Computer Science 799DD, Learning to See

http://www.cs.umass.edu/∼elm/learning2see/

Assignment 2

Warm-ups. (You don't need to turn these in.)

A. Define the entropy of a discrete random variable (without looking it up). If you can't do this, spend some time to memorize it. Be careful about notation. Make sure to distinguish among random variables, the individual values a random variable can take on, and the set of values a random variable can take on.

B. The entropy is an expectation, with respect to the distribution of the random variable, of a certain quantity. What is this quantity?

Problems.

1. *Entropy of a Gaussian.* Derive the formula for the differential entropy of a one-dimensional Gaussian distribution as a function of its mean and variance. The derivation will be a bit easier if you use the natural logarithm rather than the base 2 logarithm. (20 points)
Hints: Remember that

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}}\,dx = 1$$

and that

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}}\,dx = \sigma^2.$$

2. *Independence and entropy.* (A) Give the entropy, in bits, of four fair, 8-sided dice. (B). Suppose you drill a hole in each die, and tie them all together with a string. Will the entropy of the dice be higher or lower than the answer from part (A)? Why? (10 points).

3. *Effect of changing units.* (A) A die is labeled from 1 to 6. Assuming it's fair, what is the entropy of the roll? The die is relabeled with the even numbers from 2 to 12. What is its entropy now? (5 points)

(B) A spinner is labeled from 0-1. Assuming the spinner has an equal chance of landing on any value, what is the differential entropy of the random variable which represents the spinner's outcome? The spinner is now relabeld from to have numbers from 0-2. Now what is its differential entropy? (5 points)

(C) Can you make a general statement about coordinates (the labelling of a space) and discrete entropy? Differential entropy? (5 points)