Applied Information Theory 691GG

http://www.cs.umass.edu/∼elm/IT/

Assignment 1

1. *Computing entropy.* (2 points each) In this problem, you are to compute the entropy of various probability distributions, represented by sets of numbers in brackets. If there are K numbers in brackets, then there are K possible values of the corresponding random variable, whose probabilities are given by the numbers. In other words, a fair die would be written $[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}]$. Compute the entropy of the following distributions (don't use a calculator unless the problem says you can):

a $[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}]$

b $[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, 0]$

c $[\frac{1}{4}, \frac{1}{4}, \frac{1.1}{4}, \frac{0.9}{4}]$ (you can use a calculator for this one)

d $[\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}]$

e $[\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}]$

2. A die is labeled from 1 to 6. Assuming it's fair, what is the entropy of the roll (you can leave a "log" in your answer)? The die is relabeled with the even numbers from 2 to 12. What is its entropy now? (2 points)

3. *Independence and entropy.* (A) Give the entropy, in bits, of four fair, 8-sided dice. (B). Suppose you drill a hole in each die, and tie them all together with a string. Will the entropy of the dice be higher or lower than the answer from part (A)? Why? (10 points).

4. Problem 2.1, part (a) (10 points). All problems below are from Cover and Thomas, SECOND EDITION.

5. Problem 2.2 (5 points).

6. Problem 2.3 (5 points).

7. Problem 2.4 (5 points).

8. Problem 2.6 (5 points).

9. Problem 2.12, parts (a)-(e) (2 points each).

10. Problem 2.37. Justify your answer in detail. (10 points)

11. Problem 2.39. (10 points) For extra credit (up to 10 additional points), prove that your answer is one of the best possible answers.

12. Problem 2.43. (5 points)