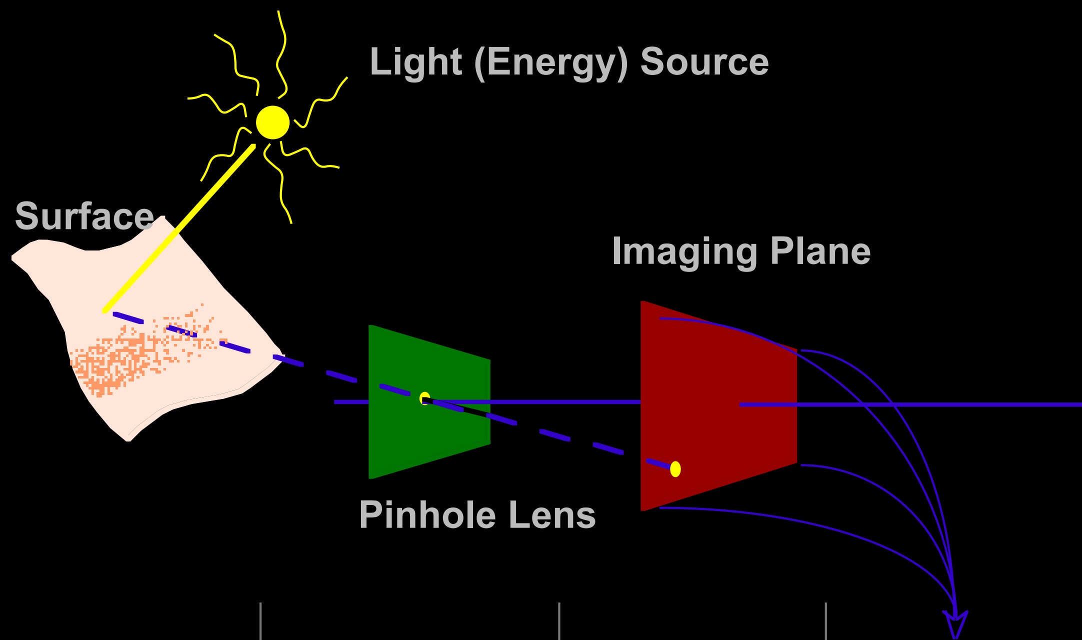
World → Optics → Sensor →
Signal → Digitizer →
Digital Representation

World	reality
Optics	focus {light} from world on sensor
Sensor	converts {light} to {electrical energy}
Signal	representation of incident light as continuous electrical energy
Digitizer	converts continuous signal to discrete signal
Digital Rep.	final representation of reality in computer memory

Factors in Image Formation

- Geometry
  - concerned with the relationship between points in the three-dimensional world and their images
- Radiometry
  - concerned with the relationship between the amount of light radiating from a surface and the amount incident at its image
- Photometry
  - concerned with ways of measuring the intensity of light
- Digitization
  - concerned with ways of converting continuous signals (in both space and time) to digital approximations

Image Formation



World	Optics	Sensor	Signal
		B&W Film	Silver Density
		Color Film	Silver density in three color layers
		TV Camera	Electrical

Geometry

- Geometry describes the projection of:

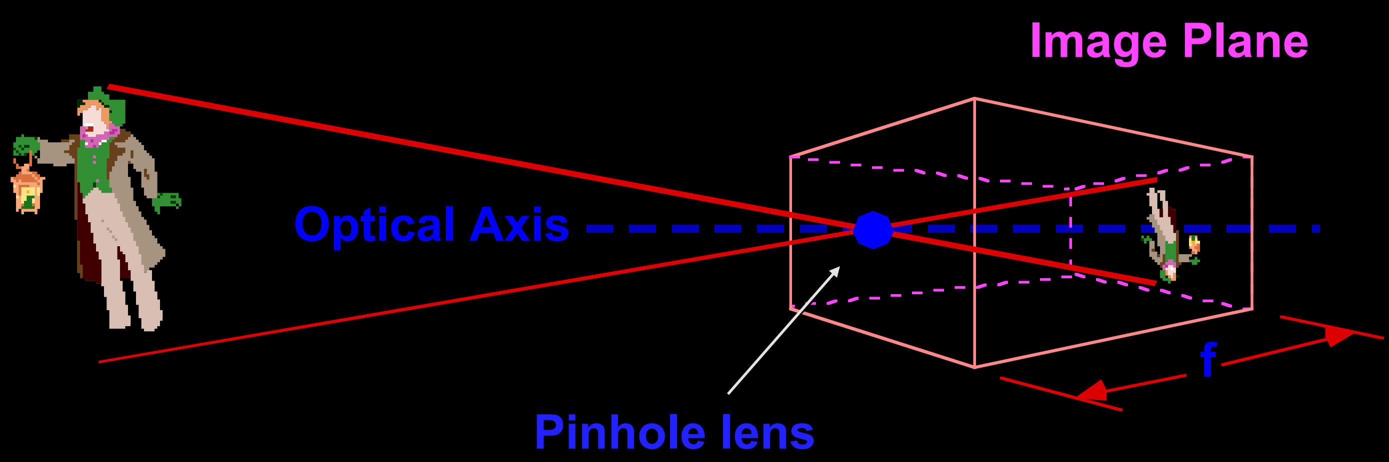
three-dimensional (3D) world → two-dimensional (2D) image plane.

- Typical Assumptions
  - Light travels in a straight line
- Optical Axis: the perpendicular from the image plane through the pinhole (also called the central projection ray)
- Each point in the image corresponds to a particular direction defined by a ray from that point through the pinhole.
- Various kinds of projections:
  - - perspective - oblique
  - - orthographic - isometric
  - - spherical

Basic Optics

- Two models are commonly used:
  - Pin-hole camera
  - Optical system composed of lenses
- Pin-hole is the basis for most graphics and vision
  - Derived from physical construction of early cameras
  - Mathematics is very straightforward
- Thin lens model is first of the lens models
  - Mathematical model for a physical lens
  - Lens gathers light over area and focuses on image plane.

Pinhole Camera Model



- World projected to 2D Image
  - Image inverted
  - Size reduced
  - Image is dim
  - No direct depth information
- f called the focal length of the lens
- Known as perspective projection

Pinhole camera image

Amsterdam

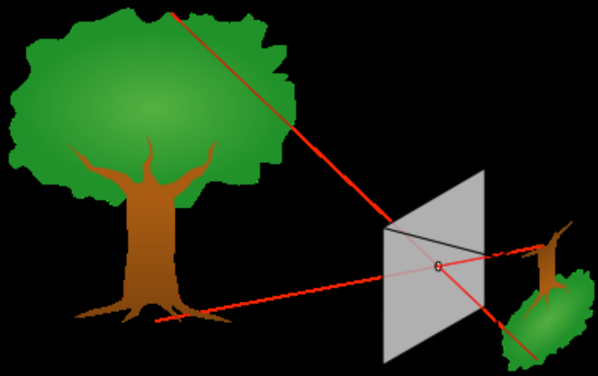
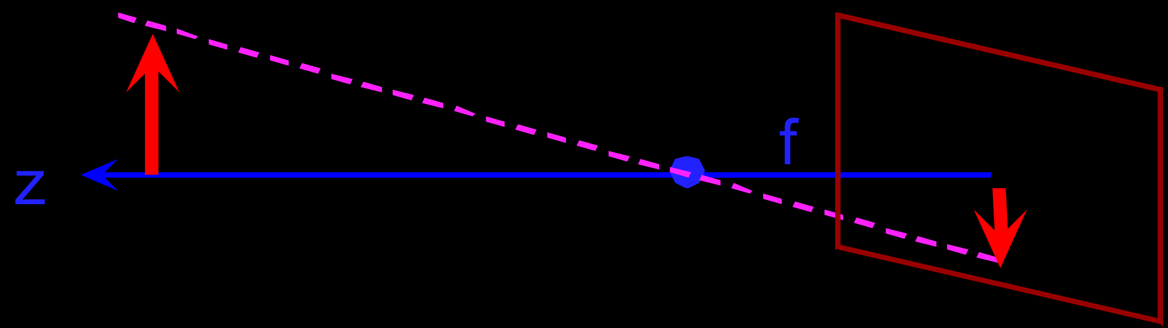


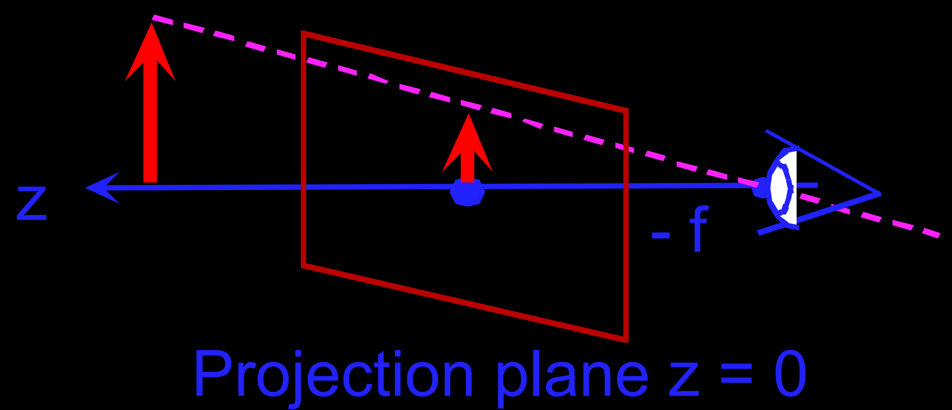
Photo by Robert Kosara, robert@kosara.net
http://www.kosara.net/gallery/pinholeamsterdam/pic01.html

Equivalent Geometry

- Consider case with object on the optical axis:



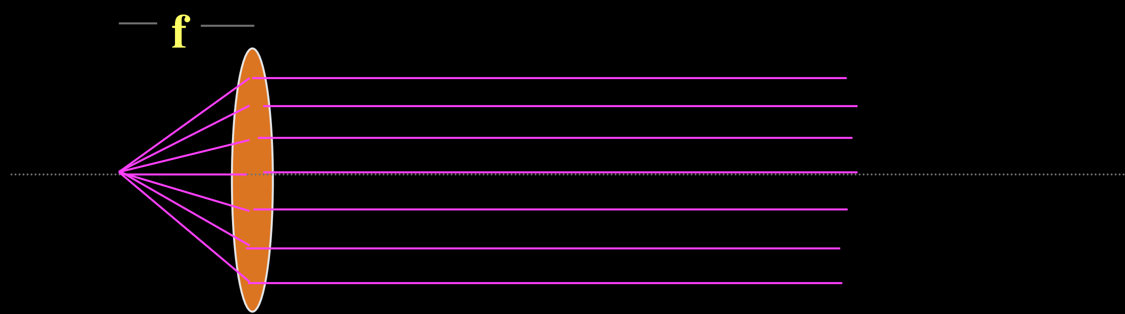
- More convenient with upright image:

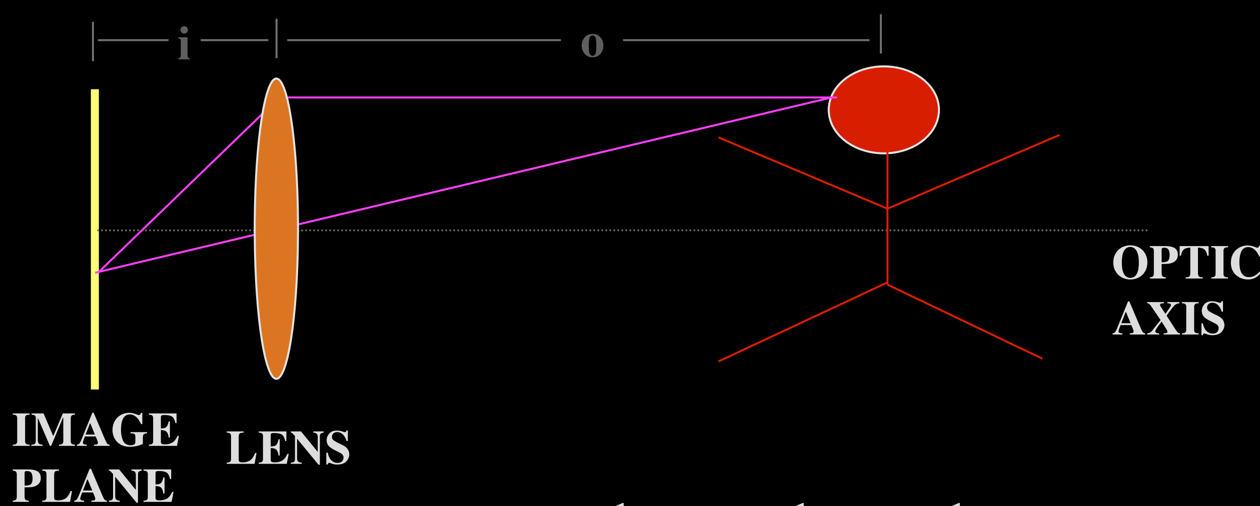


- Equivalent mathematically

Thin Lens Model

- Rays entering parallel on one side converge at focal point.
- Rays diverging from the focal point become parallel.

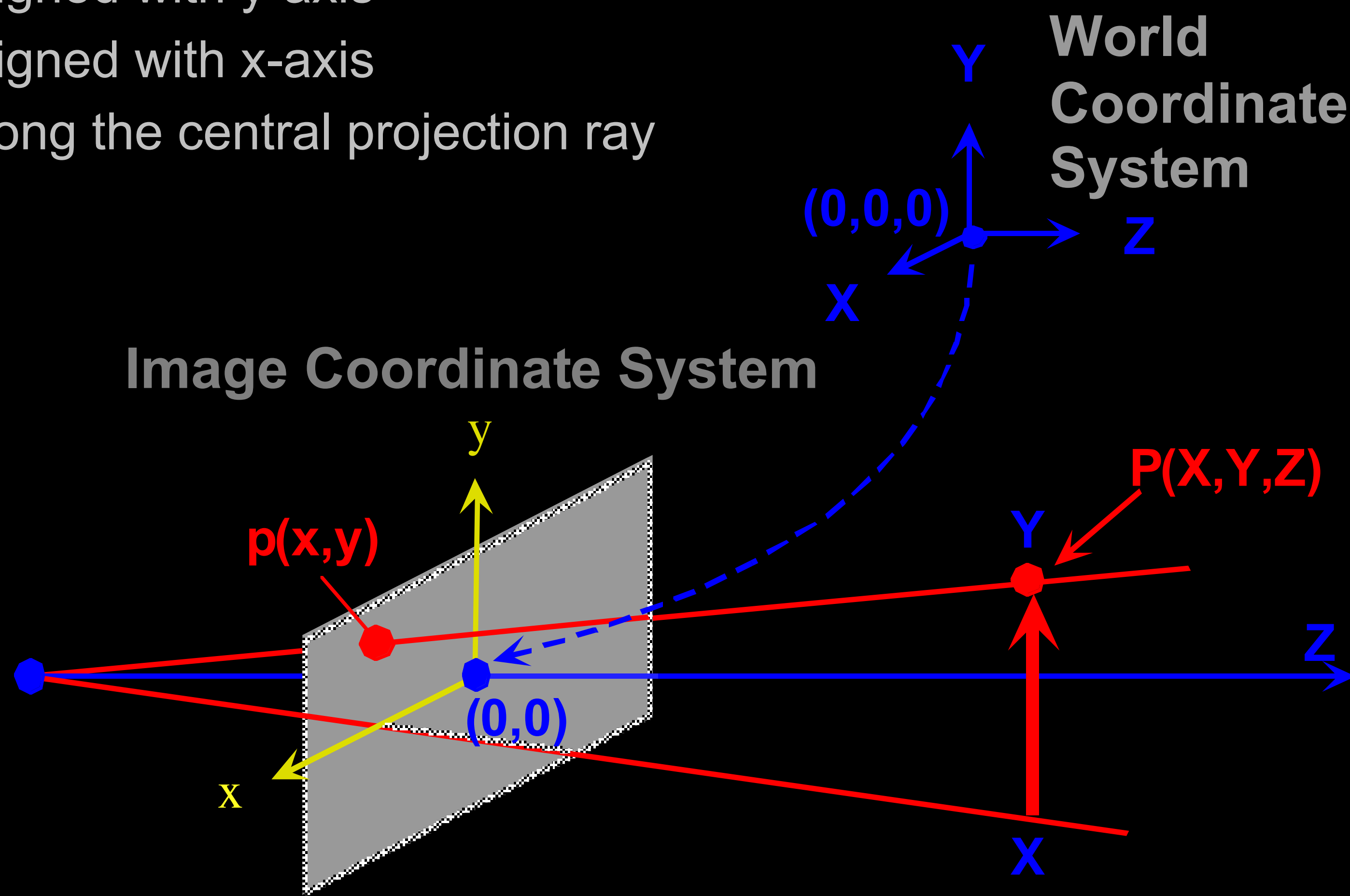




$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$ 'THIN LENS LAW'

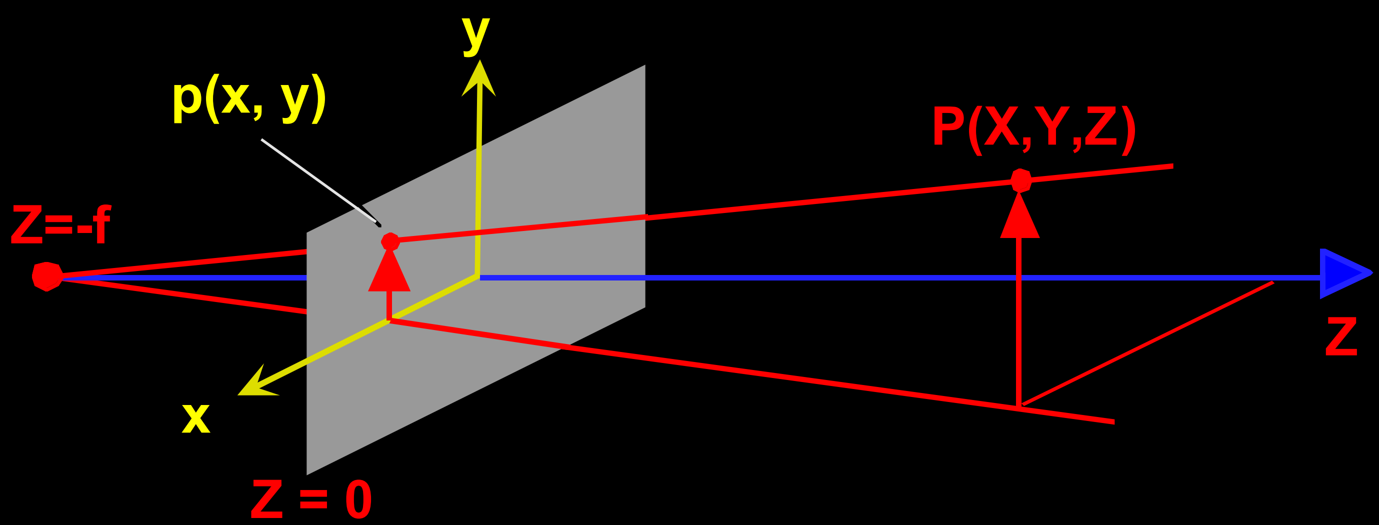
Coordinate System

- Simplified Case:
  - Origin of world and image coordinate systems coincide
  - Y-axis aligned with y-axis
  - X-axis aligned with x-axis
  - Z-axis along the central projection ray



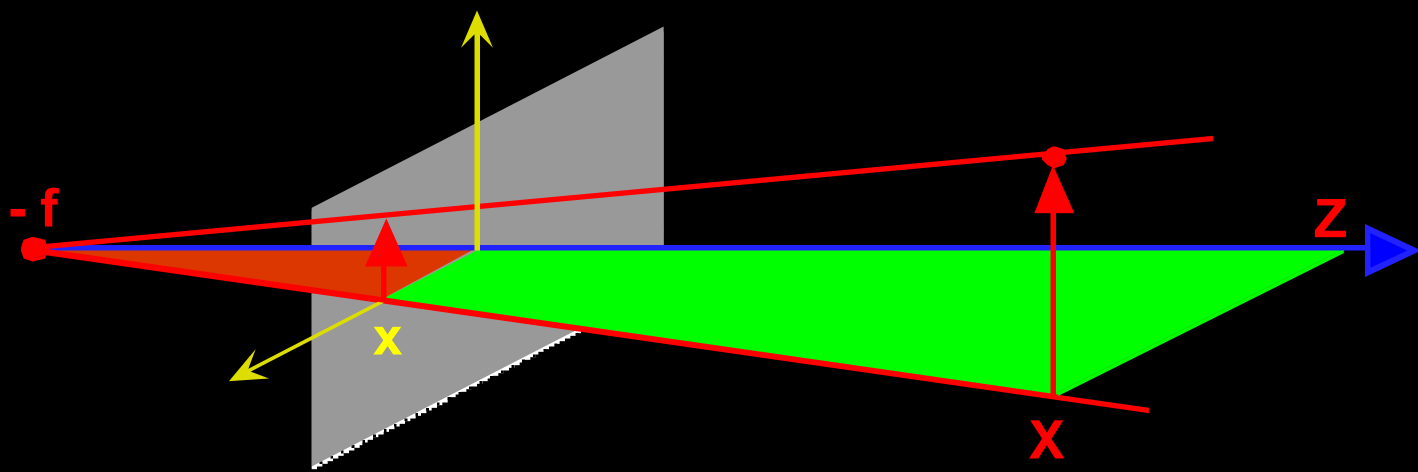
Perspective Projection

- Compute the image coordinates of p in terms of the world coordinates of P.



- Look at projections in x-z and y-z planes

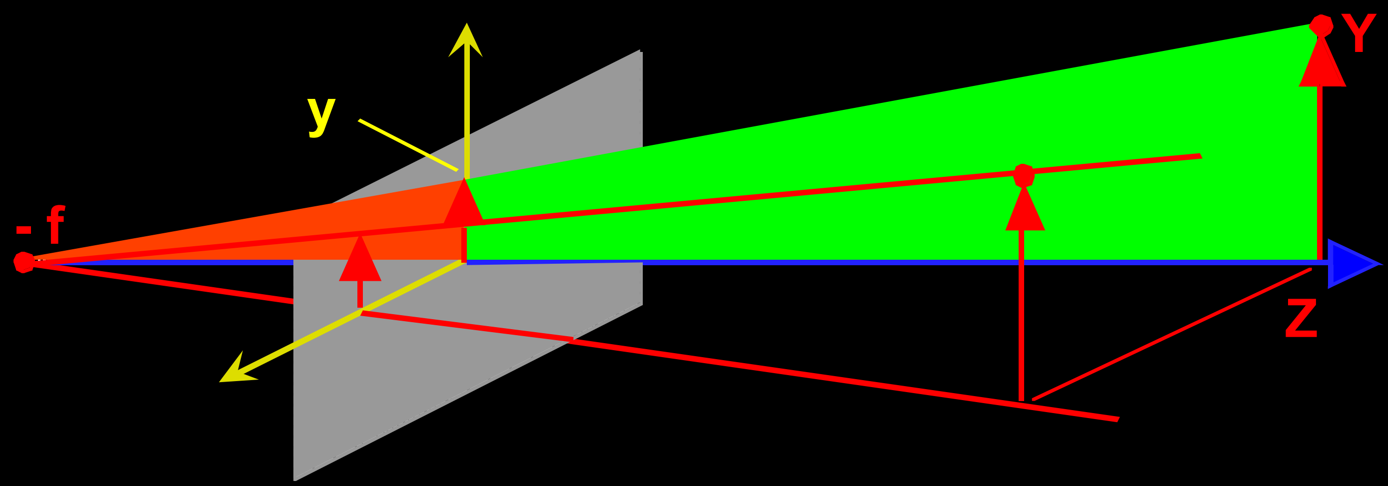
X-Z Projection



- By similar triangles: $\frac{x}{f} = \frac{X}{Z+f}$

$$x = \frac{fX}{Z+f}$$

Y-Z Projection



- By similar triangles: $\frac{y}{f} = \frac{Y}{Z+f}$

$$y = \frac{fY}{Z+f}$$

Perspective Equations

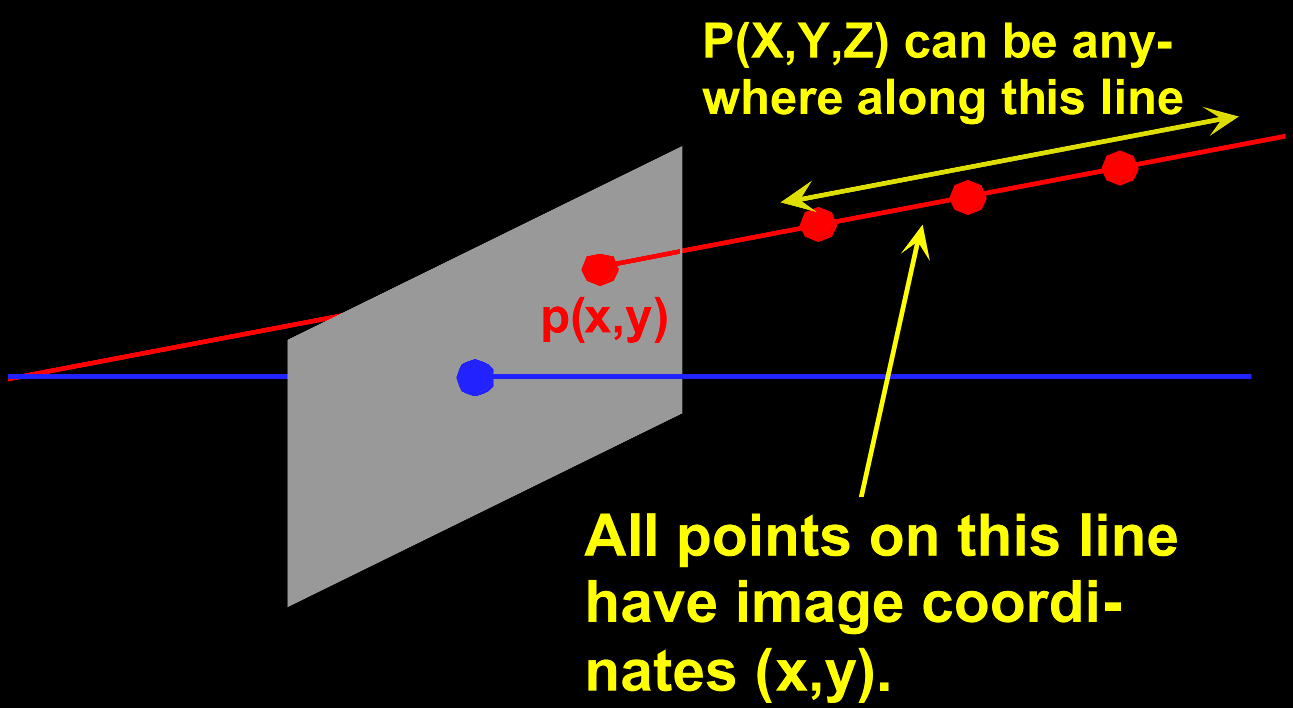
- Given point P(X,Y,Z) in the 3D world
- The two equations:

$$x = \frac{fX}{Z+f} \qquad y = \frac{fY}{Z+f}$$

- transform world coordinates (X,Y,Z) into image coordinates (x,y)

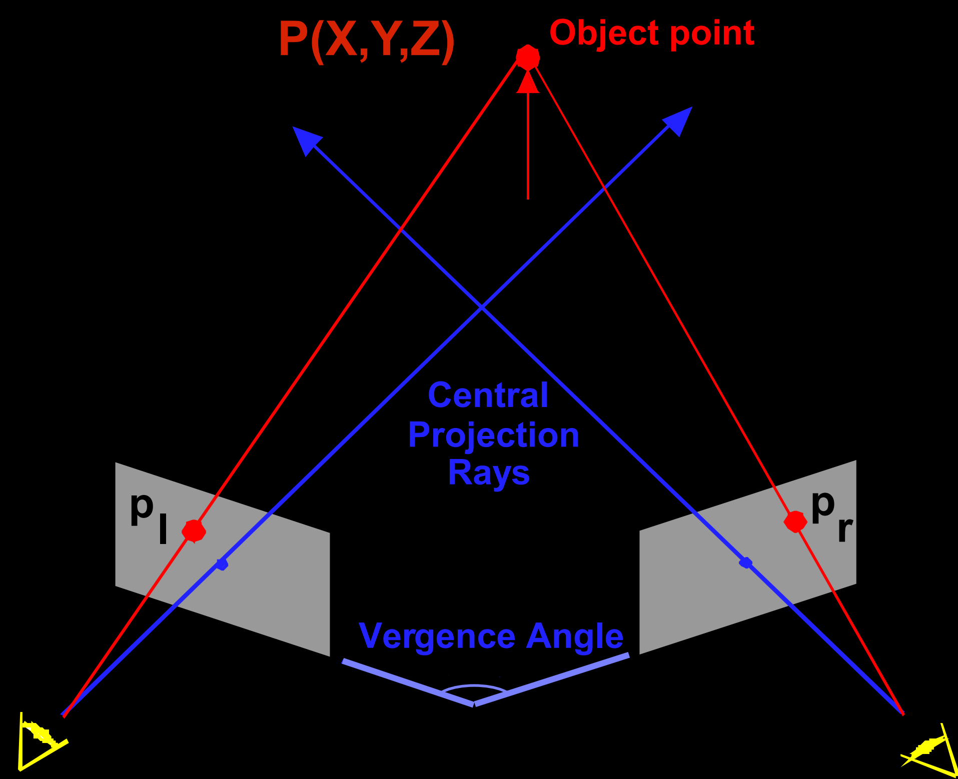
Reverse Projection

- Given a center of projection and image coordinates of a point, it is not possible to recover the 3D depth of the point from a single image.



In general, at least two images of the same point taken from two different locations are required to recover depth.

Stereo Geometry



- Depth obtained by triangulation
- Correspondence problem: p_l and p_r must correspond to the left and right projections of P, respectively.

Variability in appearance

- Consequences of image formation geometry for computer vision
  - What set of shapes can an object take on?
    - rigid
    - non-rigid
    - planar
    - non-planar
  - SIFT features
- Sensitivity to errors.

Radiometry

- Brightness: informal notion used to describe both scene and image brightness.
- Image brightness: related to energy flux incident on the image plane:

IRRADIANCE

Illuminance

- Scene brightness: brightness related to energy flux emitted (radiated) from a surface.

RADIANCE

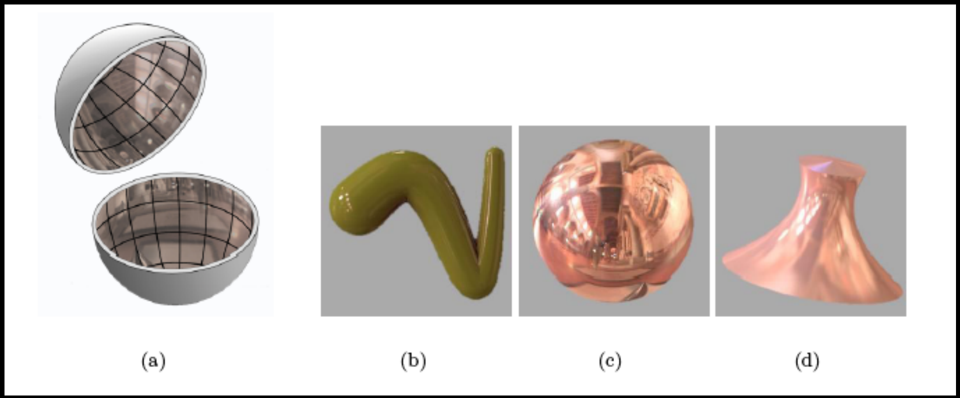
luminance

Light and Surfaces

- Reflection
  - mirrors
  - highlights
  - specularities
- Scattering
  - Lambertian
  - matte
  - diffuse

Light sources

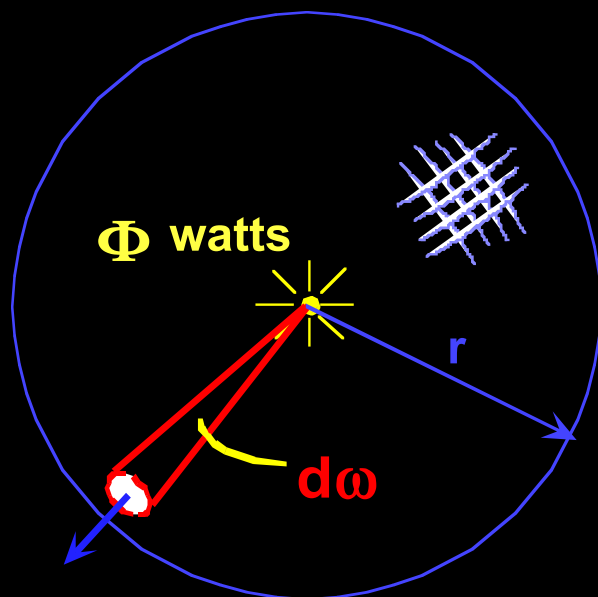
- Point source
- Extended source
- Single wavelength
- Multi-wavelength
- Uniform
- Non-uniform



Linearity of Light

- Linearity
  - definition
- For extended sources
- For multiple wavelengths
- Across time

- F watts radiated into 4p steradians



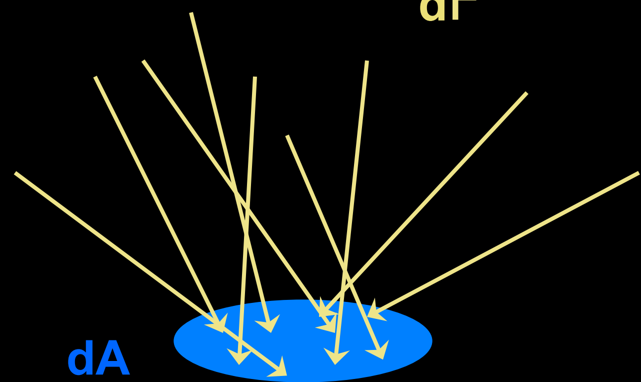
$$F = \int_{sphere} dF$$

R = Point Source Radiant Intensity = $\frac{dF}{dw}$ Watts/unit solid angle (steradian)

(of source)

Irradiance

- Light falling on a surface from all directions.
- How much?

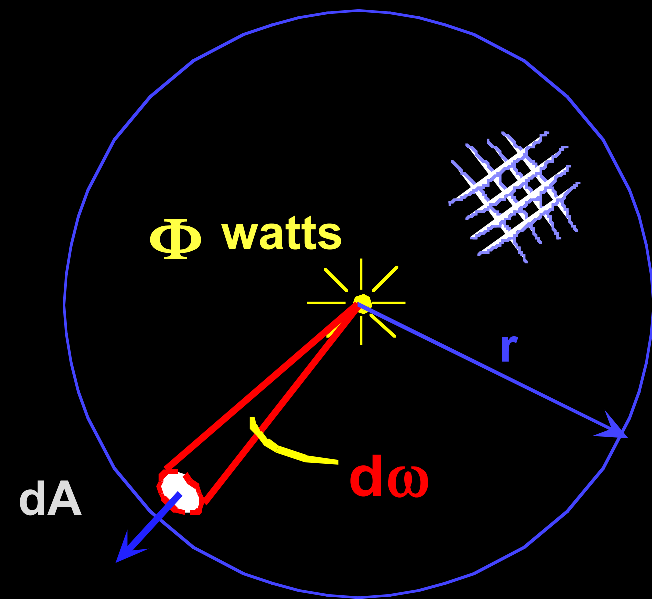


- Irradiance: power per unit area falling on a surface.

$$\text{Irradiance } E = \frac{dF}{dA} \quad \text{watts/m}^2$$

Inverse Square Law

- Relationship between point source radiance (radiant intensity) and irradiance



$$dw = \frac{dA}{r^2}$$

$$E = \frac{dF}{dA}$$

R: Radiant Intensity

E: Irradiance

F: Watts

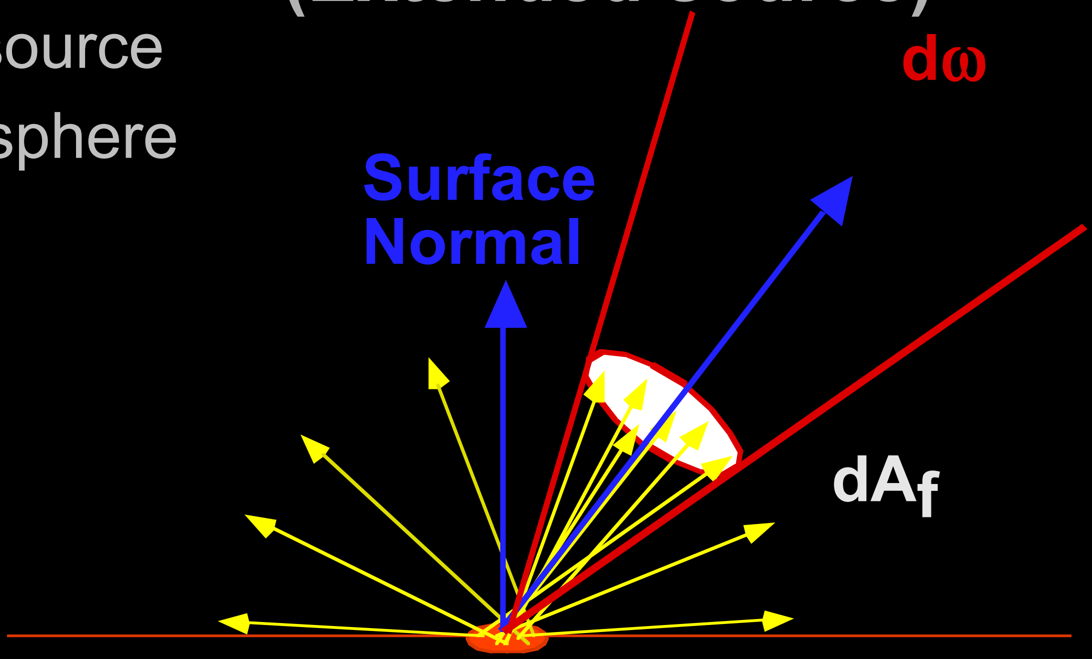
w : Steradians

$$R = \frac{dF}{dw} = \frac{r^2\, dF}{dA} = r^2 E$$

$$E = \frac{R}{r^2}$$

Surface Radiance (Extended source)

- Surface acts as light source
- Radiates over a hemisphere



R: Radiant Intensity

E: Irradiance

L: Surface radiance

- Surface Radiance: power per unit foreshortened area emitted into a solid angle

$$L = \frac{dF}{dA_f \, dw}$$

(watts/m^2 steradian)

Pseudo-Radiance

- Consider two definitions:
  - Radiance:
    power per unit foreshortened area emitted into a solid angle
  - Pseudo-radiance
    power per unit area emitted into a solid angle

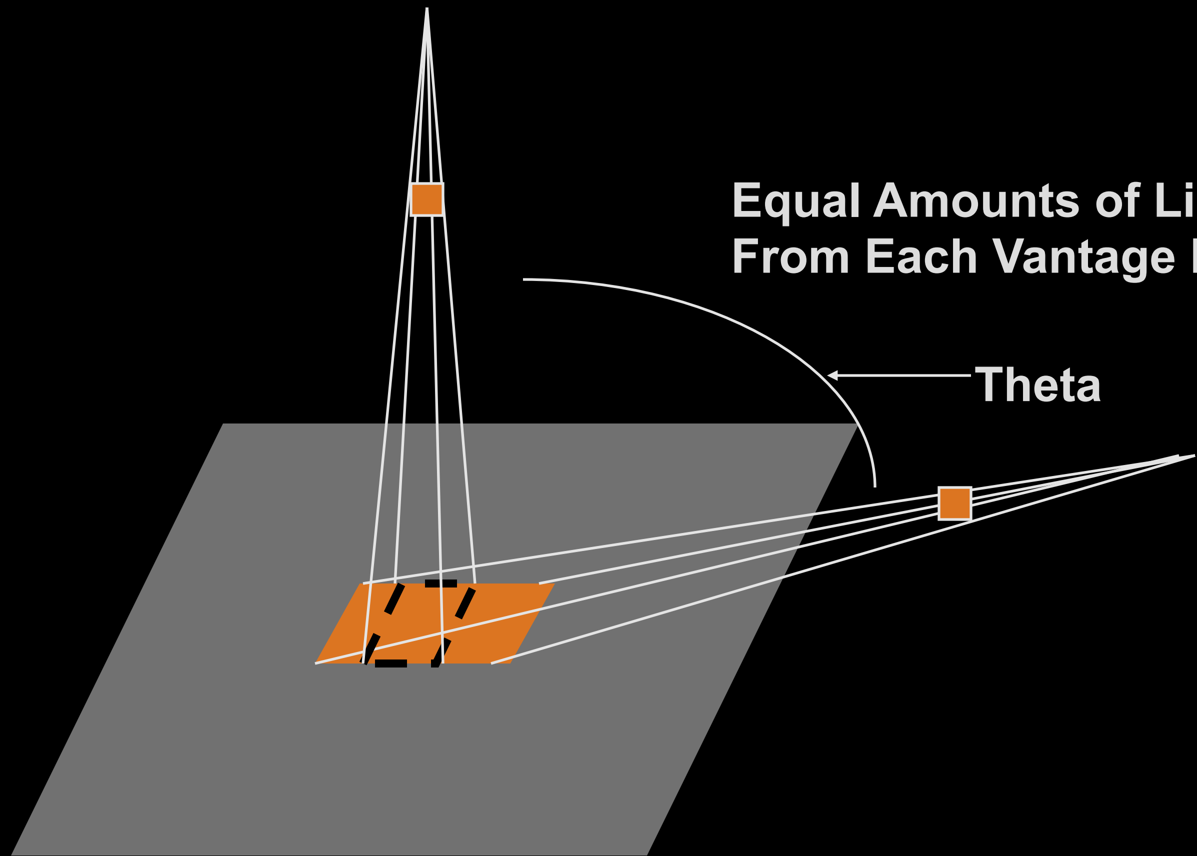
  - Why should we work with radiance rather than pseudo-radiance?
    - Only reason: Radiance is more closely related to our intuitive notion of “brightness”.

Lambertian Surfaces

- A particular point P on a Lambertian (perfectly matte) surface appears to have the same brightness no matter what angle it is viewed from.
  - Piece of paper
  - Matte paint
- Doesn’t depend upon incident light angle.
- What does this say about how they emit light?

Lambertian Surfaces

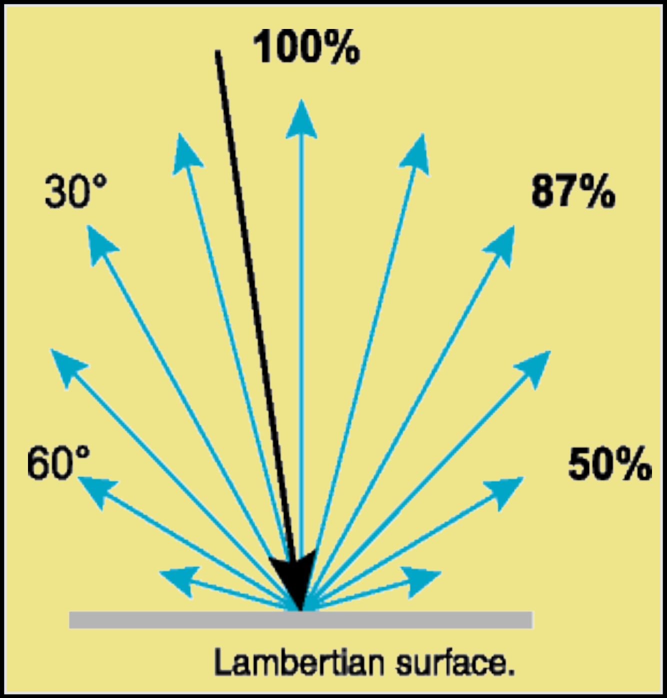
Equal Amounts of Light Observed
From Each Vantage Point



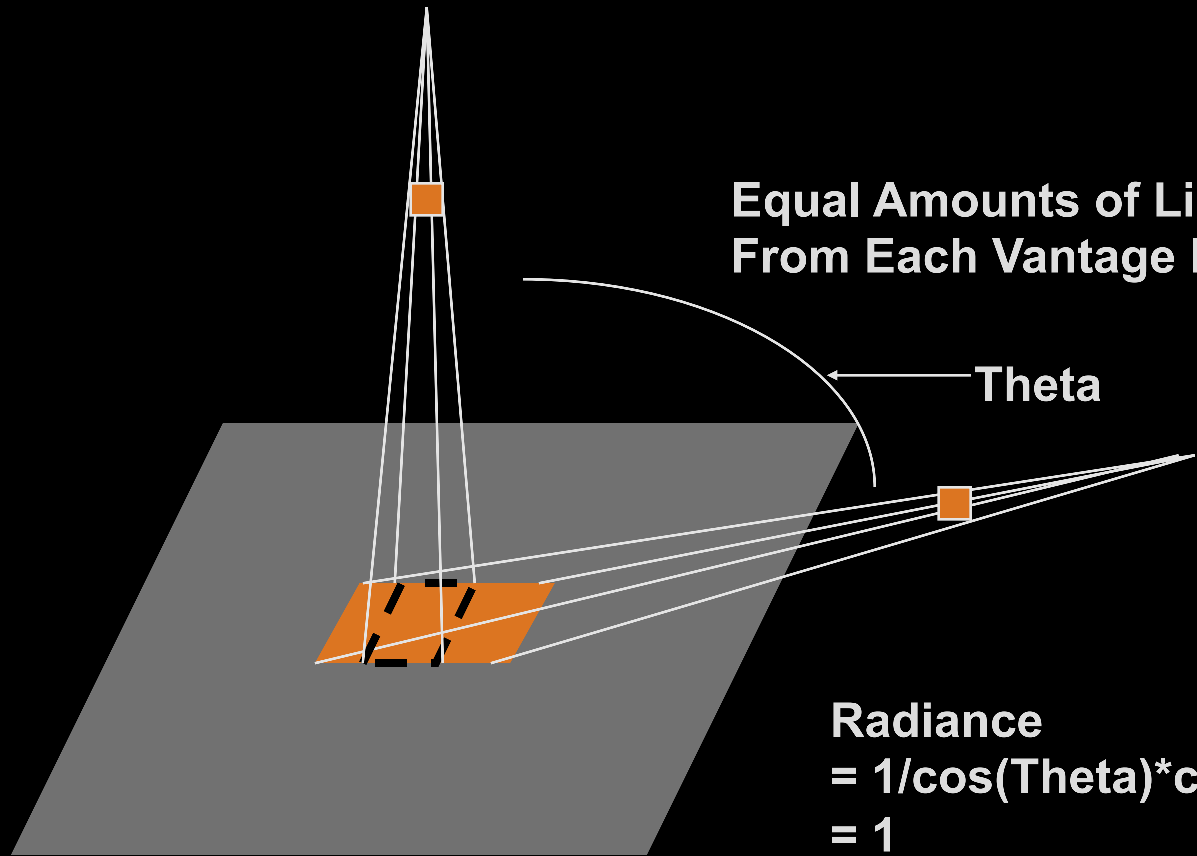
Area of black box = 1
Area of orange box = 1/cos(Theta)
Foreshortening rule.

Lambertian Surfaces

Relative magnitude of light scattered in each direction.
Proportional to cos (Theta).



Lambertian Surfaces



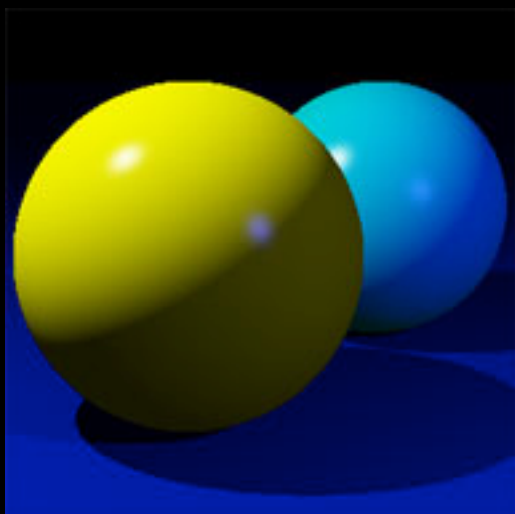
Equal Amounts of Light Observed
From Each Vantage Point

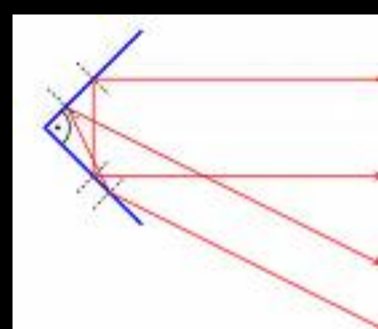
Radiance
= 1/cos(Theta)*cos(Theta)
= 1

Area of black box = 1
Area of orange box = 1/cos(Theta)
Foreshortening rule.

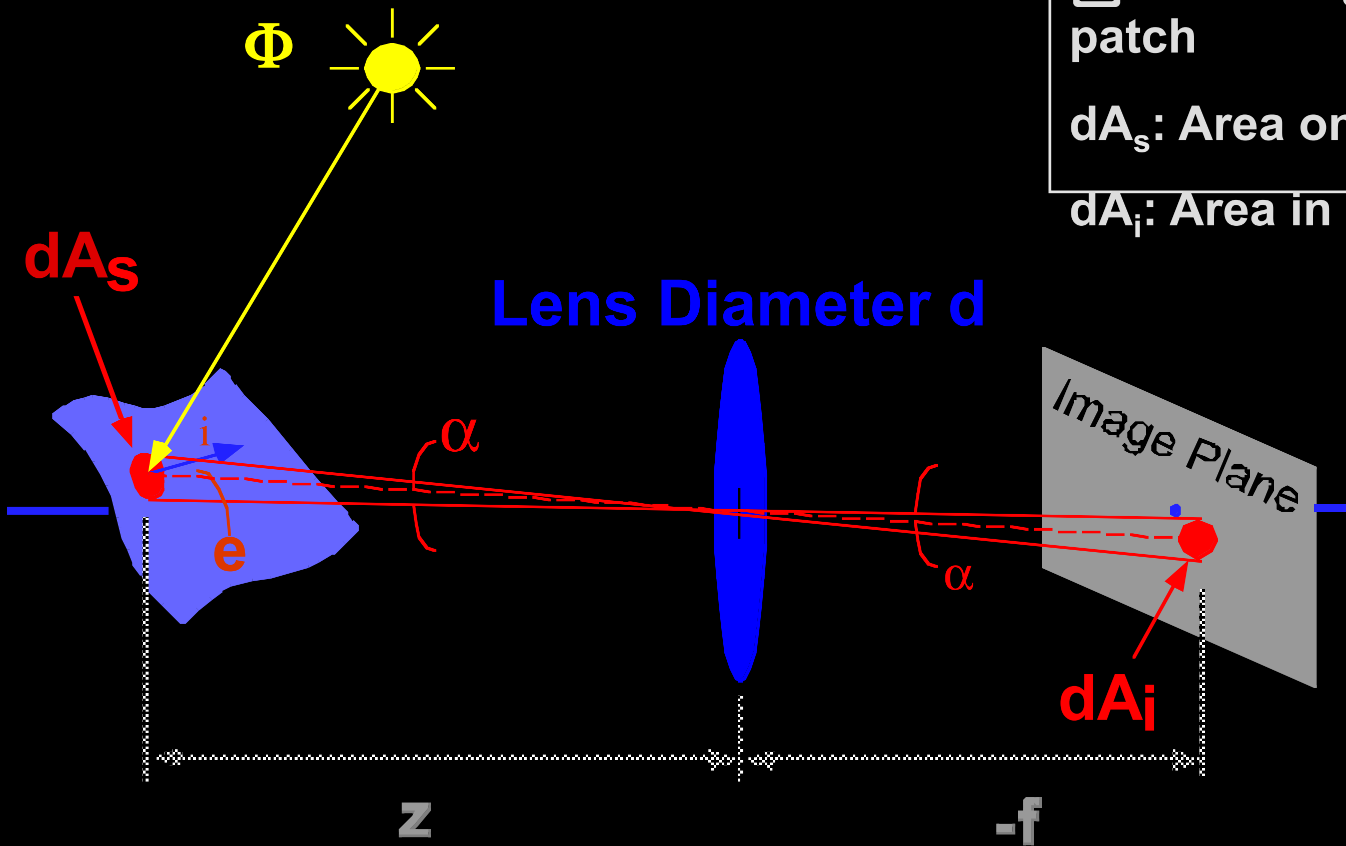
The BRDF

The bidirectional reflectance distribution function.





- Goal: Relate the radiance of a surface to the irradiance in the image plane of a simple optical system.



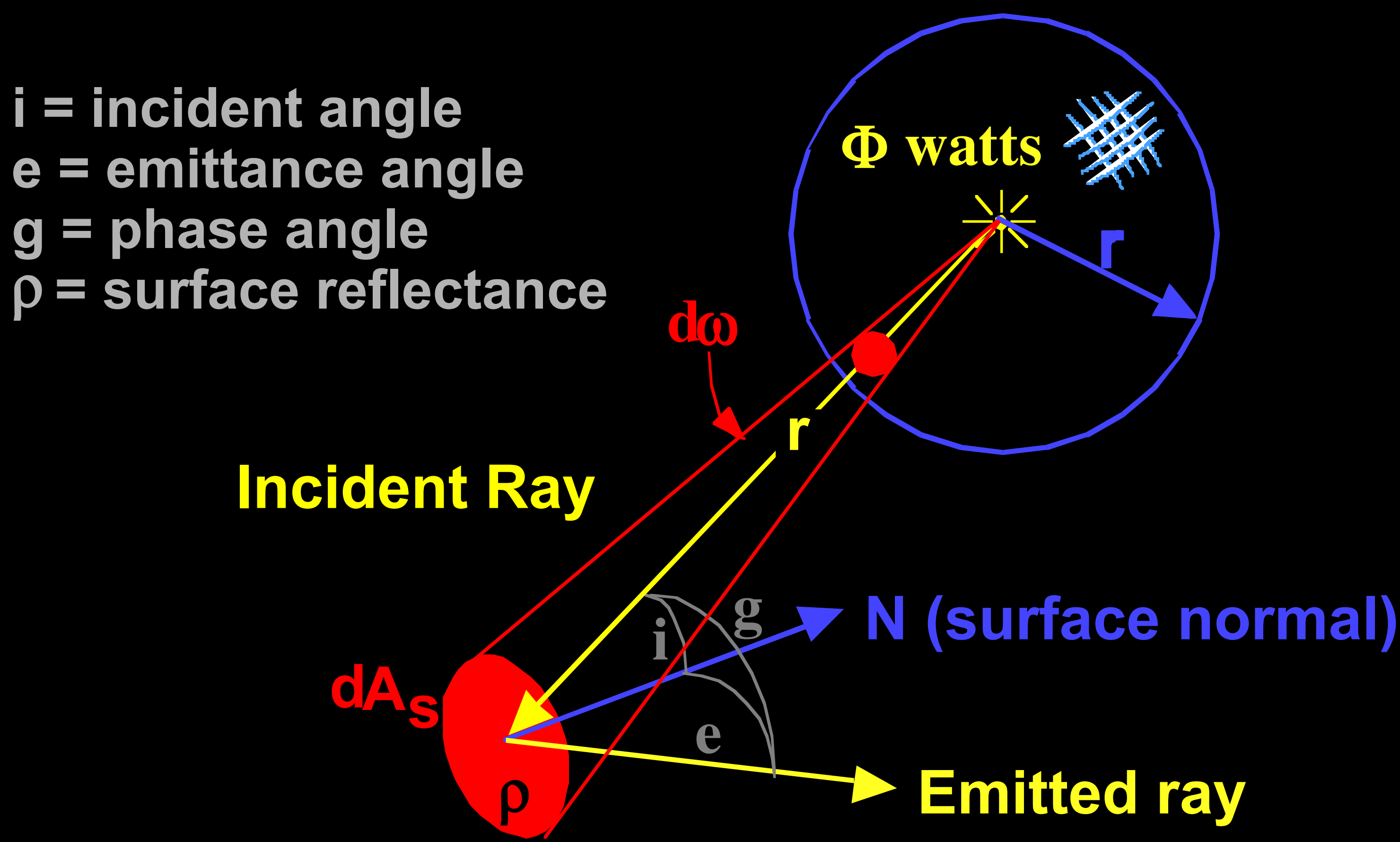
[symbol]: Solid angle of patch

dA_s: Area on surface

dA_i: Area in image

Light at the Surface

- E = flux incident on the surface (irradiance) = $\frac{dF}{dA}$



- We need to determine dF and dA

Reflections from a Surface I

dA

- dA = dA_scos i {foreshortening effect in direction of light source}

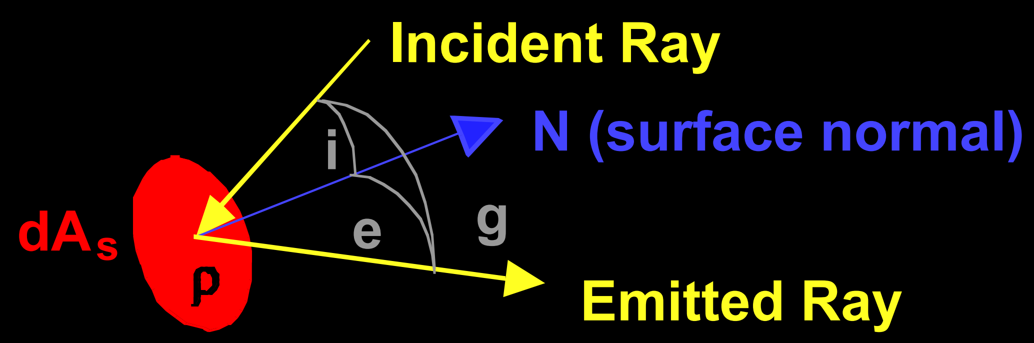
dF

- dF = flux intercepted by surface over area dA
  - dA subtends solid angle dw = dA_s cos i / r^2
  - dF = R dw = R dA_s cos i / r^2
  - E = dF / dA_s

Surface Irradiance: E = R cos i / r^2

Reflections from a Surface II

- Now treat small surface area as an emitter
  - ….because it is bouncing light into the world
- How much light gets reflected?

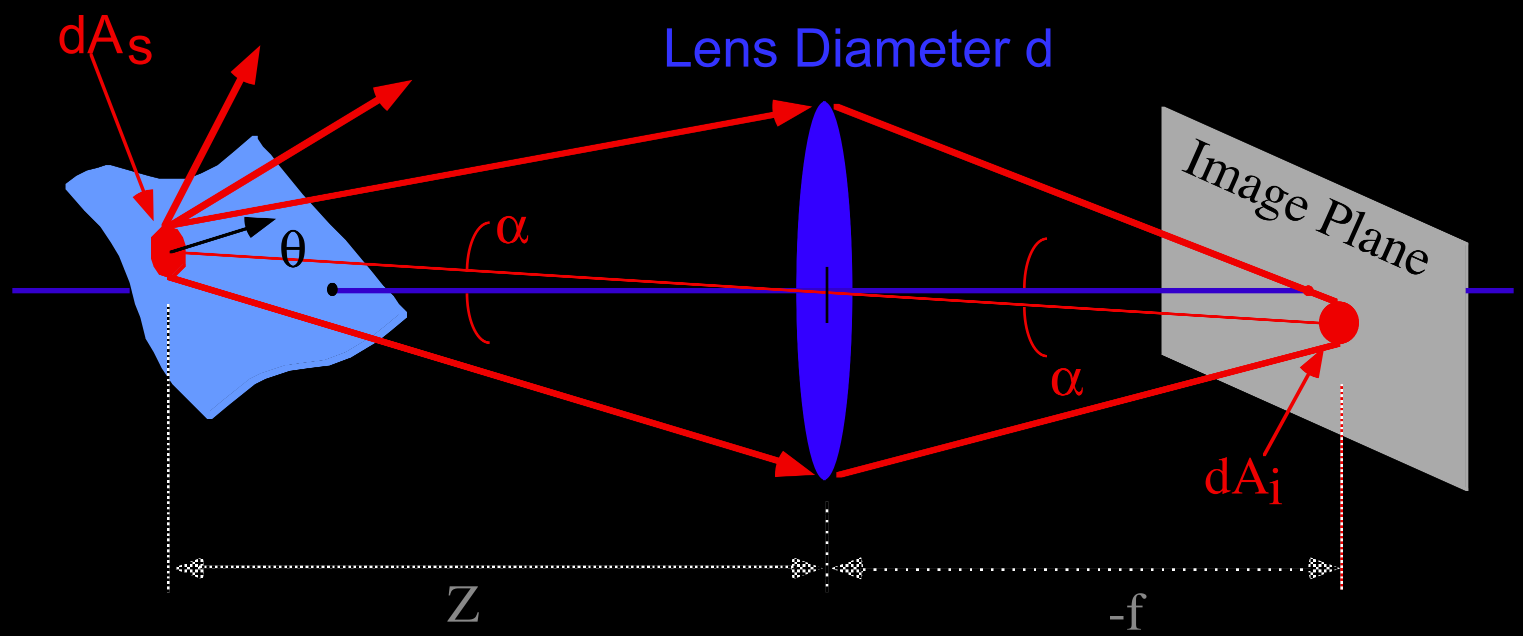


- E is the surface irradiance
- L is the surface radiance = luminance
- They are related through the surface reflectance function:

$$\frac{L_s}{E} = r(i,e,g,l)$$

May also be a function of the wavelength of the light

Power Concentrated in Lens



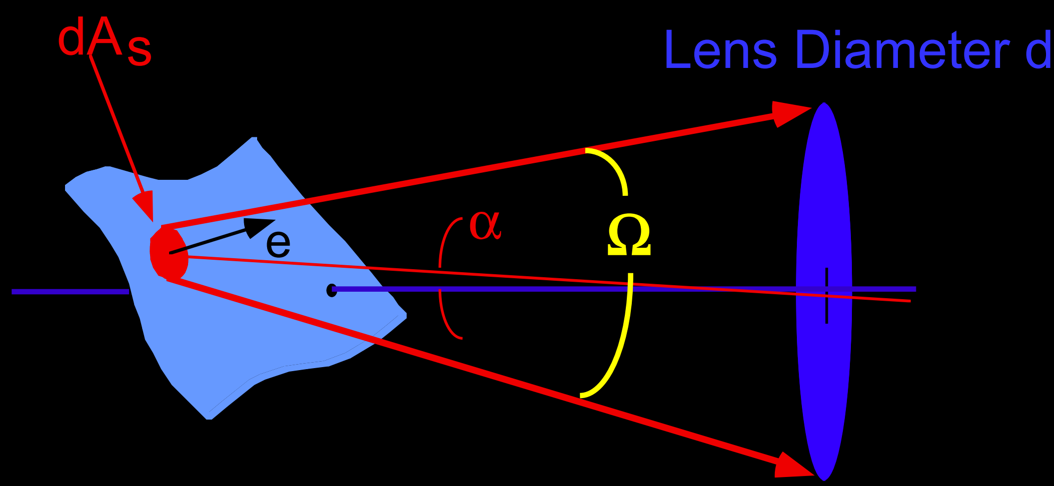
$$L_s = \frac{d^2F}{dA_s dw}$$

Luminance of patch (known from previous step)

What is the power of the surface patch as a source in the direction of the lens?

$$d^2F = L_s \, dA_s dw$$

Through a Lens Darkly



- In general:
  - L_s is a function of the angles i and e.
  - Lens can be quite large
  - Hence, must integrate over the lens solid angle to get dF

$$dF = dA_s \int_W L_s \, dW$$

Simplifying Assumption

- Lens diameter is small relative to distance from patch

$$dF = dA_s \int_W L_s \, dW$$

L_s is a constant and can be removed from the integral

$$dF = dA_s L_s \int_W dW$$

Surface area of patch in direction of lens

$$= dA_s \cos e$$

Solid angle subtended by lens in direction of patch

$$= \frac{\text{Area of lens as seen from patch}}{(\text{Distance from lens to patch})^2}$$

$$= \frac{p\,(d/2)^2 \cos a}{(z / \cos a)^2}$$

Putting it Together

$$dF = dA_s \int_W L_s \, dW$$

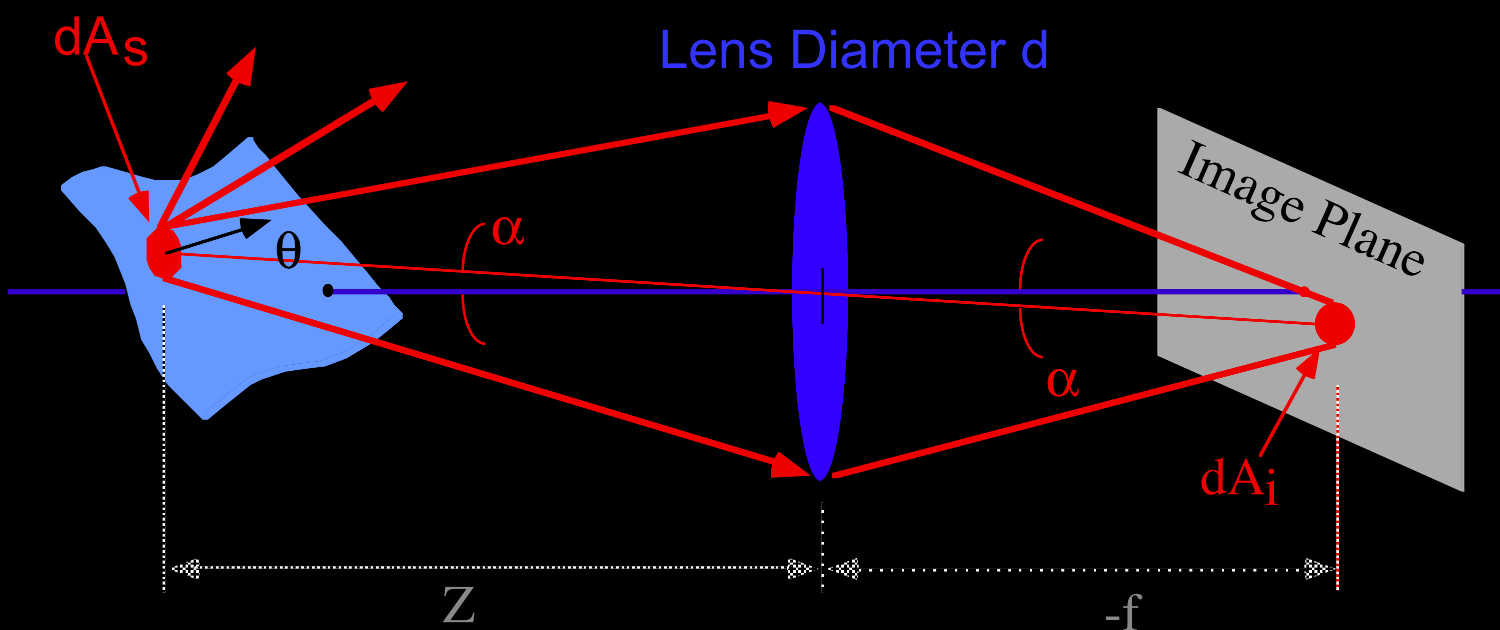
$$= dA_s \cos e \; L_s \frac{p\,(d/2)^2 \cos a}{(z / \cos a)^2}$$

- Power concentrated in lens:

$$dF = \frac{p}{4} L_s \, dA_s \left[\frac{d}{Z}\right]^2 \cos e \cos^3 a$$

- Assuming a lossless lens, this is also the power radiated by the lens as a source.

Through a Lens Darkly

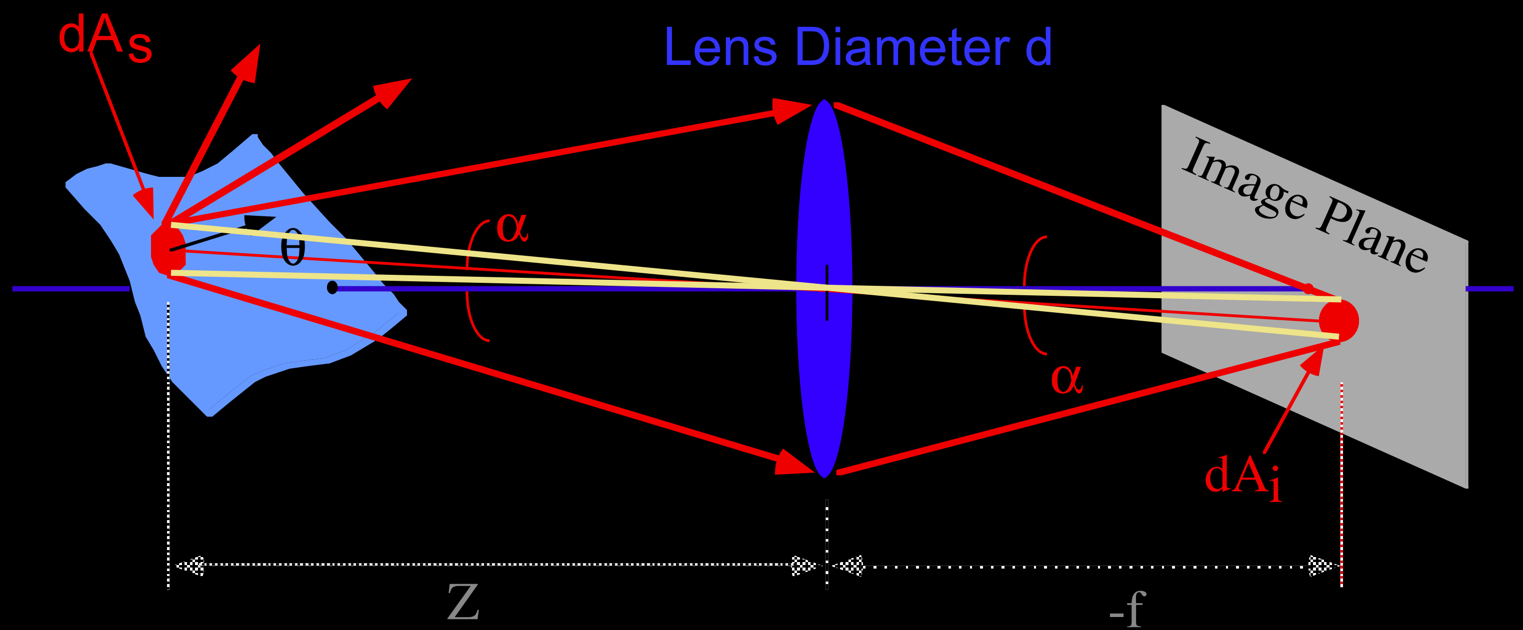


- Image irradiance at $dA_i = \dfrac{dF}{dA_i} = E_i$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{p}{4} \left[\frac{d}{Z}\right]^2 \cos e \cos^3 a$$

ratio of areas

Patch ratio



The two solid angles are equal

$$\frac{dA_s \cos e}{(Z / \cos a)^2} = \frac{dA_i \cos a}{(-f / \cos a)^2} \Rightarrow \frac{dA_s}{dA_i} = \frac{\cos a}{\cos e}\left(\frac{Z}{-f}\right)^2$$

The Fundamental Result

- Source Radiance to Image Sensor Irradiance:

$$\frac{dA_s}{dA_i} = \frac{\cos a}{\cos e}\left(\frac{Z}{-f}\right)^2$$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{p}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3 a$$

$$E_i = L_s \frac{\cos a}{\cos e}\left(\frac{Z}{-f}\right)^2 \frac{p}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3 a$$

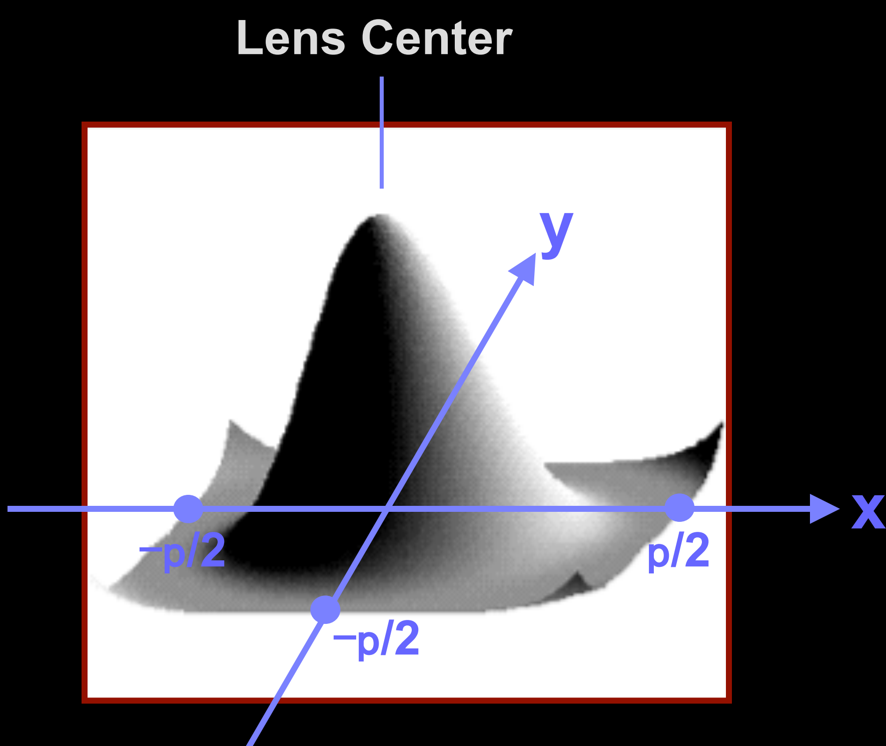
$$E_i = L_s \frac{p}{4}\left(\frac{d}{-f}\right)^2 \cos^4 a$$

Radiometry Final Result

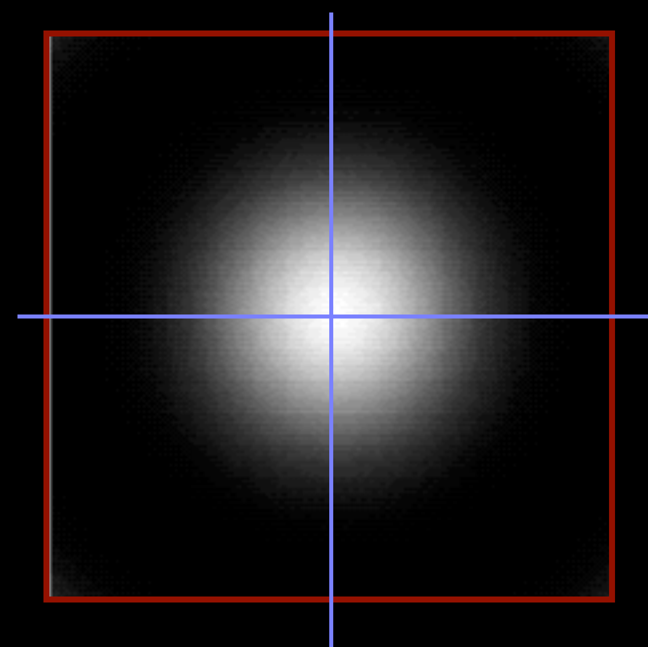
$$E_i = L_s \frac{p}{4}\left(\frac{d}{-f}\right)^2 \cos^4 a$$

- Image irradiance is a function of:
  - Scene radiance L_s
  - Focal length of lens f
  - Diameter of lens d
    - f/d is often called the 'effective focal length' of the lens
  - Off-axis angle a

$Cos^4 a$ Light Falloff

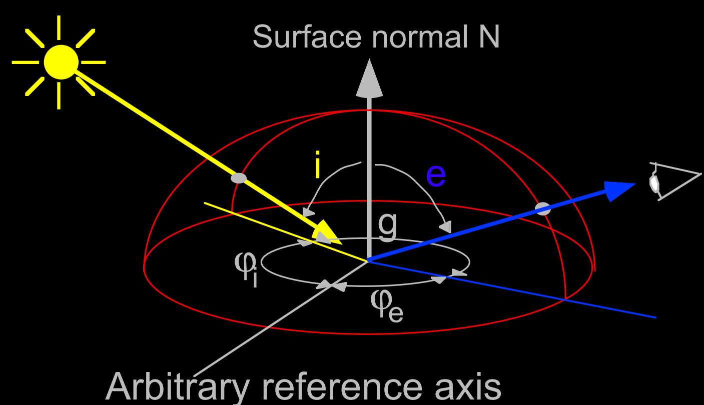


Top view shaded by height



Limitation of Radiometry Model

- Surface reflection r can be a function of viewing and/or illumination angle

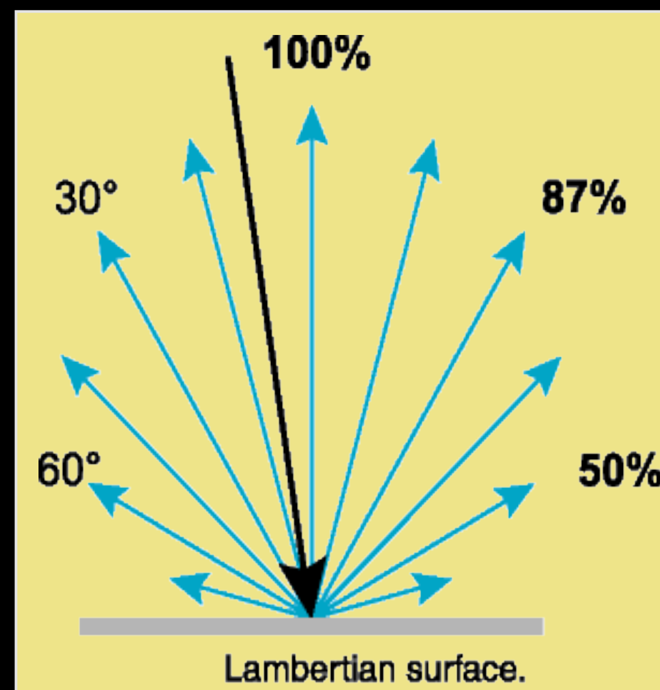


$$r(i,e,g,j_i,j_e) = \frac{dL(e,j_e)}{dE(i,j_i)}$$

- r may also be a function of the wavelength of the light source
- Assumed a point source (sky, for example, is not)

Lambertian Surfaces

- The BRDF for a Lambertian surface is a constant
  - $r(i,e,g,j_e,j_i) = k$
  - function of cos e due to the foreshortening effect
  - k is the 'albedo' of the surface
  - Good model for diffuse surfaces
- Other models combine diffuse and specular components (Phong, Torrance-Sparrow, Oren-Nayar)



BRDF

- Ron Dror's thesis

Reflection parameters

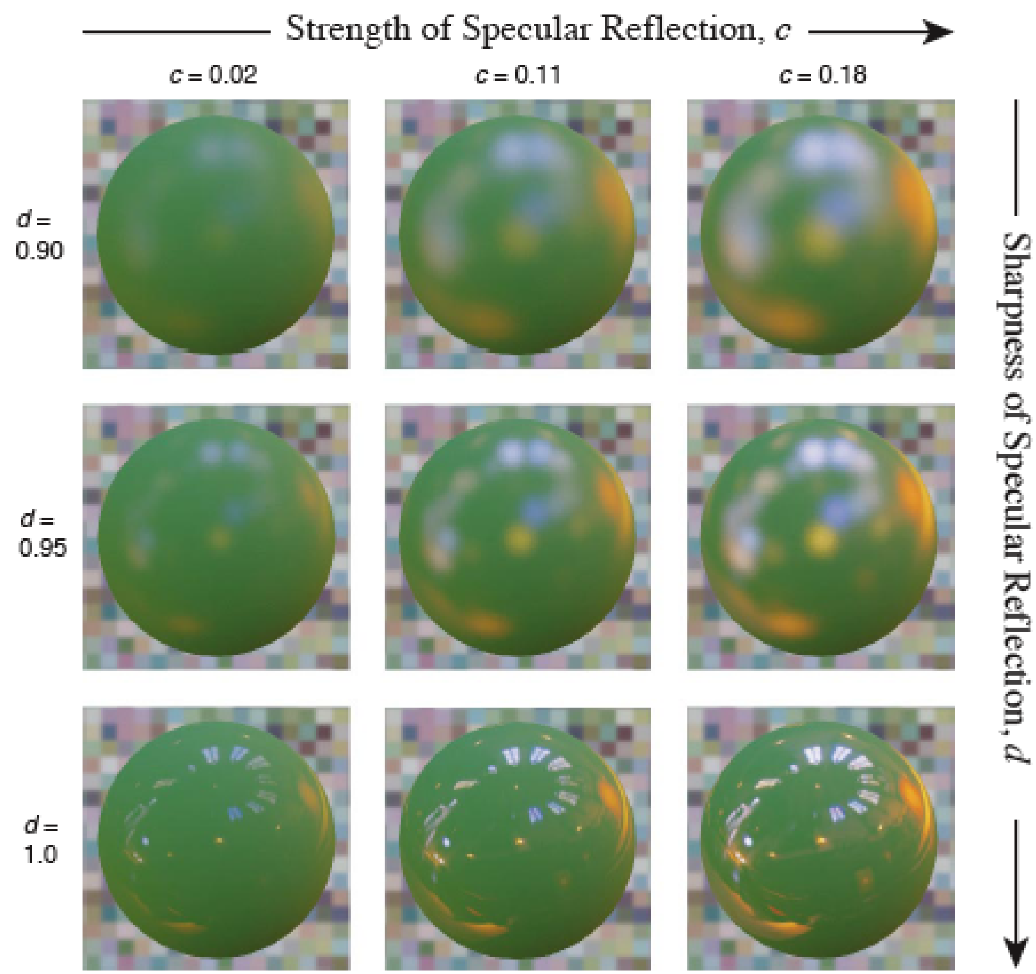
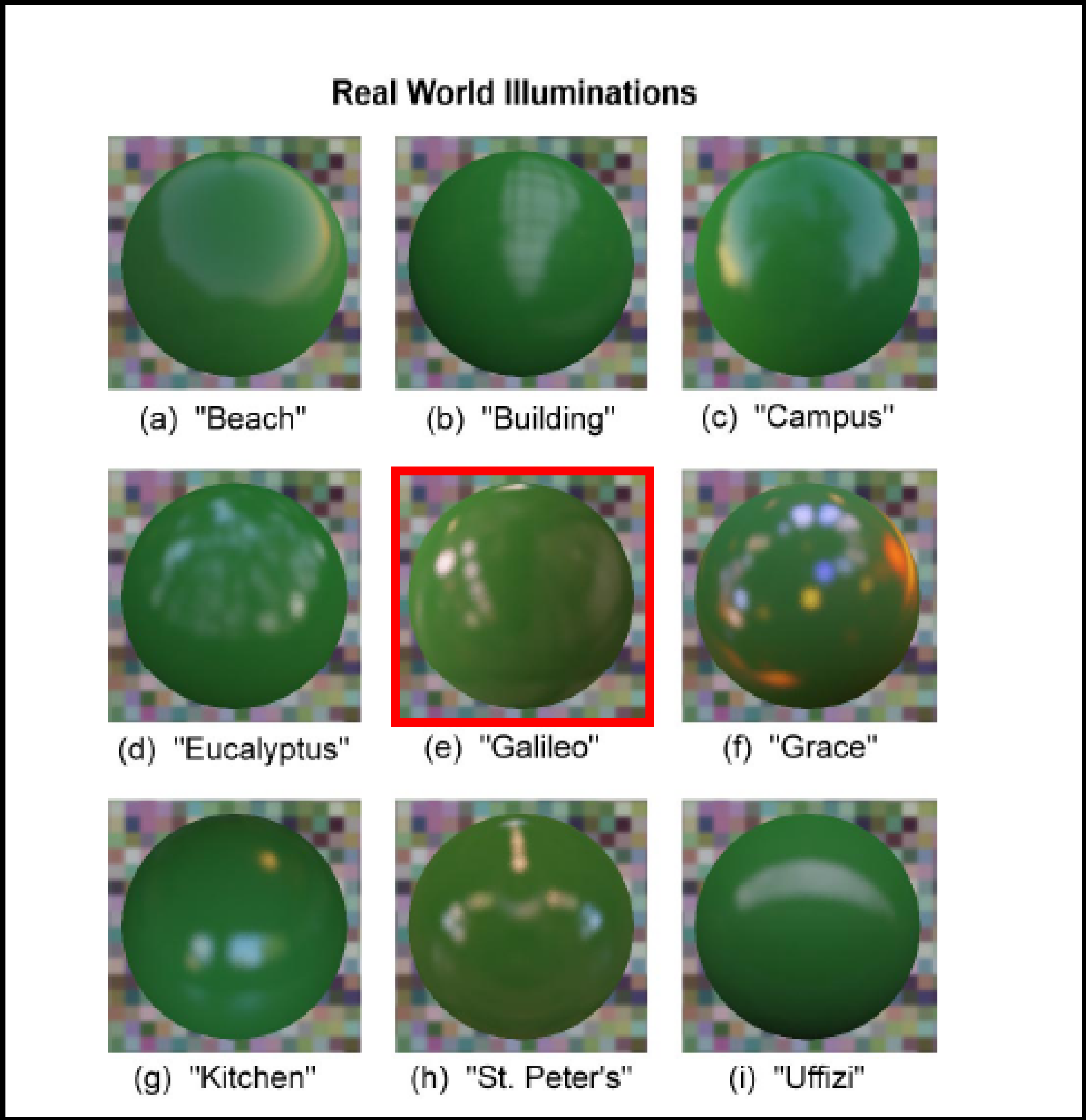
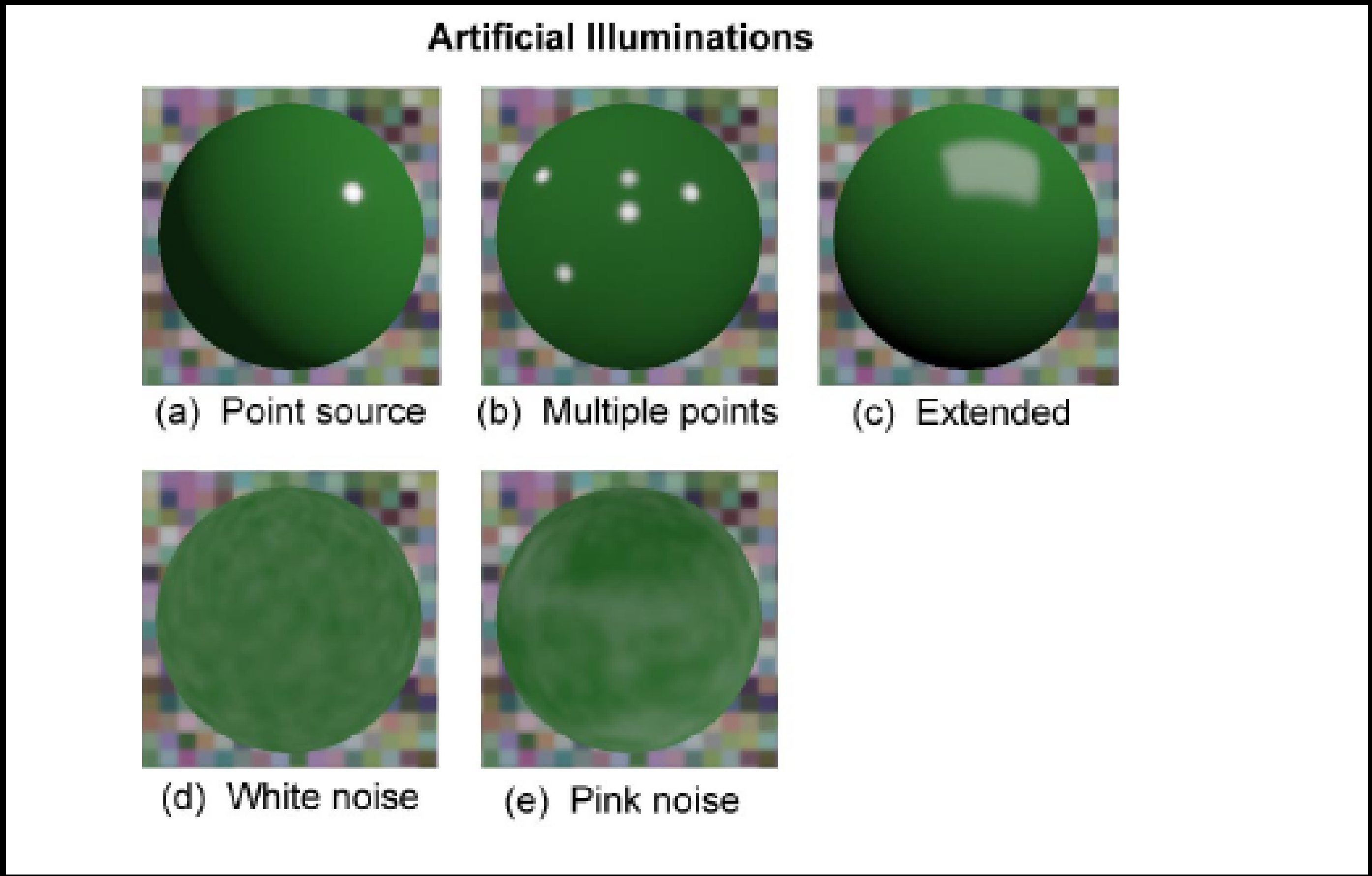


Figure 2.6. Grid showing range of reflectance properties used in the experiments for a particular real-world illumination map. All the spheres shown have an identical diffuse component. In Pellacini's reparameterization of the Ward model, the specular component depends on the c and d parameters. The strength of specular reflection, c, increases with ρ_s, while the sharpness of specular reflection, d, decreases with α. The images were rendered in *Radiance*, using the techniques described in Appendix B.

Real World Light Variation



Fake Light and Real Light



Simple and Complex Light

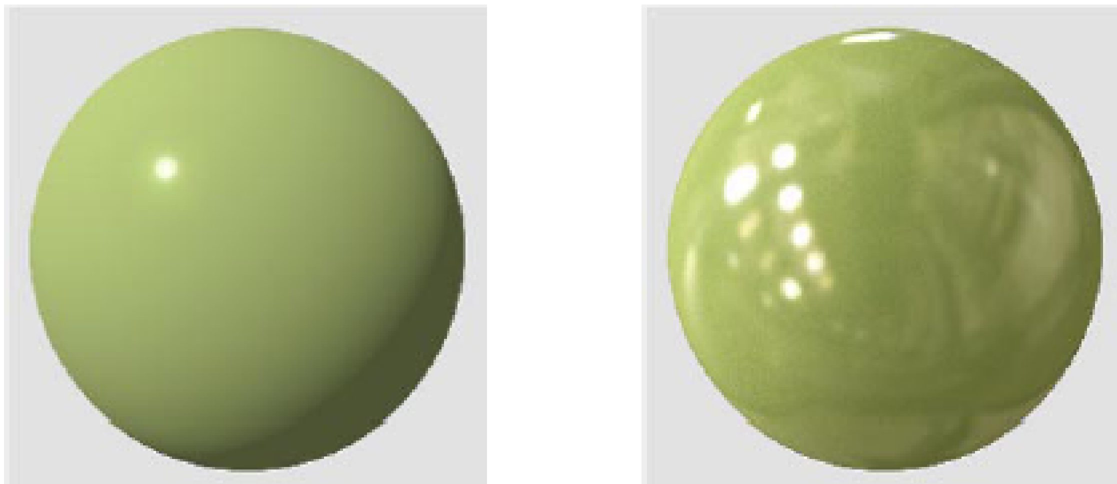

(a) (b)

Figure 2.9. (a) A shiny sphere rendered under illumination by a point light source. (b) The same sphere rendered under photographically-acquired real-world illumination. Humans perceive reflectance properties more accurately in (b).

BRDF again

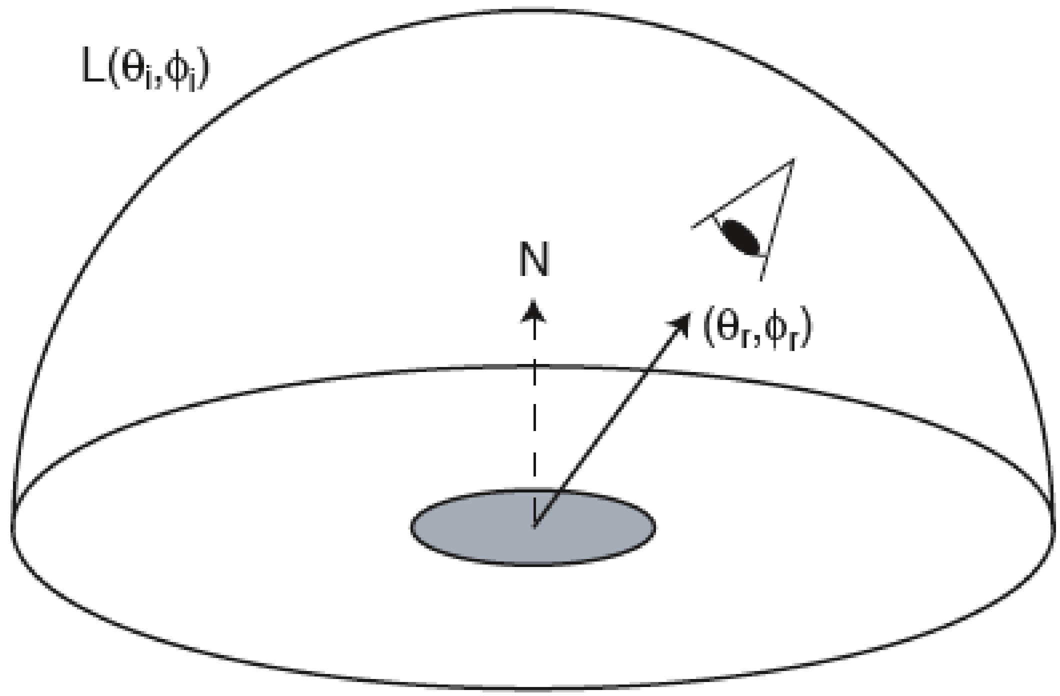


Figure 3.1. A viewer observes a surface patch with normal N from direction (θ_r, ϕ_r). $L(\theta_i, \phi_i)$ represents radiance of illumination from direction (θ_i, ϕ_i). The coordinate system is such that N points in direction $(0, 0)$.

$$B(\theta_r, \phi_r) = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\pi/2} L(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \sin\theta_i \, d\theta_i \, d\phi_i,$$

Likelihood of material

Figure 3.6. A photograph of a matte sphere, shown against a uniform gray background. This image could also be produced by a chrome sphere under appropriate illumination, but that scenario is highly unlikely.

Classifying Under Uncertainty

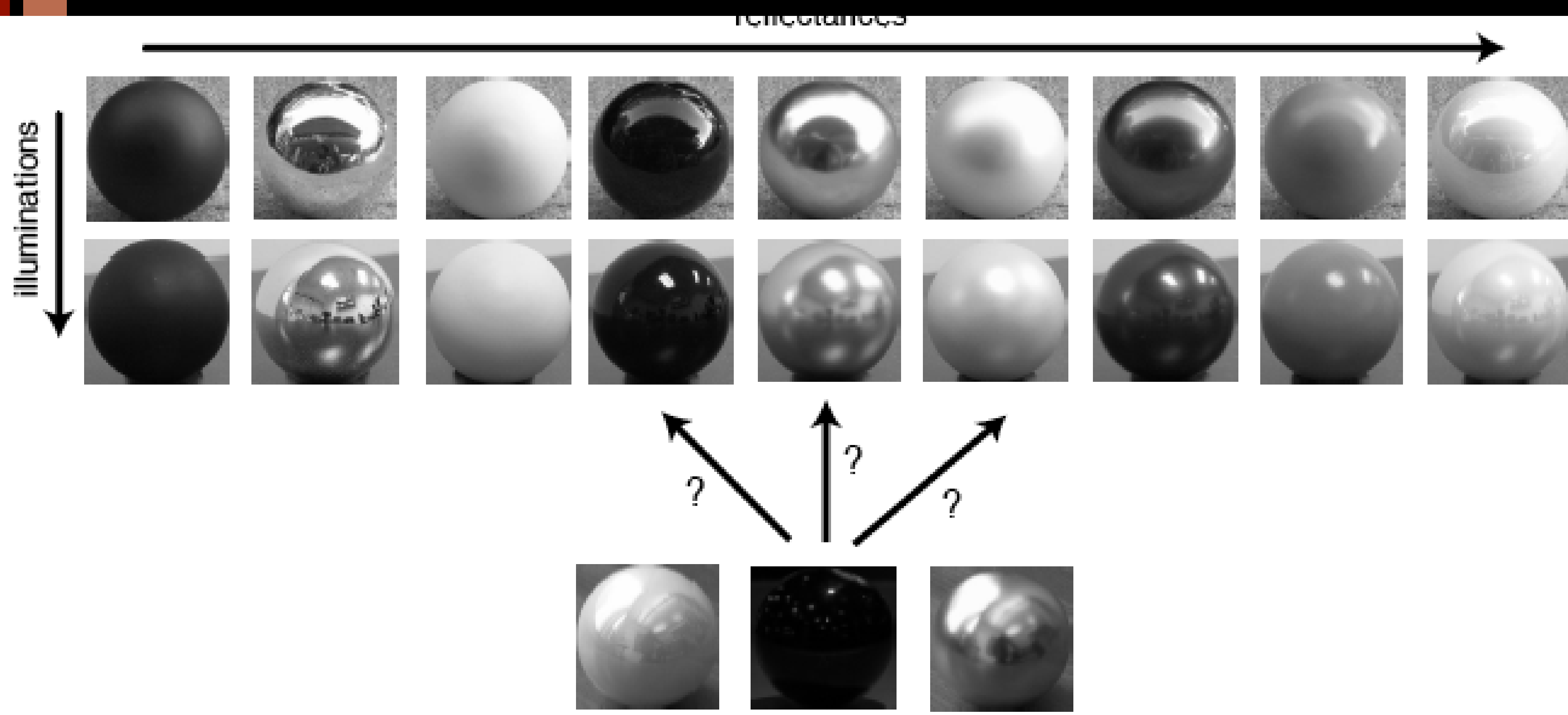
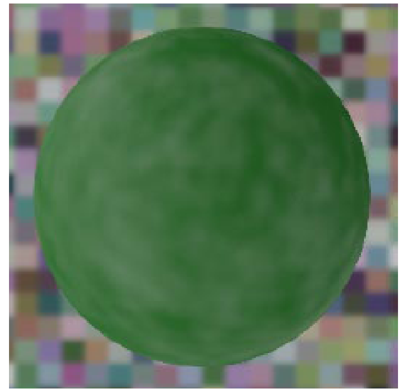


Figure 3.7. The problem addressed by a classifier of Chapter 6, illustrated using a database of photographs. Each of nine spheres was photographed under seven different illuminations. We trained a nine-way classifier using the images corresponding to several illuminations, and then used it to classify individual images under novel illuminations.

More Fake Light

(a) Original



(b) $1/f^2$ power spectrum

(c) Heeger and Bergen texture

(d) Portilla and Simoncelli texture

Figure 4.14. Spheres of identical reflectance properties rendered under a photographically-acquired illumination map (a) and three synthetic illumination maps (b-d). The illumination in (b) is Gaussian noise with a $1/f^2$ power spectrum. The illumination in (c) was synthesized with the procedure of Heeger and Bergen [43] to match the pixel histogram and marginal wavelet histograms of the illumination in (a). The illumination in (d) was synthesized using the technique of Portilla and Simoncelli, which also enforces conditions on the joint wavelet histograms. The illumination map of (a) is due to Debevec [24].

Illumination Maps

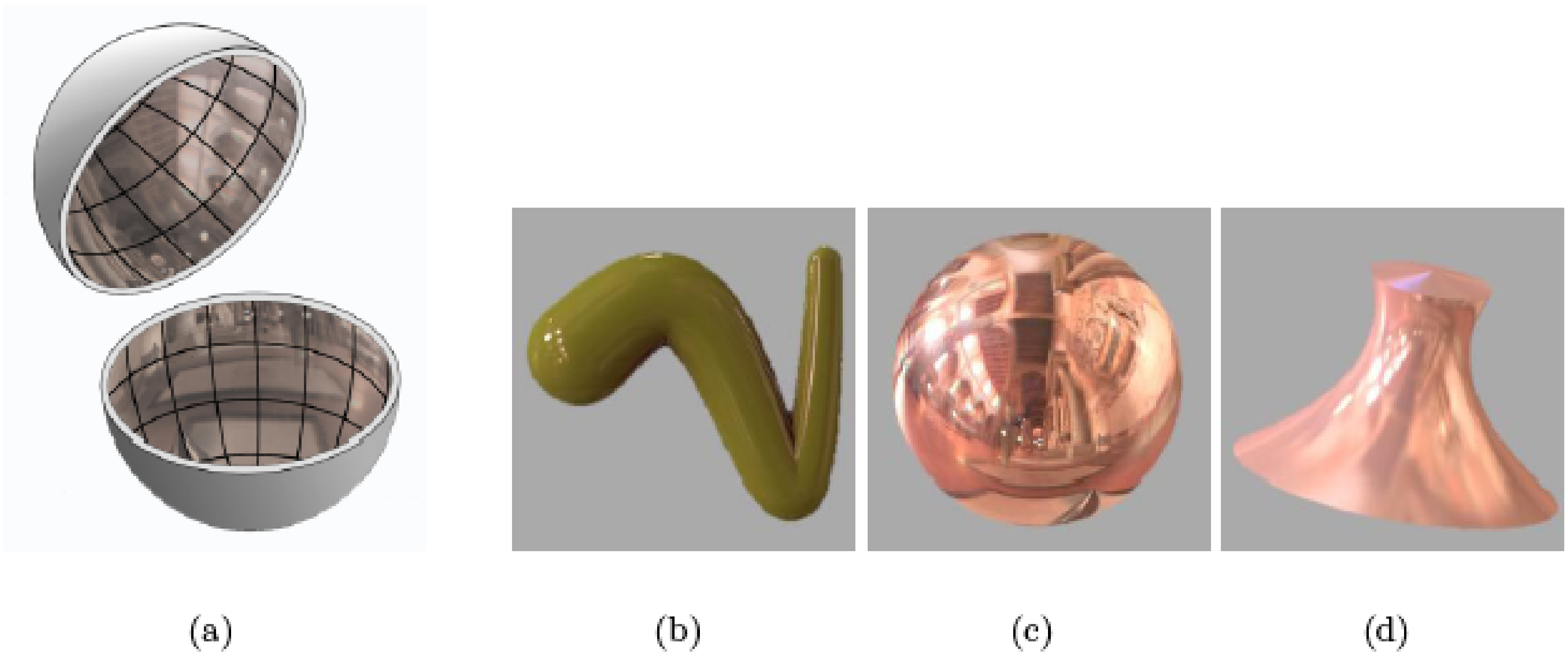


Figure 5.2. (a) A photographically-acquired illumination map, illustrated on the inside of a spherical shell. The illumination map is identical to that of Figure 4.1d. (b-d) Three surfaces of different geometry and reflectance rendered under this illumination map using the methods of Appendix B.

Classification

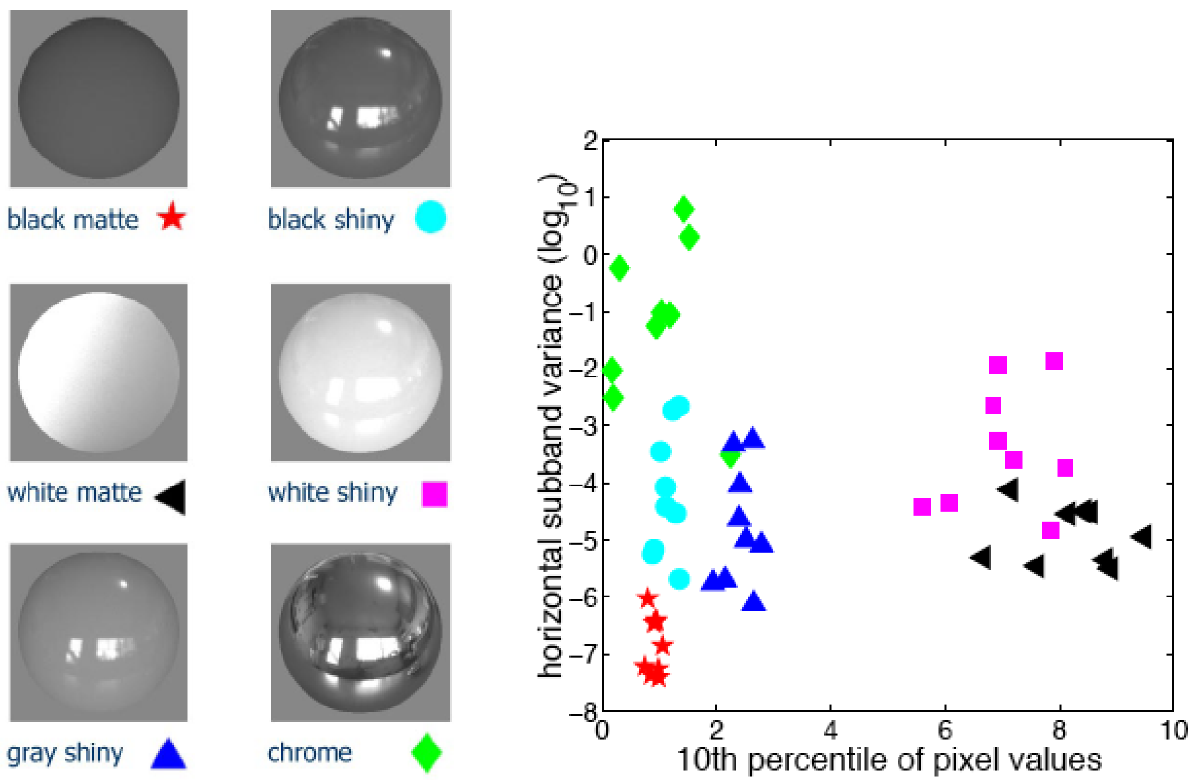


Figure 5.11. At left, synthetic spheres of 6 different reflectances, each rendered under one of Debevec's illumination maps. Ward model parameters are as follows: black matte, $\rho_d = .1$, $\rho_s = 0$; black shiny, $\rho_d = .1$, $\rho_s = .1$, $\alpha = .01$; white matte, $\rho_d = .9$, $\rho_s = 0$; white shiny, $\rho_d = .7$, $\rho_s = .25$, $\alpha = .01$; chrome, $\rho_d = 0$, $\rho_s = .75$, $\alpha = 0$; gray shiny, $\rho_d = .25$, $\rho_s = .05$, $\alpha = .01$. We rendered each sphere under the nine photographically-acquired illuminations depicted in Figure 2.7 and plotted a symbol corresponding to each in the two-dimensional feature space at right. The horizontal axis represents the 10th percentile of pixel intensity, while the vertical axis is the log variance of horizontally-oriented QMF wavelet coefficients at the second-finest scale, computed after geometrically distorting the original image as described in Section 6.1.2.

Introduction to Computer Vision

Classification Continued

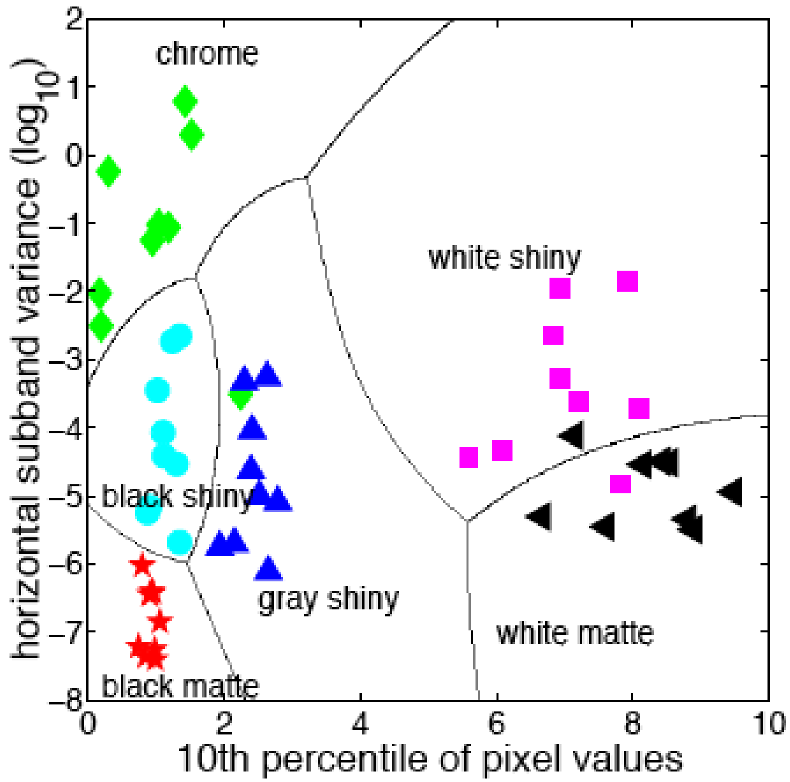


Figure 5.12. The curved lines separate regions assigned to different reflectances by a simple classifier based on two image features. The training examples are the images described in Figure 5.11. The classifier is a one-versus-all support vector machine, described in Section 6.1.1. Using additional image features improves classifier performance.

Lighting direction vs. Viewer direction

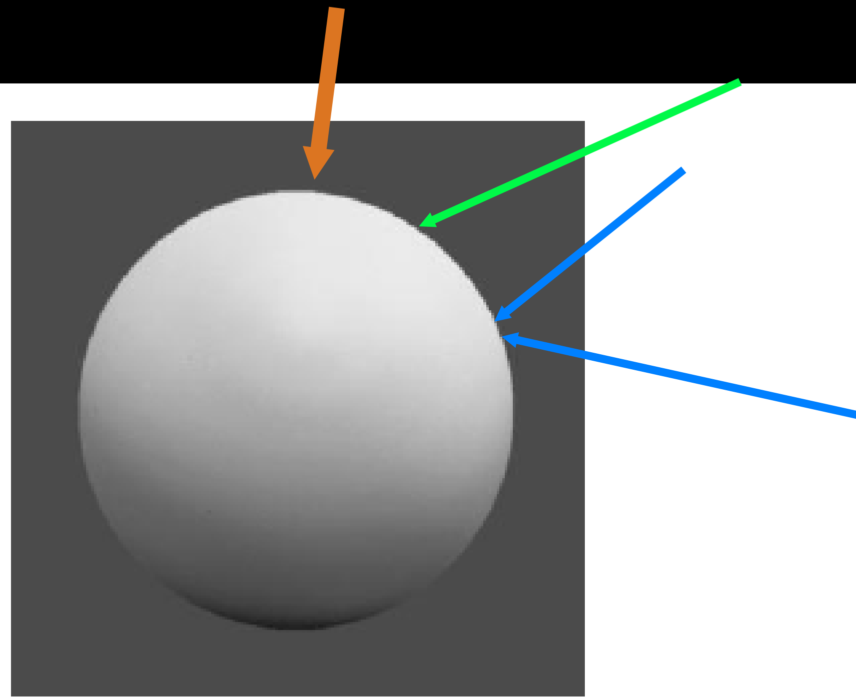


Figure 3.6. A photograph of a matte sphere, shown against a uniform gray background.

For Lambertian surface:

Viewer direction is irrelevant

Lighting direction is *very relevant*

Lambertian Sphere

Orthogonal Projection,
Infinitely Distant Point Light from -90 to +90 degrees