Light and Motion

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Abstract

1 Analyzing the movement of light

The goal of this section is to make sure that you understand some basic ideas about the emission of light, its interaction with surfaces, and its propagation through lenses. Some ideas, like the "inverse square" law, should be easy to remember once you understand the intuition behind them.

1.1 Point sources of light

We start with a discussion of *point light sources*. This is a simplified model of any small or distant light source, such as a distant light bulb, a star, or perhaps the sun, in which we treat it as though it were an infinitesimal point. Of course, we know that no light source is infinitely small, but assuming that it is a point can make certain analyses simpler.

Any light source has associated with it a *power* output, which is the amount of energy consumed per unit of time (for example, joules per second). A common unit of power associated with light sources is the *watt*. For example, most of us are familiar with using 60-watt or 100-watt light bulbs. While this power rating refers to the amount of power *consumed* by the bulb, rather than the amount of power produced as light, for this discussion, we will assume that the power consumed and the power output as light are the same. As another example, the wattage of the sun is about 3.846×10^{26} watts.

Assume that a point light source of W watts emits energy equally in all directions. If a point light source is surrounded by a sphere of radius 1 meter, then all of the light output by the source will land upon the sphere. Since a sphere of radius 1 meter has a surface area of 4π square meters, then the number of watts of power irradiating each square meter is be given by

$$R = \frac{W\ watts}{4\pi\, m^2}.$$

Steradians. One square unit of area of a unit sphere is considered one unit of *solid angle*, or one *steradian*. One steradian represents a cone of directions large enough to intersect one unit of area of a unit sphere, when the tip of the cone is placed at the center of the sphere. Since there are 4π units of area on the surface of a unit sphere, there are 4π steradians of solid angle in a sphere. Compare this to the number of radians in a circle (2π), which is equal to the circumference of a unit circle.

The amount of light falling on a surface is a function not only of the power of the light, but also of our distance from it. However, the amount of light emitted into a unit solid angle by a light source is not related to one's distance from a light source. Hence, the emitting power of point light sources, or their *radiance*, is often measured in *watts per unit solid angle* or *watts per steradian*. Alternatively, it may be simply given in watts, which can be thought of as *watts per* 4π *steradians* if desired.

Now imagine that instead of surrounding the light source by a sphere of radius one meter, we have surrounded it with a sphere of radius 3 meters. The amount of light radiated by the source has not changed. However, now the amount of light falling on each square meter of the surrounding sphere is

$$R = \frac{W\ watts}{4\pi \cdot 3^2\ m^2} = \frac{W\ watts}{36\pi\, m^2}.$$

Thus, increasing the *distance* of a surface from the light source by a factor of 3 reduces the power radiated on each unit of that surface by a factor of 9. This is a simple consequence of the fact that the surface area of a sphere is inversely proportional to the square of its radius. The falloff in the power of light per unit of area with the inverse square of distance is an example of an *inverse square law*.

Example 1. Suppose you have just mounted 10 square meters of solar panels on your house, and that they are oriented so that they are perfectly perpendicular to the direction of the sun. (Assume the sun is a point light source.) Discounting the absorption and scattering of the atmosphere, compute the number of watts of power being radiated upon your solar panels, assuming the sun is exactly 150 million kilometers from your solar panels.

2 Extended light sources

Most real light sources are better modeled as *extended light sources* rather than point light sources. That is, the light source has some extent, whether it is small or large. Common examples of extended light sources are the entire sky on an overcast day, the light coming in through an office window (again, on a cloudy day), or a large fluorescent bulb. In these cases, the power of the light emanating from a single infinitesimal point may be considered negligible or even 0. Rather, it is the power emanating over a finite area that is of interest. Hence, for extended light sources, we measure power not just as watts per steradian, but rather, as *watts per steradian per unit area*, where the area refers to the area of the *emitter*, not to the surface on which light is falling.

Example 2. Imagine that a light meter records one watt of light emanating from a fluorescent bulb when half of the bulb is covered with opaque electrical tape. If the tape is removed, we expect the light meter to record two watts, since the surface area of the emitter has doubled.

This discussion is particularly germaine to computer vision, since each element of the sensor in a video camera (either a CCD or a CMOS device), or each rod or cone cell in the retina is effectively measuring the *total light energy* emanating from a very small patch of a scene. To understand total light reaching a particular sensor element, we must think of the surface as an emitter of light, and integrate the output of that surface (in the direction of the viewer) over the area of the surface which reaches the sensor element. If a video camera is pointed directly as a light source, such as a fluorescent bulb, then we are simply measuring the direct output of the light source. However, if the video camera is aimed at a passive non-emitting surface, like a table, then we are measuring the *reflected light* from the surface. However, this reflected light can still be conceptualized as a light source.

3 Lenses and cameras

Recall the theoretical pinhole camera model discussed in class. The pinhole camera has an infinitely small hole which lets light through, forming an inverted image of the environment on the image plane at the back of the camera.

Let's calculate how much light, in watts, from an extended source falls onto a particular element E of a sensor mounted at the image plane of our pinhole camera. Let's assume our camera is pointed at a flat fluorescent lamp which is $10mm$ wide and $1000mm$ long, and so has an emitting area of $10,000mm^2$ as viewed from our camera.

Furthermore, let's assume that the fluorescent lamp outputs 0.001 watts per steradian per mm^2. Without going into the details of exactly how large the imaging elements of our sensor are, or how far away the lamp is, let's assume that an area A of the lamp which is $10mm$ by $10mm$, or $100mm^2$, is imaged by the single element E of our sensor array. That is, for each point in the small area A of the fluorescent lamp, each ray traced from A, through the camera's pinhole, lands on sensor element E.

We can now see that the total number of watts W_E landing on sensor element E is given by:

$$\begin{aligned} W_E\ watts &= 0.0001 \frac{watts}{steradian\ mm^2} \times 100mm^2 \times S\ steradians && (1) \\ &= 0.01 \frac{watts}{steradian} \times S\ steradians. && (2) \end{aligned}$$

Thus, all we need to know to complete the calculation is the size S in steradians of the solid angle of light which makes it through the pinhole in the camera. But since the pinhole is infinitely small, the answer to this question is $S = 0\ steradians$, and hence

$$W_E\ watts = 0\ watts.$$

Here, we have demonstrated a well-known result, which is that ideal pinhole cameras, whose apertures are infinitely small, cannot capture any finite amount of light. There are two options for solving this problem. One is to give our pinhole a finite extent, so that the S in the above equation is non-zero. This is a simple solution, but will introduce blurring, or de-focus into our images. Alternatively, we can construct a camera with a lens, which can capture light from a large solid angle of directions from a source, and focus that light to a single point again.

Now let's redo our calculation of W_E assuming we have replaced our pinhole camera with a camera using a certain lens. Let the lens have ideal optics, meaning that it does not absorb or scatter light, and that it focuses all of the light coming into it onto the image plane. Suppose the lens has a radius of $10mm$ and is $10m$, or $10,000mm$, from the fluorescent light source. The cone of directions subtended (or "taken up") by the lens, as measured in steradians, from a point on the light source can be obtained by setting up a proportionality express in the following way:

$$\frac{area\ of\ lens}{area\ of\ 10m\ radius\ sphere} = \frac{steradians\ S\ subtending\ lens}{steradians\ in\ a\ sphere},$$

or

$$\frac{\pi 100mm^2}{\pi 10^8 mm^2} = \frac{S}{4\pi}.$$

Hence, the number of steradians S subtended by the lens, is about $\frac{10^6}{4\pi} \approx 1.26 \times 10^{-5}$. Substituting this into Eq.2 gives $1.26 \times 10^{-7}\ watts$ landing on the sensor element E.