Computer Vision 691A: Joint Alignment

Erik Learned-Miller, with Vidit Jain, Andras Ferencz, Gary Huang, Lilla Zollei, Sandy Wells, ....
Congealing *(CVPR 2000, PAMI 2006)*

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![Image of handwritten digit 7]
Five Applications

- Image factorizations
  - For transfer learning, learning from one example
- Alignment for Data Pooling
  - 3D MR registration
  - EEG registration
- Artifact removal
  - Magnetic resonance bias removal
- Improvements to recognition algorithms
  - Alignment before recognition
- Defining anchor points for registration
  - Find highly repeatable regions for future registrations
Congealing

- Process of joint “alignment” of sets of arrays (samples of continuous fields).

- 3 ingredients
  - A set of arrays in some class
  - A parameterized family of continuous transformations
  - A criterion of joint alignment
Congealing Binary Digits

- 3 ingredients
  - A set of arrays in some class:
    - Binary images
  - A parameterized family of continuous transformations:
    - Affine transforms
  - A criterion of joint alignment:
    - Entropy minimization
Congealing
Criterion of Joint Alignment

- Minimize sum of pixel stack entropies by transforming each image.

Note: Mutual Information doesn’t make sense here.
An Image Factorization

Observed Image → "Latent Image" → Transform

(Previous work by Grenander,, Frey and Jojic.)
\arg\max_{T \in \mathcal{T}} P(T|I) \overset{(a)}{=} \arg\max_{T \in \mathcal{T}} P(I|T) P(T) \\
\overset{(b)}{=} \arg\max_{T \in \mathcal{T}} P(I|T) \\
\overset{(c)}{=} \arg\max_{T \in \mathcal{T}} P(L(I, T)) \\
= \arg\max_{T \in \mathcal{T}} \prod_{x,y} \prod_{i=1}^{N} p_{x,y}(L_i(x,y)) \\
= \arg\max_{T \in \mathcal{T}} \sum_{x,y} \sum_{i=1}^{N} \log p_{x,y}(L_i(x,y)) \\
\overset{(d)}{=} \arg\min_{T \in \mathcal{T}} \sum_{x,y} H(p_{x,y}) \\
\overset{(e)}{=} \arg\min_{T \in \mathcal{T}} \sum_{x,y} \hat{H}_{\text{Vasicek}}(L_1(x,y), \ldots, L_N(x,y))
Why Minimize Entropy?

- Negative entropy is just the average log likelihood of points under their own distribution.

\[
\text{Min entropy} = \text{maximum non-parametric likelihood}
\]
The Independent Pixel Assumption

- Model assumes independent pixels
- A poor generative model:
  - True image probabilities don’t match model probabilities.
  - Reason: heavy dependence of neighboring pixels.
- However! This model is great for alignment and separation of causes!
  - Why?
  - Relative probabilities of “better aligned” and “worse aligned” are usually correct.
- Once components are separated, a more accurate (and computationally expensive) model can be used to model each component.
Each pair implicitly creates a sample of the transform $T$. 
Character Models

- Latent Images
  - Congealing
  - Transforms
  - Image Kernel Density Estimator (or other estimator)
  - Transform Kernel Density Estimator (CVPR 2003)

\[ P(I_L) \]
Latent Image Probability Density for Zeroes

\[ P(T) \]
Transform Probability Density for Zeroes
How do we line up a new image?

Sequence of successively “sharper” models

step 0  step 1  step N

Take one gradient step with respect to each model.
Digit Models from One Example

Model of non-affine “A” variability

General model of affine variability

One example of each digit

0 Model
1 Model
2 Model

... ...

9 Model
Next Application: 
Alignment of 3D Magnetic Resonance Volumes

Lilla Zollei, Sandy Wells, Eric Grimson
Congealing MR Volumes: Joint Registration

- 3 ingredients
  - A set of arrays in some class:
    - Gray-scale MR volumes
  - A parameterized family of continuous transformations:
    - 3-D affine transforms
  - A criterion of joint alignment:
    - Grayscale entropy minimization

- Purposes:
  - Pooling data for fMRI studies
  - Building general purpose statistical anatomical atlases
Why Entropy?

- Drives volumes to having mass concentrated in a small number of tissues.
- Comparison to Transformed Mixture of Gaussians (Frey and Jojic).
- Convexity of entropy in distribution.
Congealing Gray Brain Volumes (ICCV 2005 Workshop)
Aligned Volumes
Validation: Synthetic Data

**Unaligned** input data sets

**Aligned** input data sets
Real Data

Data set: 28 T1-weighted MRI; [256x256x124] with (.9375, .9375, 1.5) mm³ voxels
Experiment: 2 levels; 12-param. affine; N = 2500; iter = 150; time = 1209 sec!!
Grayscale Entropy Minimization

Frequency of occurrence in image vs. Image intensity

White-gray separation?
MR Congealing Challenges

- **Big data**
  - 8 million voxels per patient
  - 100 patients
  - 12 transform *parameters*
  - 20 iterations

- **Techniques:**
  - Stochastic sampling
  - Multi-resolution techniques
  - Don’t use visual basic
Signal to Noise in Event Related Potentials

Before congealing

After congealing
Next Application: Bias Removal in Magnetic Resonance Images

Parvez Ahammad, Vidit Jain
## The Problem

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<th>Ideal Image</th>
<th>Bias Field</th>
<th>Observed Image</th>
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Bias fields have low spatial frequency content
Bias Removal in MR as a Congealing Problem

- 3 ingredients
  - A set of arrays in some class:
    - MR Scans of Similar Anatomy (2D or 3D)
  - A parameterized family of *continuous* transformations:
    - Smooth brightness transformations
  - A criterion of joint alignment:
    - Entropy minimization
Congealing with brightness transforms
Grayscale Entropy Minimization

Frequency of occurrence in image

White-gray separation?

Image intensity
Some Infant Brains
(thanks to Inder, Warfield, Weisenfeld)

- Pretty well registered (not perfect)
- Pretty bad bias fields
Fourier Basis for Smooth Bias Fields
Assumptions

- Pixels in same location, across images, are independent.
  - When is this not true?
    - Systematic bias fields.
- Pixels in same image are independent, given their location.
  - Clearly not true, but again, doesn’t seem to matter.
- Bias fields are truly bandlimited.
Some Other Recent Approaches

- Minimize entropy of intensity distribution in single image
  - Viola (95)
  - Warfield and Weisenfeld extensions (current)
- Wells (95)
  - Use tissue models and maximize likelihood
  - Use Expectation Maximization with unknown tissue type
- Fan (02)
  - Incorporate multiple images from different coils, but same patient.
Potential difficulties with single image method

- If there is a component of the brain that looks like basis set, it will get eliminated.
- Does this occur in practice?
  - Yes!
MRI Bias Removal
Summary

- Remove source of variability
  - MR bias removal
  - MR anatomical alignment
  - ERP signal alignment
  - Better alignment for recognition (hyper-features)

- Model a source of variability
  - Form factorized models (learning from one example)

- Define points of high saliency and repeatability (anchor points) for difficult registration problems