Log-Linear Models with Structured Outputs (continued)

Introduction to Natural Language Processing
Computer Science 585—Fall 2009
University of Massachusetts Amherst

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Overview

- What computations do we need?
- Smoothing log-linear models
- MEMMs vs. CRFs again
  - Action-based parsing and dependency parsing

Recipe for Conditional Training of p(y | x)

1. Gather constraints/features from training data

$$\alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)$$

2. Initialize all parameters to zero
3. Classify training data with current parameters; calculate expectations $E_\Theta[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_\Theta(y'|x_j) f_{iy}(x_j, y')$
4. Gradient is $\tilde{E}[f_{iy}] - E_\Theta[f_{iy}]$
5. Take a step in the direction of the gradient
6. Repeat from 3 until convergence

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EM!

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Gradient-Based Training

- λ <- λ + rate * Gradient(F)
- After all training examples? (batch)
- After every example? (on-line)
- Use second derivative?
- A big field: numerical optimization

Overfitting

- If we have too many features, we can choose weights to model the training data perfectly
- If we have a feature that only appears in spam training, not ham training, it will get weight ∞ to maximize p(spam | feature) at 1.
- These behaviors
  - Overfit the training data
  - Will probably do poorly on test data

Solutions to Overfitting

- Throw out rare features.
  - Require every feature to occur > 4 times, and > 0 times with ling, and > 0 times with spam.
- Only keep, e.g., 1000 features.
  - Add one at a time, always greedily picking the one that most improves performance on held-out data.
- Smooth the observed feature counts.
- Smooth the weights by using a prior.
  - max $p(\lambda|\text{data})$ = max $p(\lambda, \text{data}) = p(\lambda)p(\text{data}|\lambda)$
  - decree $p(\lambda)$ to be high when most weights close to 0

Smoothing with Priors

- What if we had a prior expectation that parameter values wouldn't be very large?
- We could then balance evidence suggesting large (or infinite) parameters against our prior expectation.
- The evidence would never totally defeat the prior, and parameters would be smoothed (and kept finite)
- We can do this explicitly by changing the optimization objective to maximum posterior likelihood:

  $\log P(y, \lambda \mid x) = \log P(\lambda) + \log P(y \mid x, \lambda)$

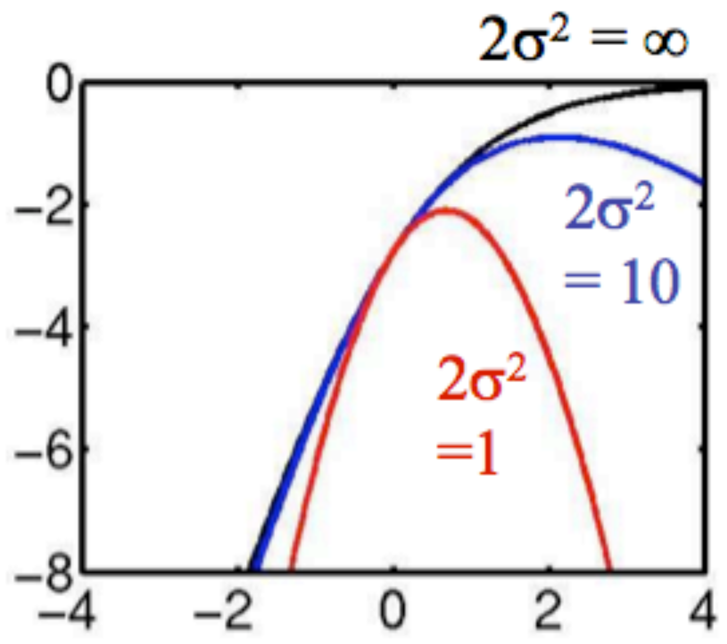
  Posterior Prior Likelihood

Smoothing: Priors

- Gaussian, or quadratic, priors:
  - Intuition: parameters shouldn't be large.
  - Formalization: prior expectation that each parameter will be distributed according to a gaussian with mean μ and variance σ^2.

$$P(\lambda_i) = \frac{1}{\sigma_i\sqrt{2\pi}}\exp\left(-\frac{(\lambda_i-\mu_i)^2}{2\sigma_i^2}\right)$$

  - Penalizes parameters for drifting to far from their mean prior value (usually μ=0).
  - $2\sigma^2$=1 works surprisingly well.



Parsing as Structured Prediction

Shift-reduce parsing

Stack	Input remaining	Action
()	Book that flight	shift
(Book)	that flight	reduce, Verb → book, (Choice #1 of 2)
(Verb)	that flight	shift
(Verb that)	flight	reduce, Det → that
(Verb Det)	flight	shift
(Verb Det flight)		reduce, Noun → flight
(Verb Det Noun)		reduce, NOM → Noun
(Verb Det NOM)		reduce, NP → Det NOM
(Verb NP)		reduce, VP → Verb NP
(Verb)		reduce, S → V
(S)		SUCCESS!

Ambiguity may lead to the need for backtracking.

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Train log-linear model of p(action | context)

Word Dependency Parsing

Raw sentence

He reckons the current account deficit will narrow to only 1.8 billion in September.

slide adapted from Yuji Matsumoto

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POS-tagged sentence

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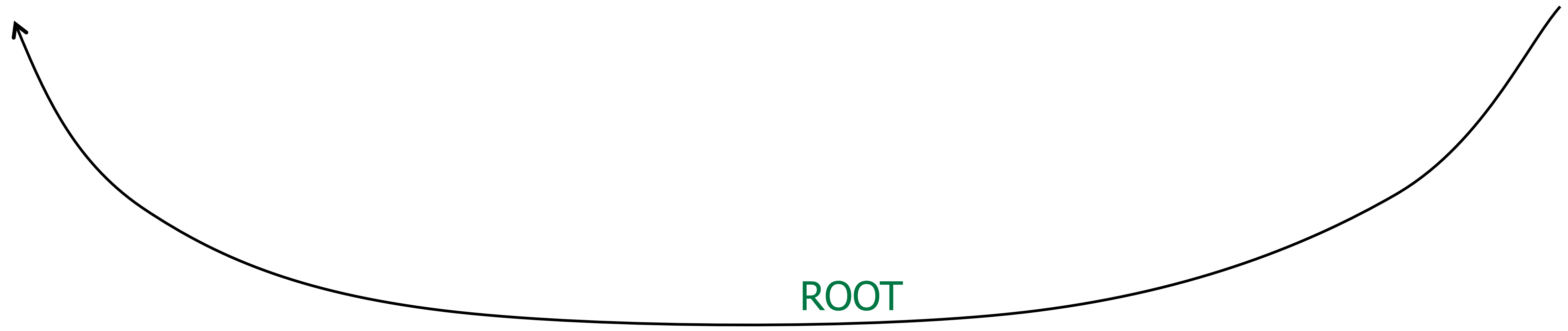

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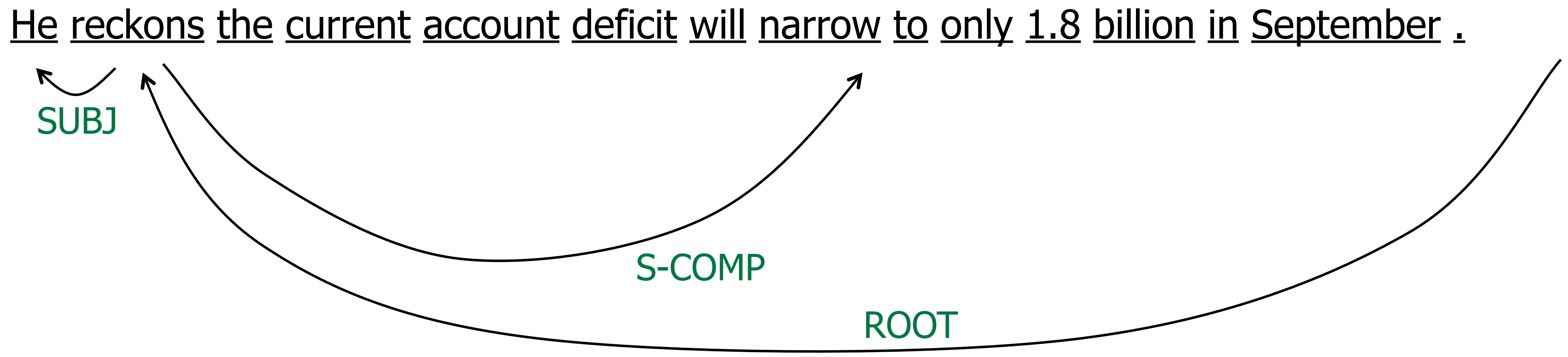
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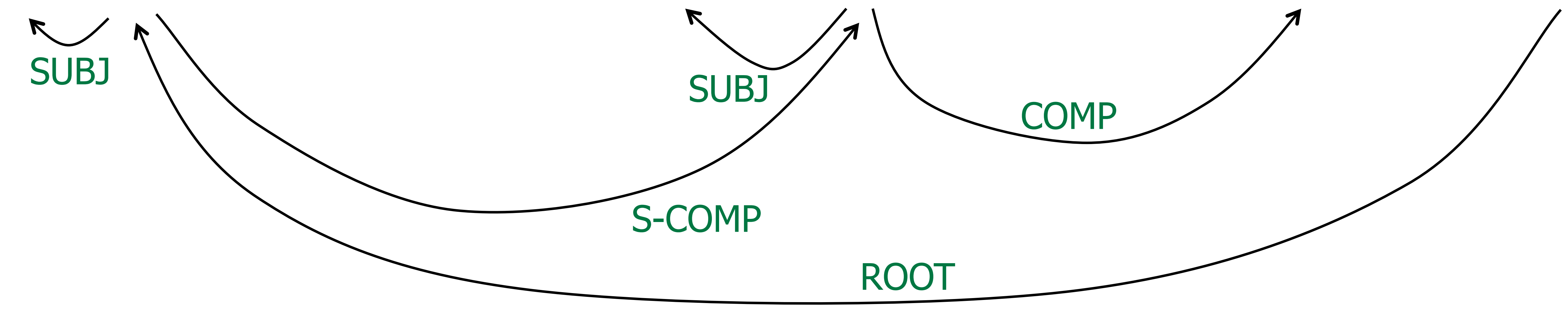

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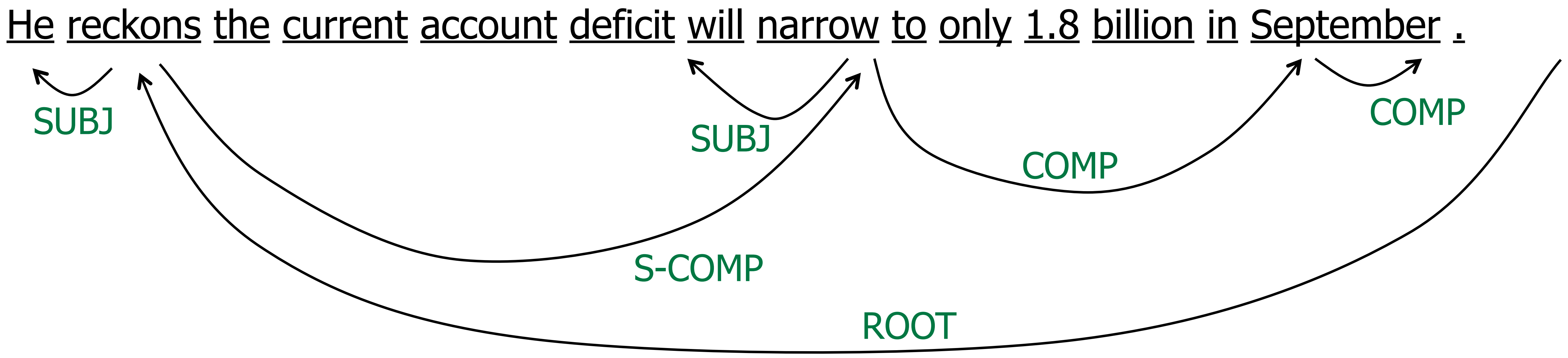
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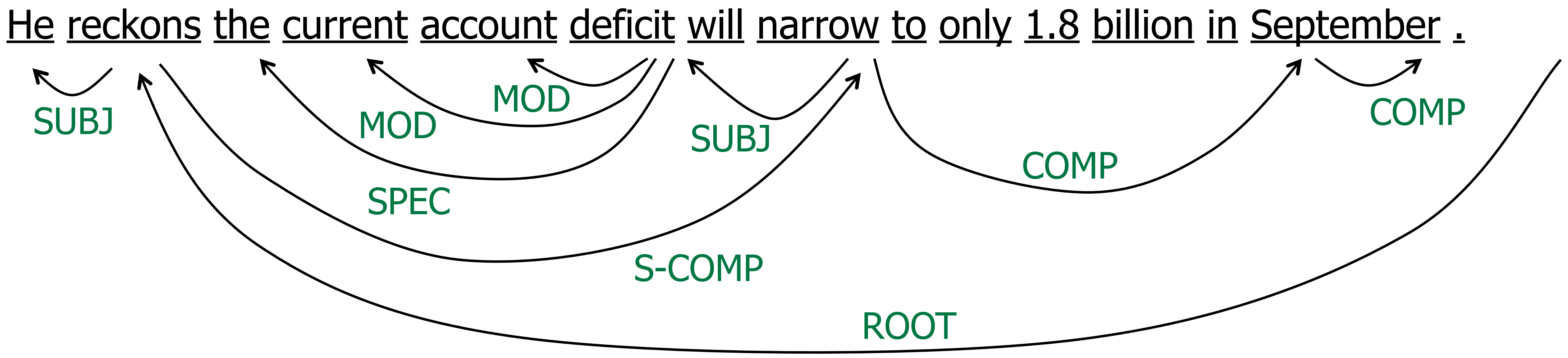
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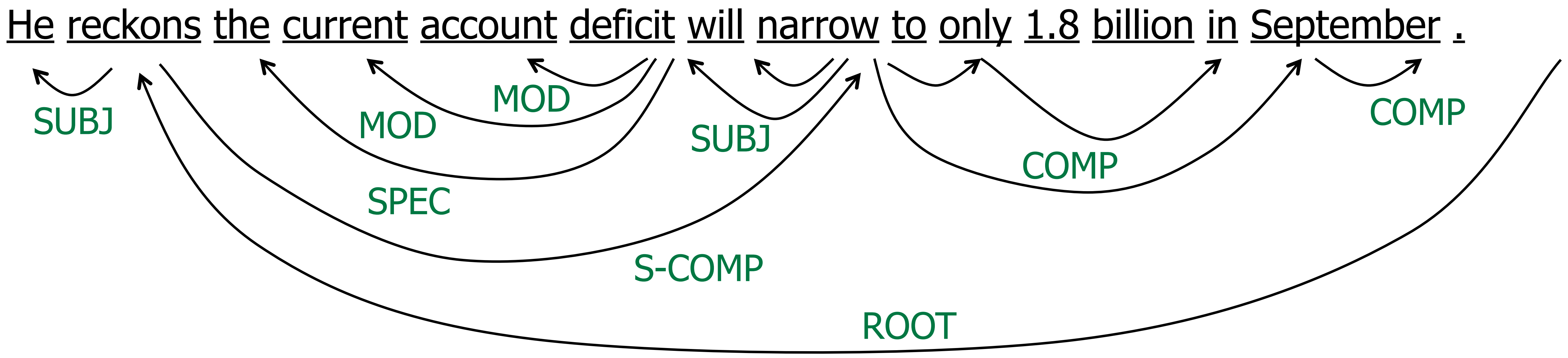
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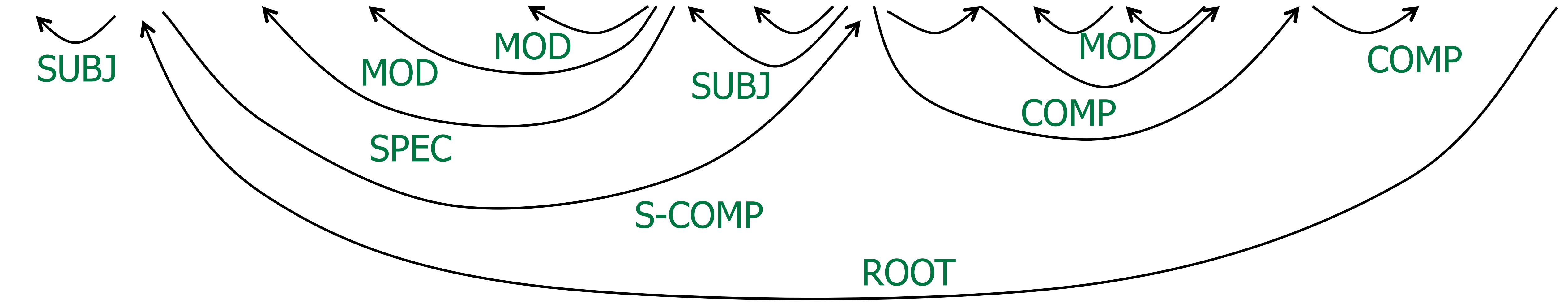

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Great ideas in NLP: Log-linear models

(Berger, della Pietra, della Pietra 1996; Darroch & Ratcliff 1972)

- In the beginning, we used generative models.

$$p(A) * p(B \mid A) * p(C \mid A,B) * p(D \mid A,B,C) * \ldots$$

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but which dependencies to allow? $p(D \mid A,\cancel{B},C)$?

what if they're all worthwhile? $p(D \mid A,B,\cancel{C})$?

$\ldots p(D \mid A,B) * p(C \mid \cancel{A},B,D)$?

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$* \Phi(D,A,B) * \Phi(D,B,C) * \Phi(D,A,C) *$

throw them all in!

…

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- Solution: Log-linear (max-entropy) modeling

$$(1/Z) * \Phi(A) * \Phi(B,A) * \Phi(C,A) * \Phi(C,B)$$

$$* \boxed{\Phi(D,A,B) * \Phi(D,B,C) * \Phi(D,A,C)} * \ldots$$

throw them all in!

  - Features may interact in arbitrary ways
  - **Iterative scaling** keeps adjusting the feature weights until the model agrees with the training data.

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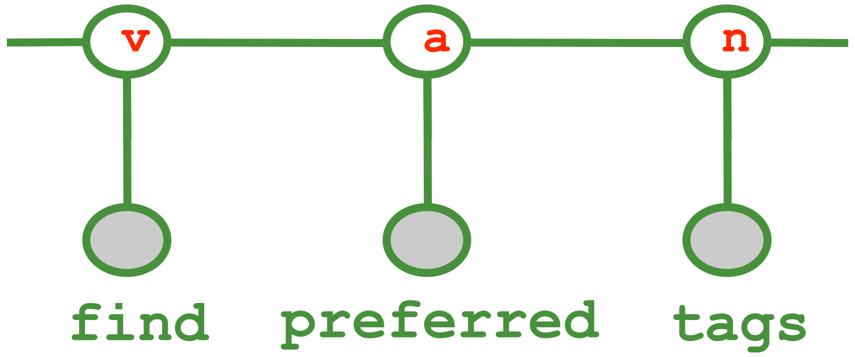

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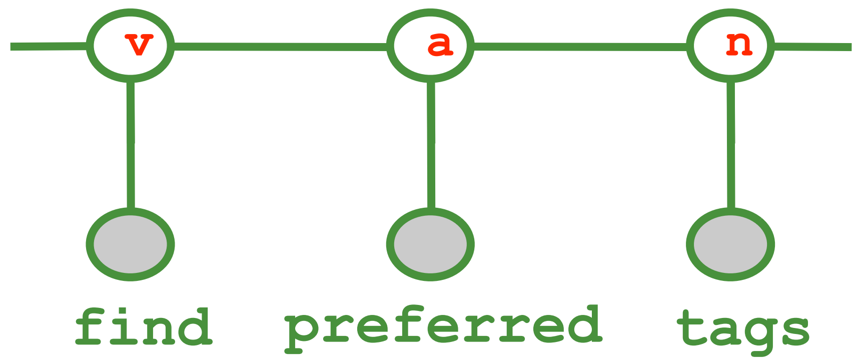

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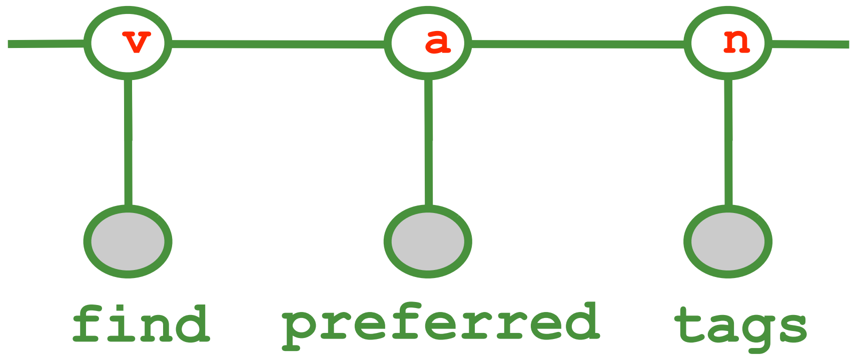

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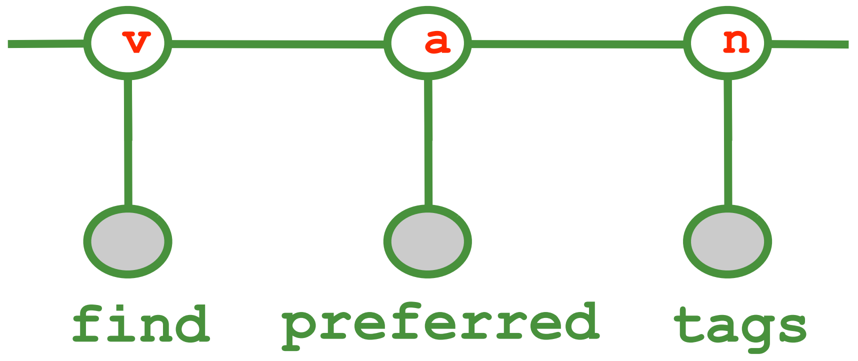

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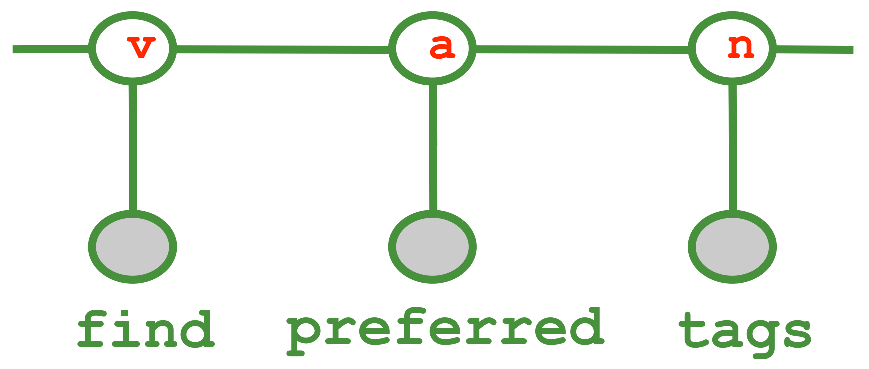

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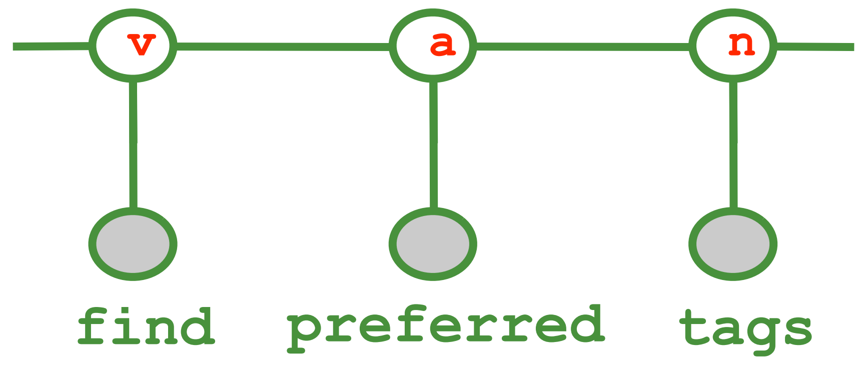

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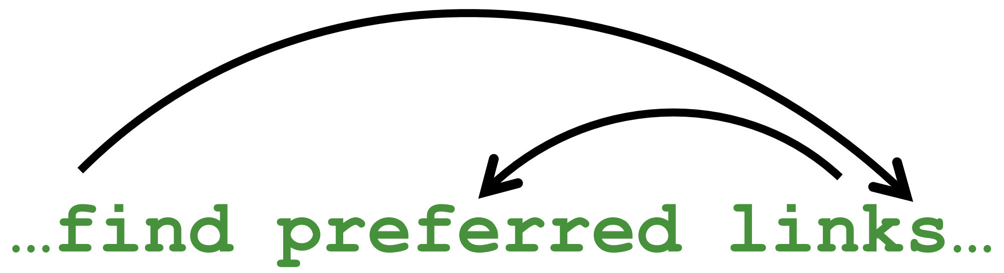

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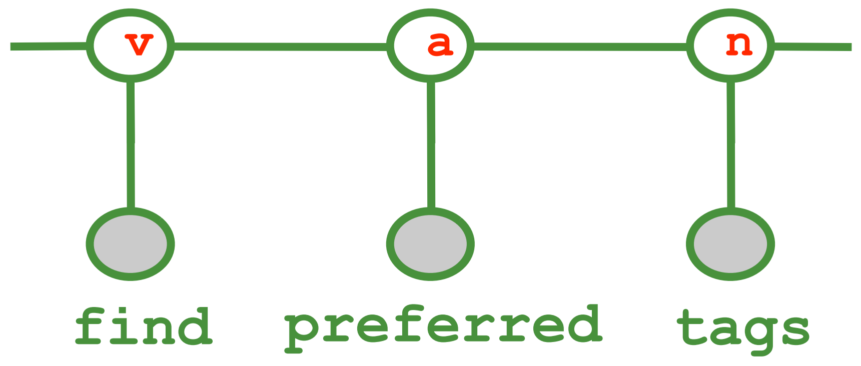

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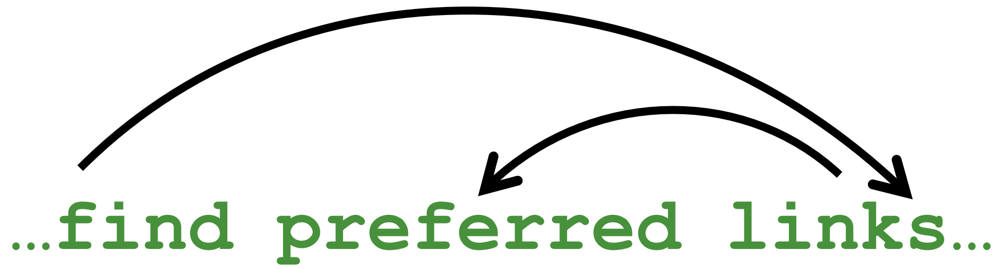

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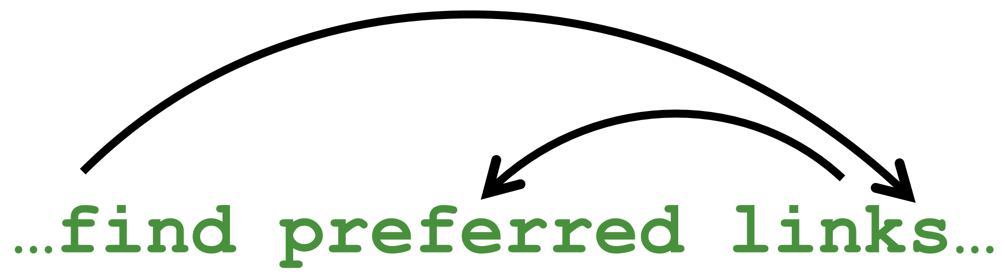

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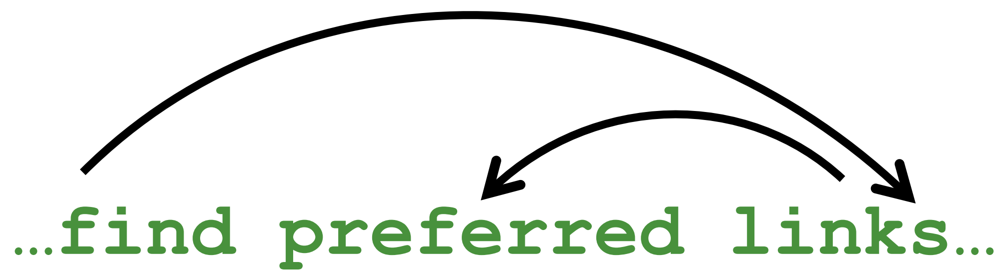

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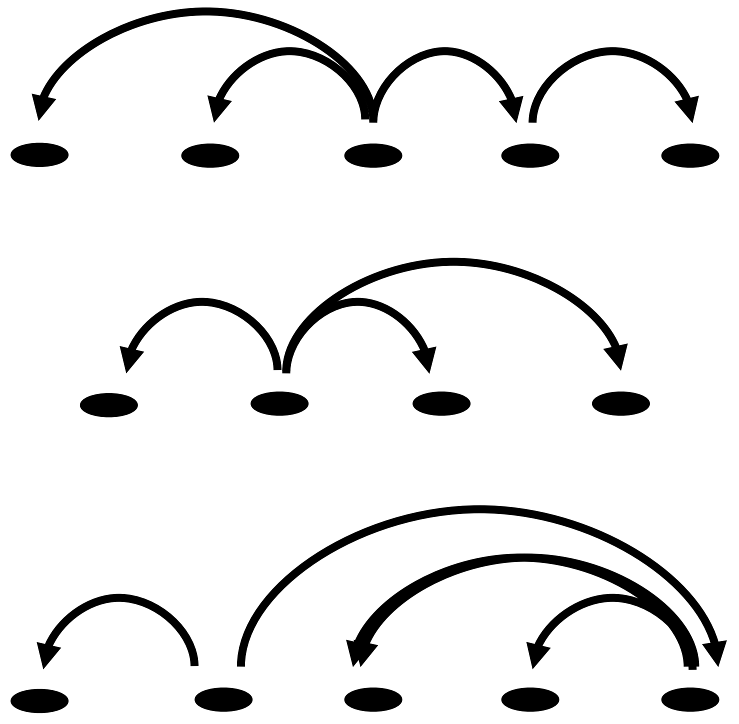

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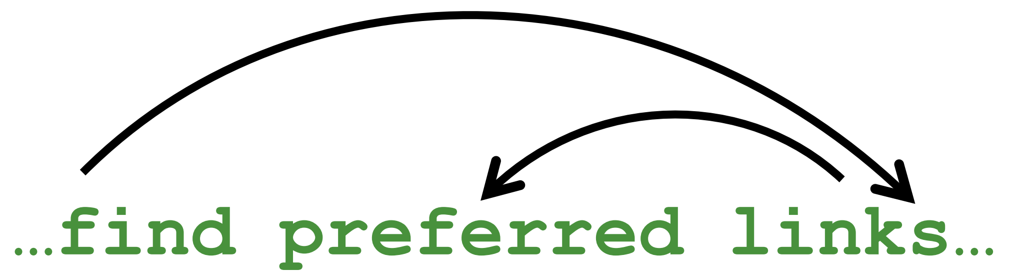

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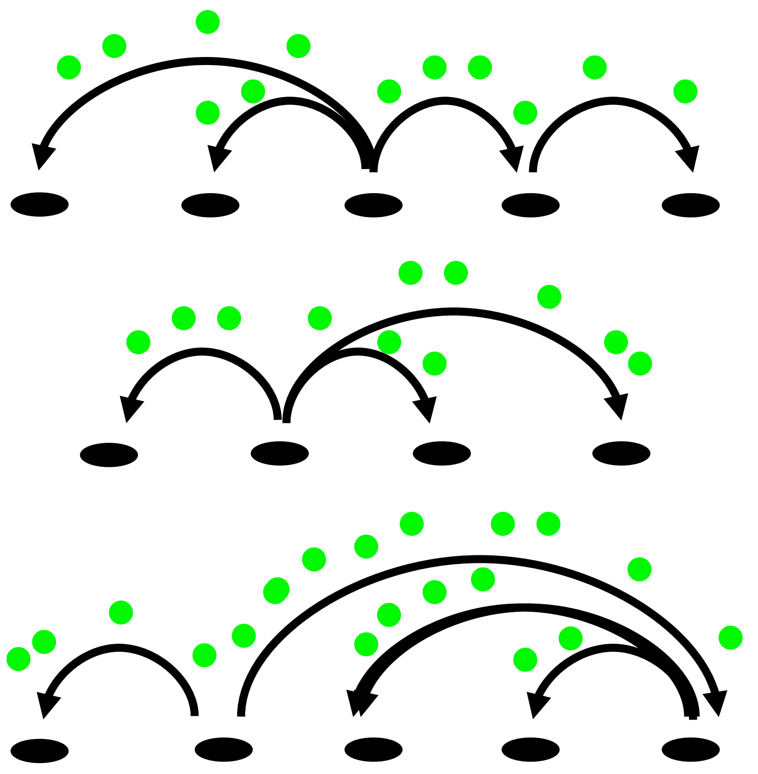

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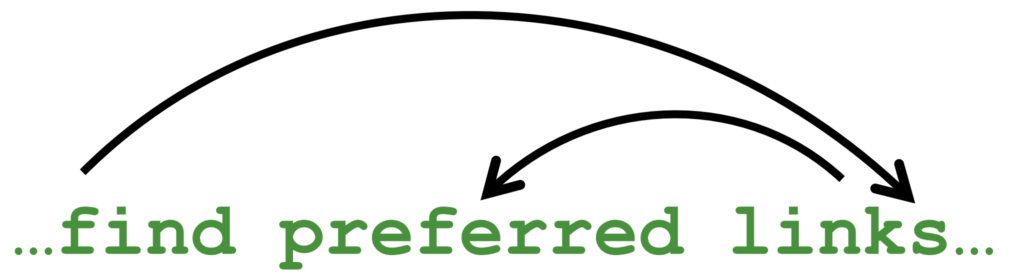

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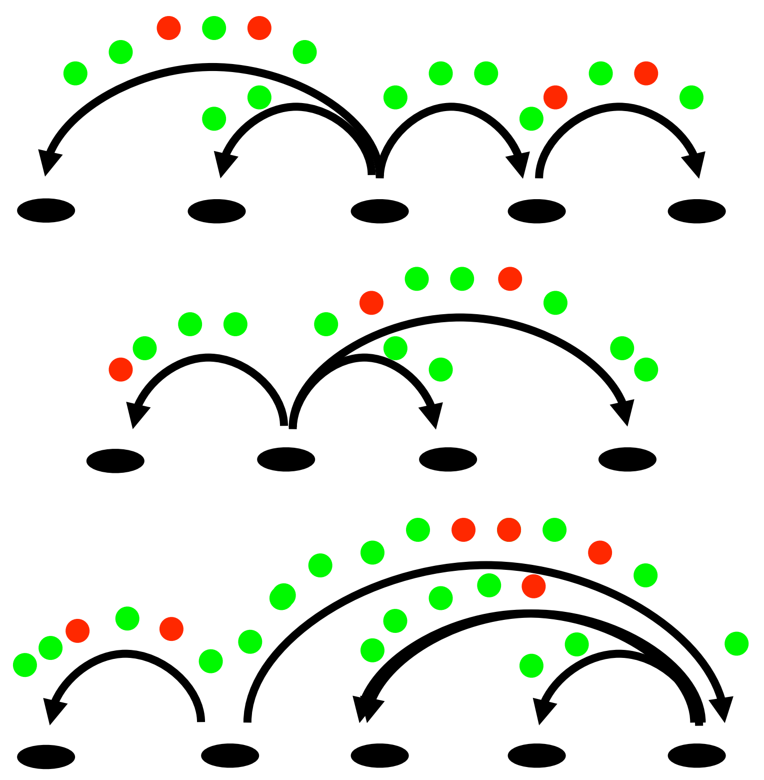

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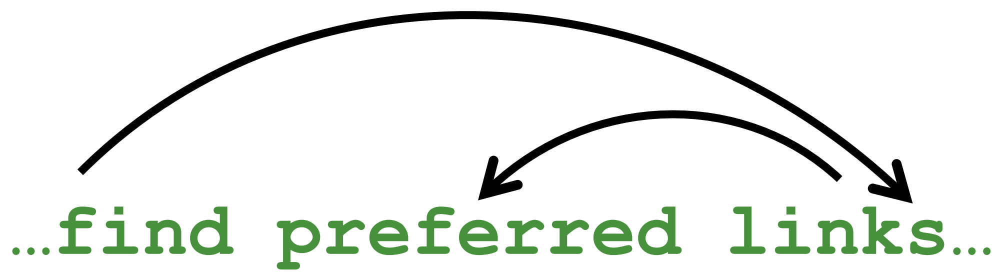

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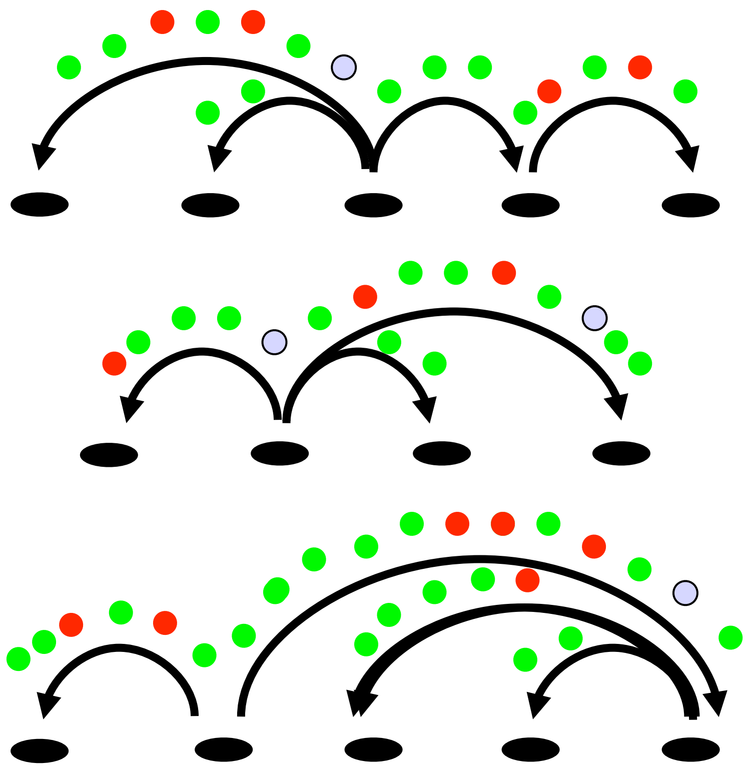

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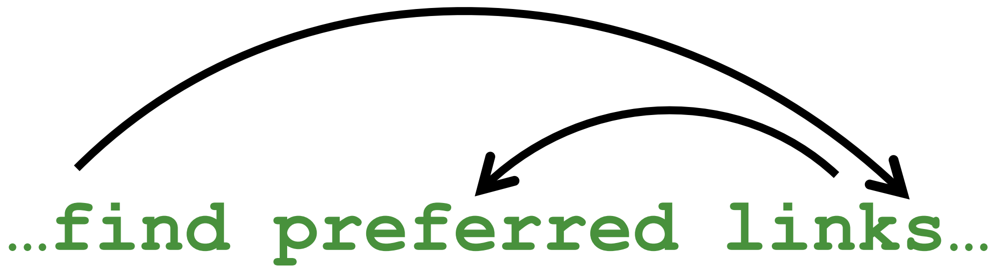

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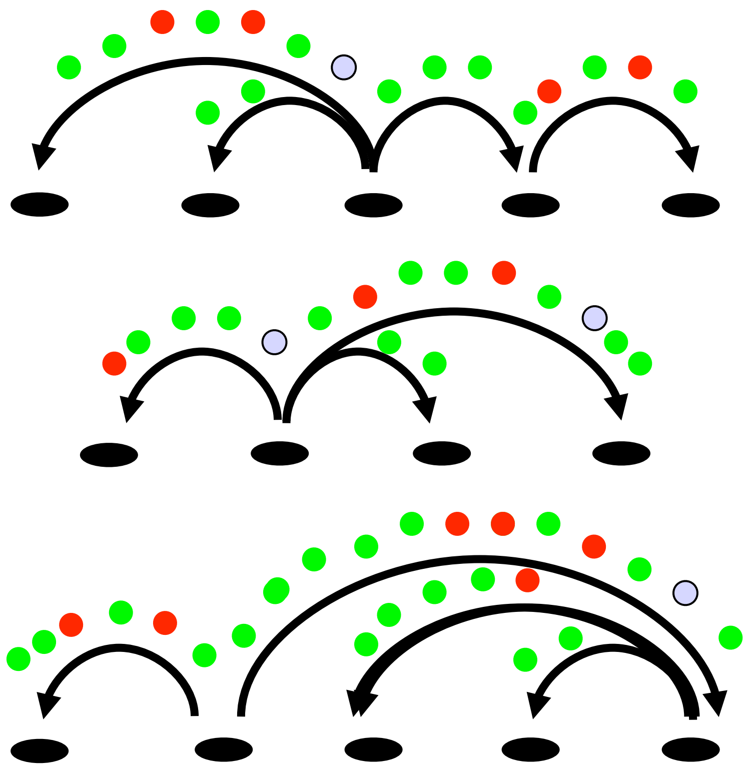
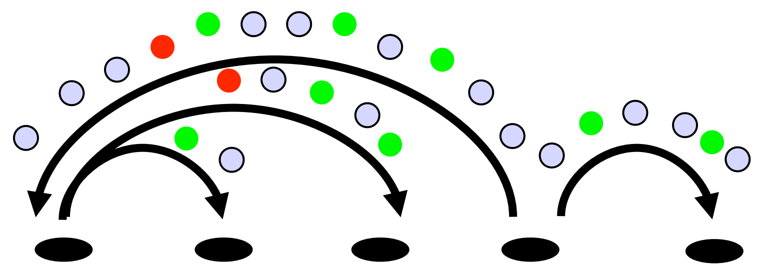
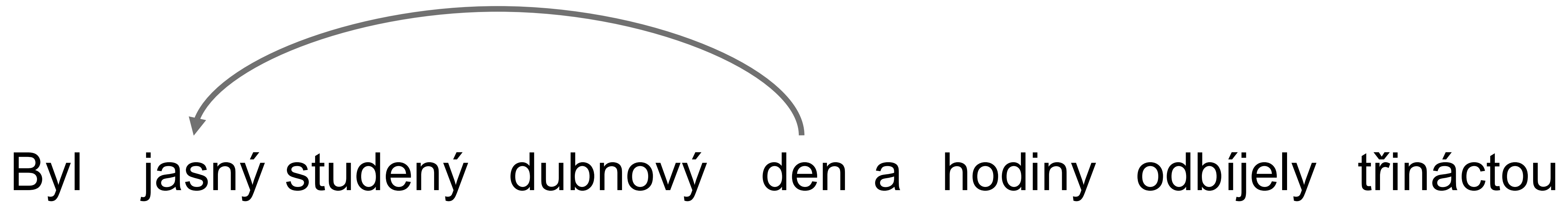




Edge-Factored Parsers (McDonald et al. 2005)

- Is this a good edge?



"It was a bright cold day in April and the clocks were striking thirteen"

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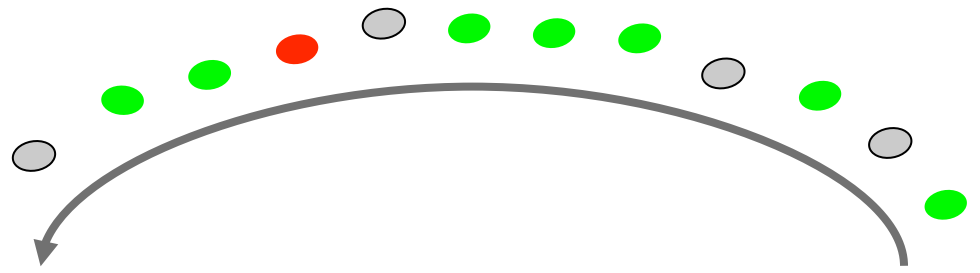
yes, lots of green ...

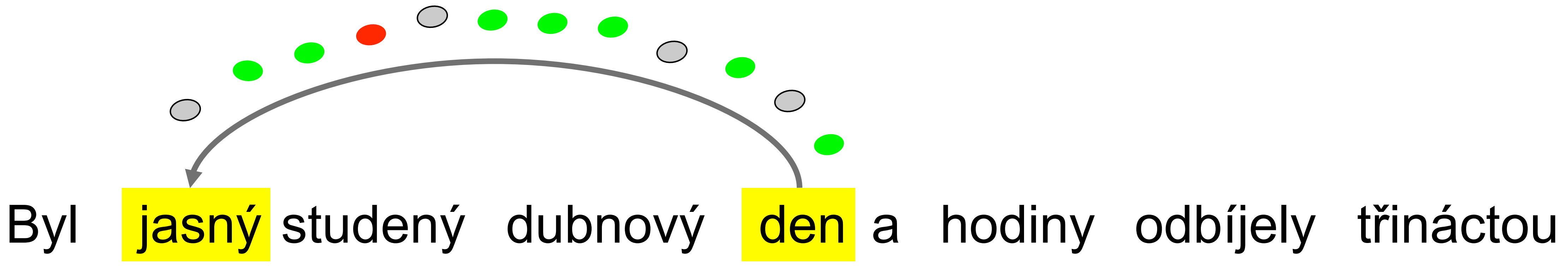


Byl jasný studený dubnový den a hodiny odbíjely třináctou

“It was a bright cold day in April and the clocks were striking thirteen”

Edge-Factored Parsers (McDonald et al. 2005)

- Is this a good edge?



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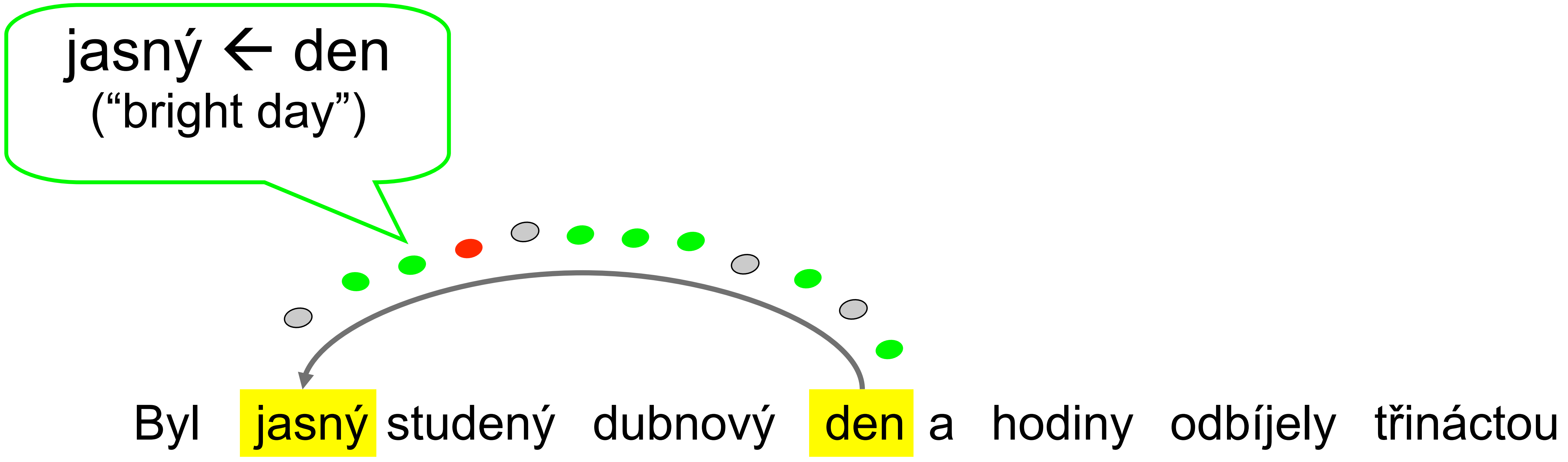

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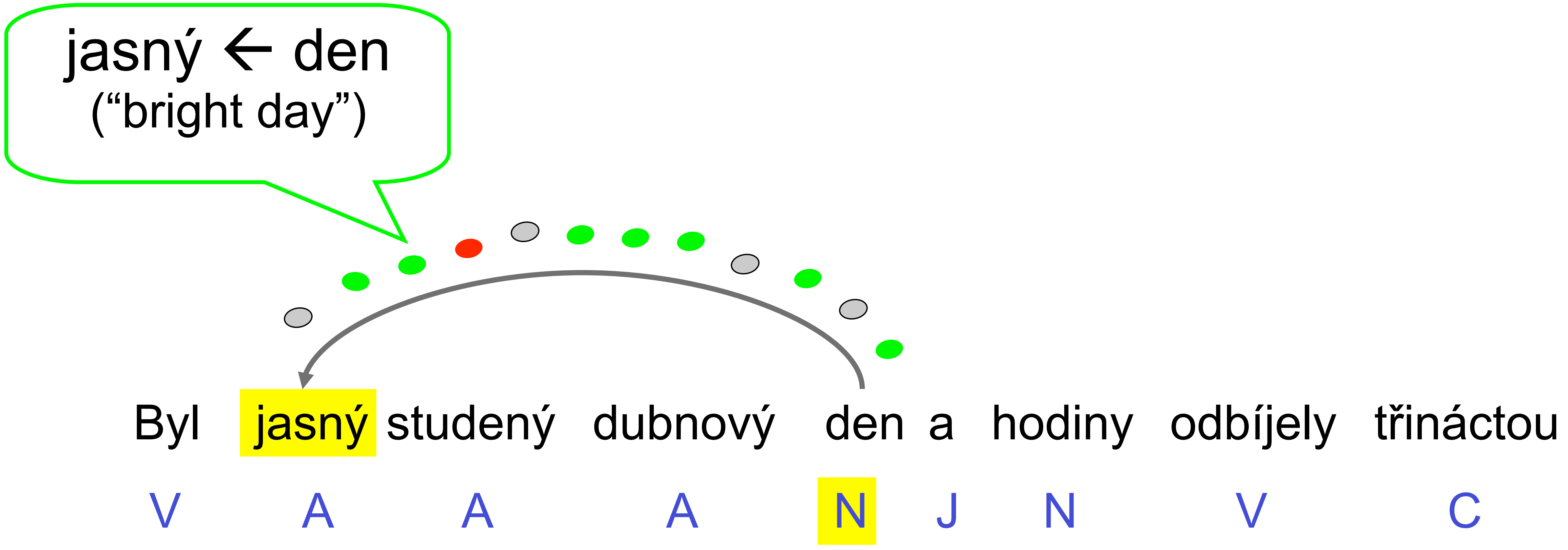

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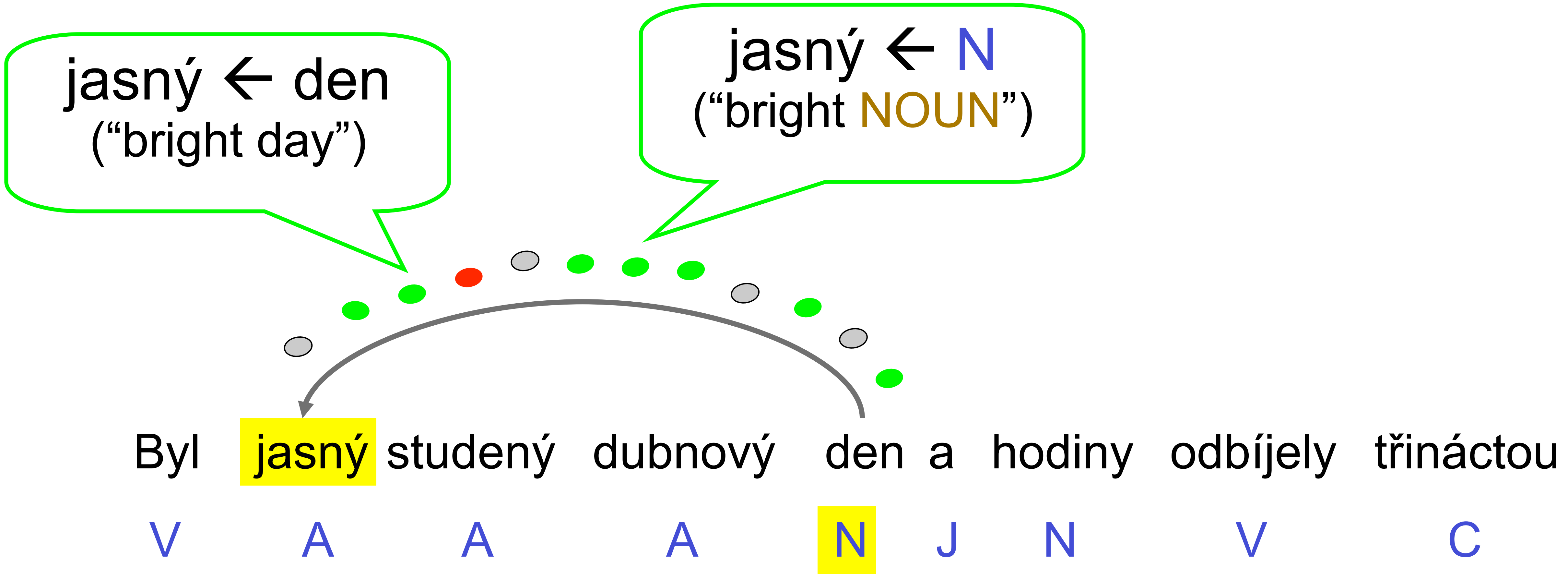

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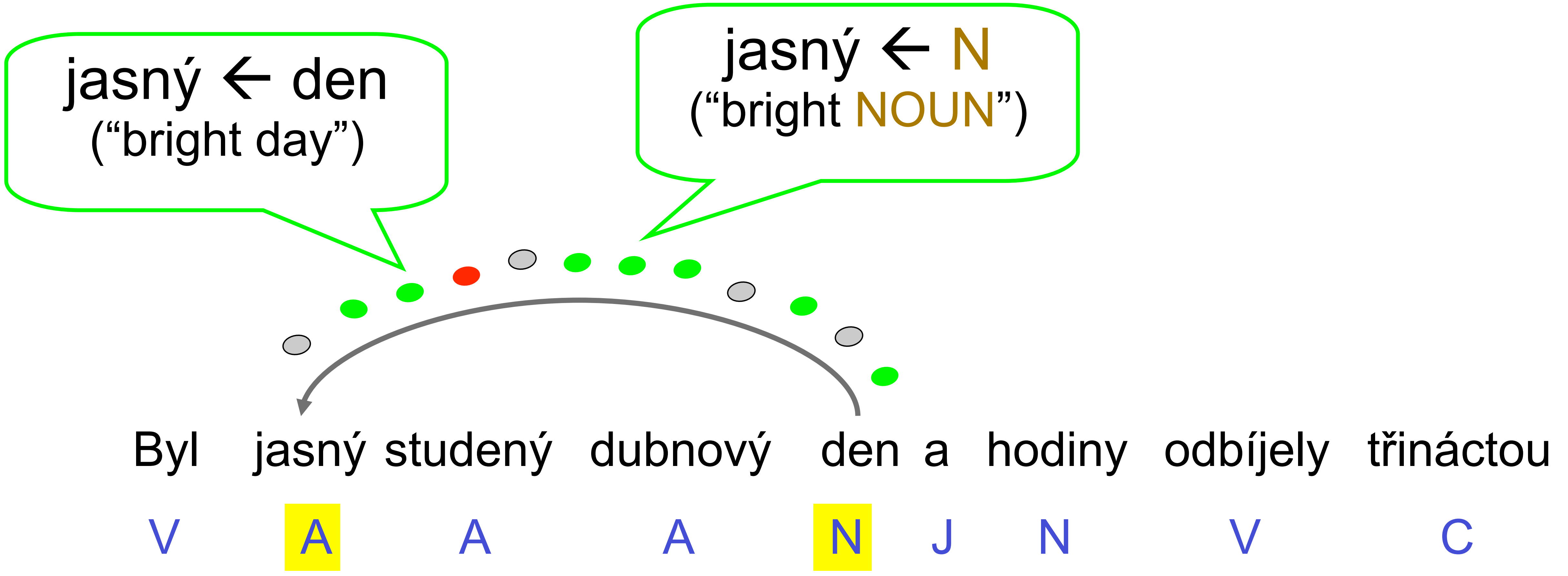

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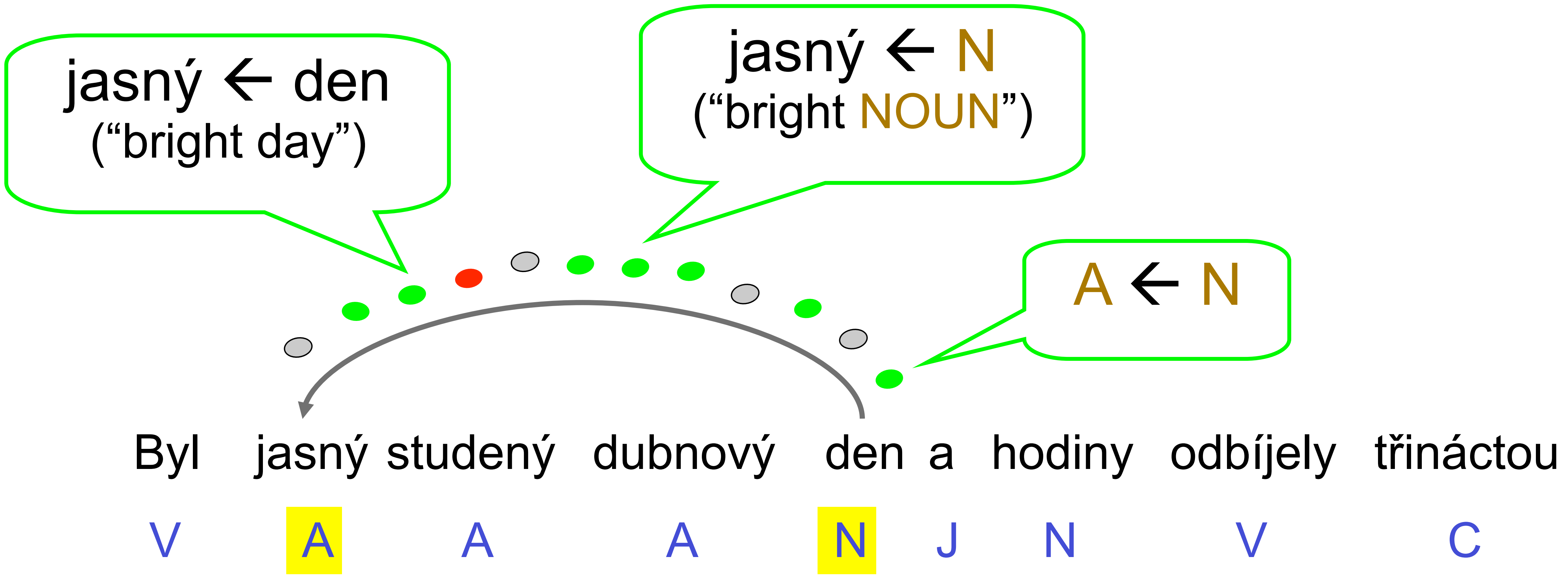

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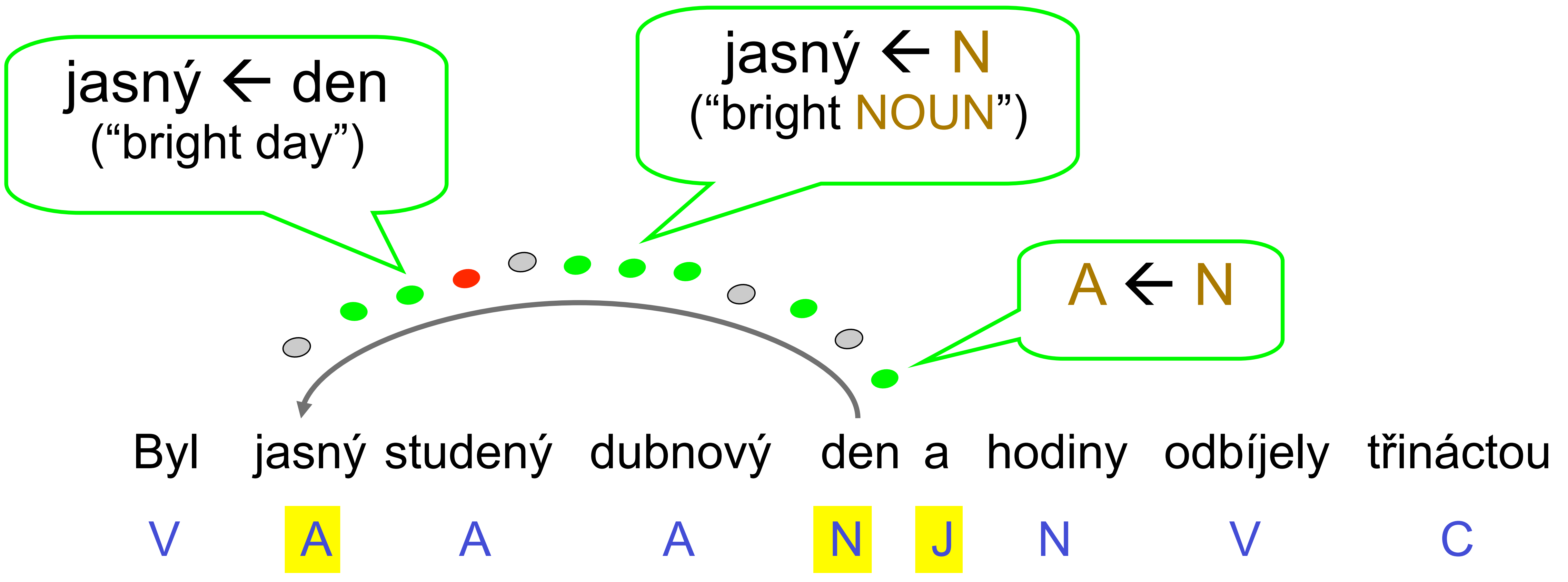

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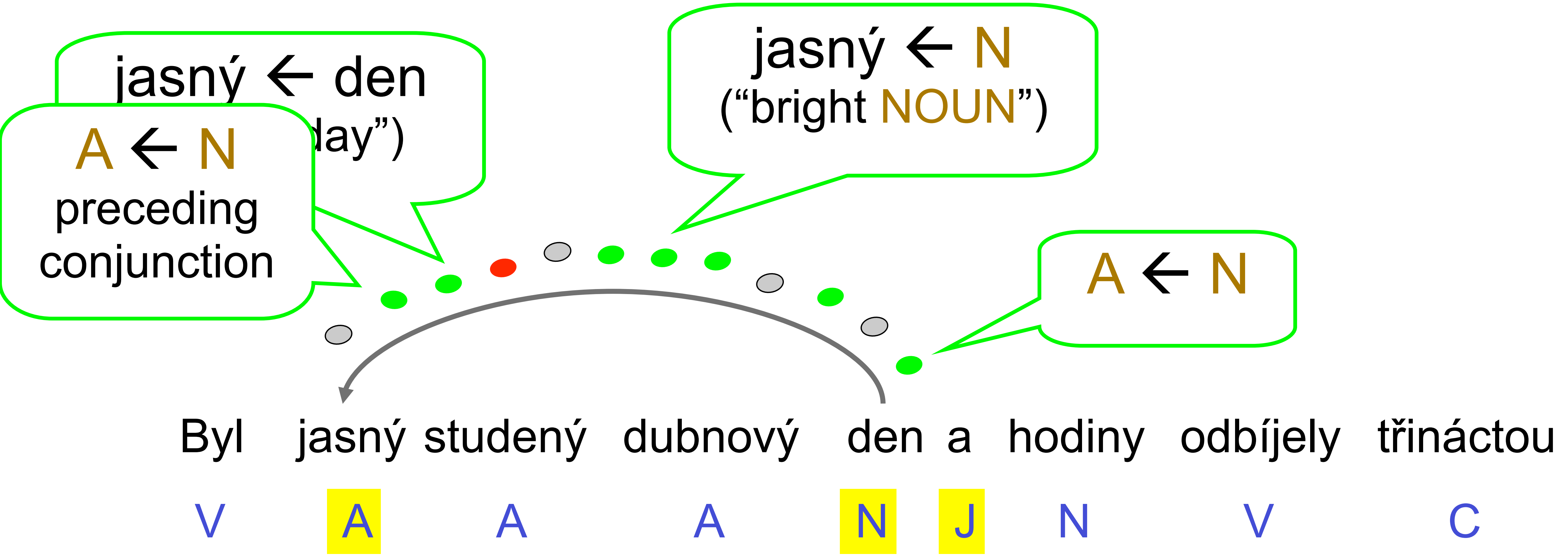
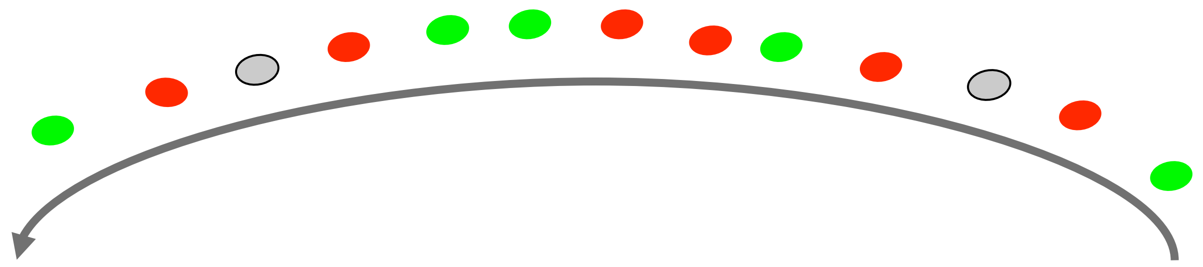


"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

- How about this competing edge?



Byl	jasný	studený	dubnový	den	a	hodiny	odbíjely	třináctou
V	A	A	A	N	J	N	V	C

"It was a bright cold day in April and the clocks were striking thirteen"

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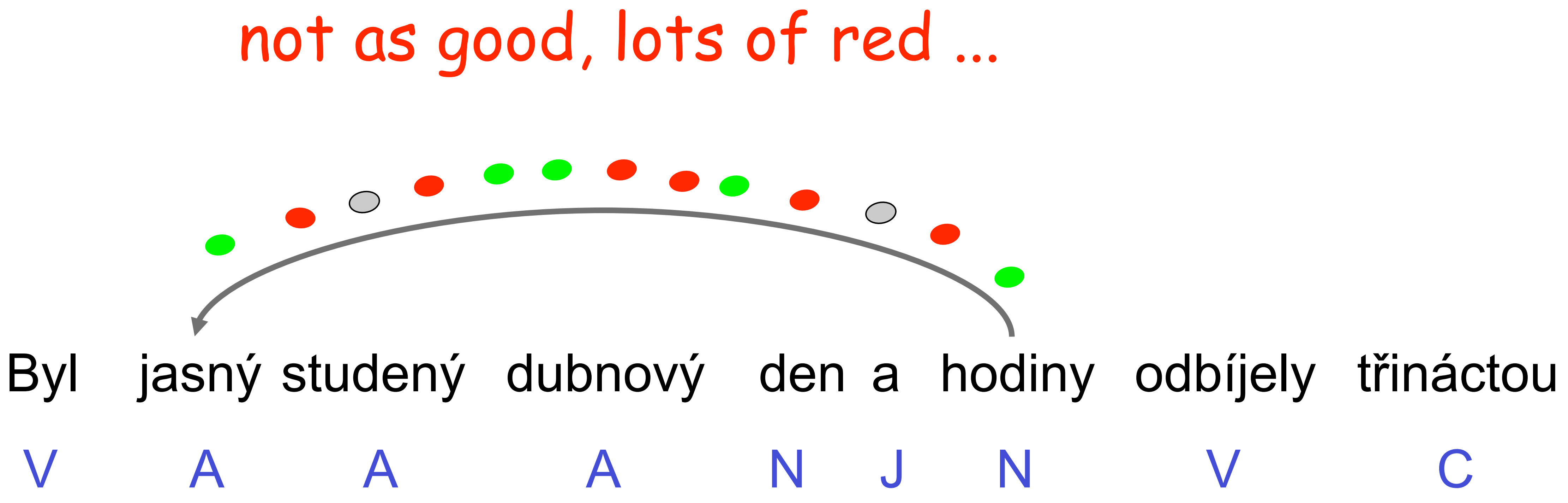

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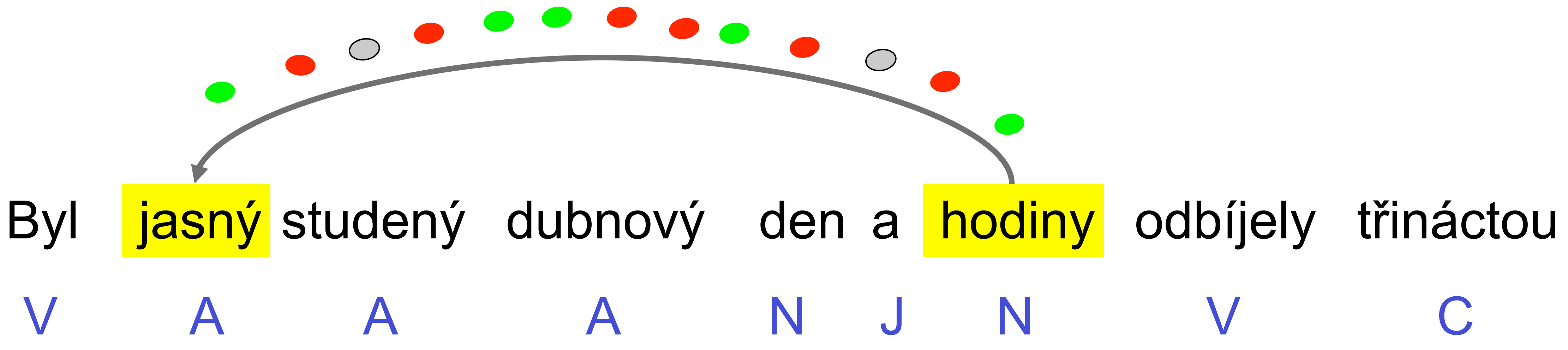

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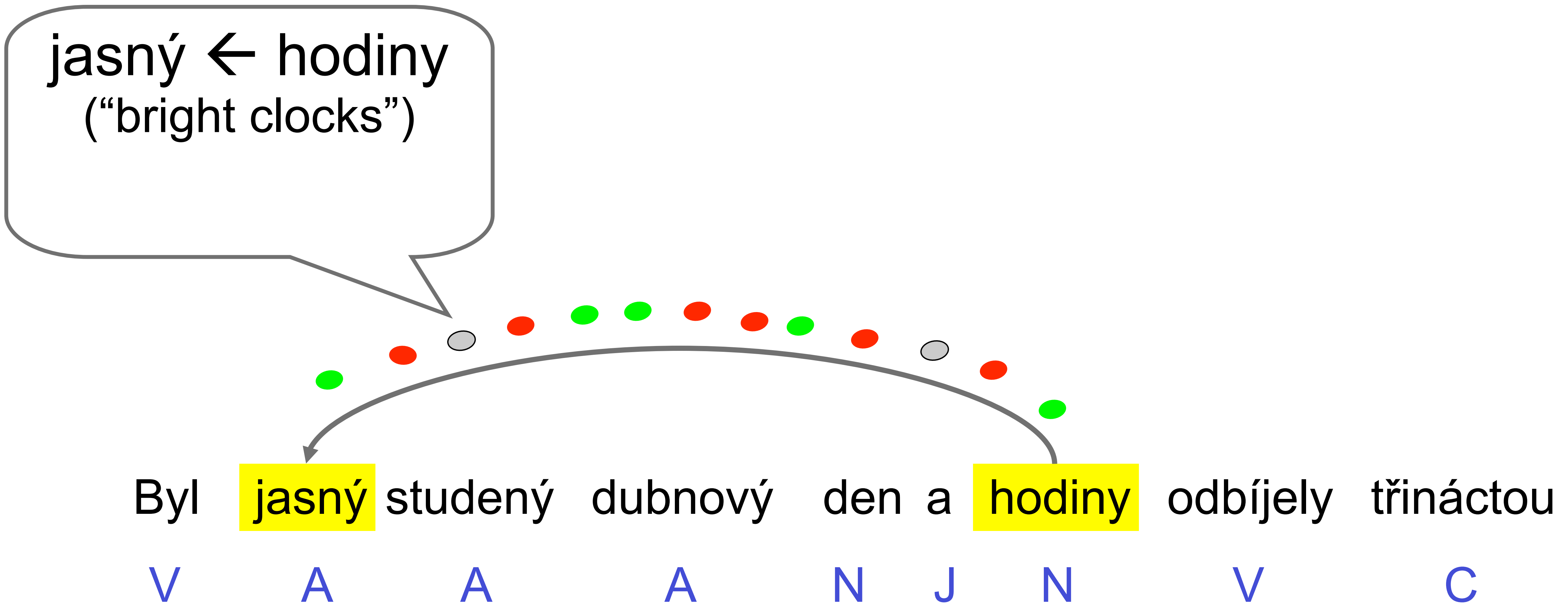

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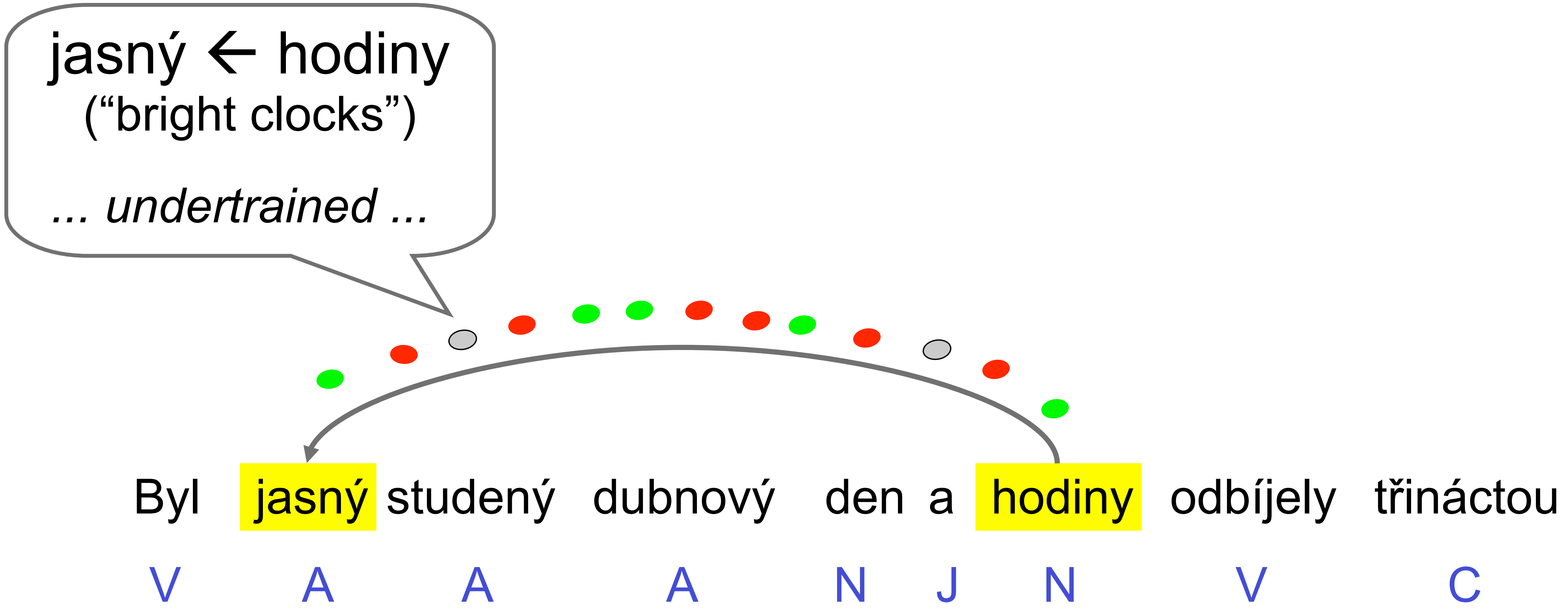

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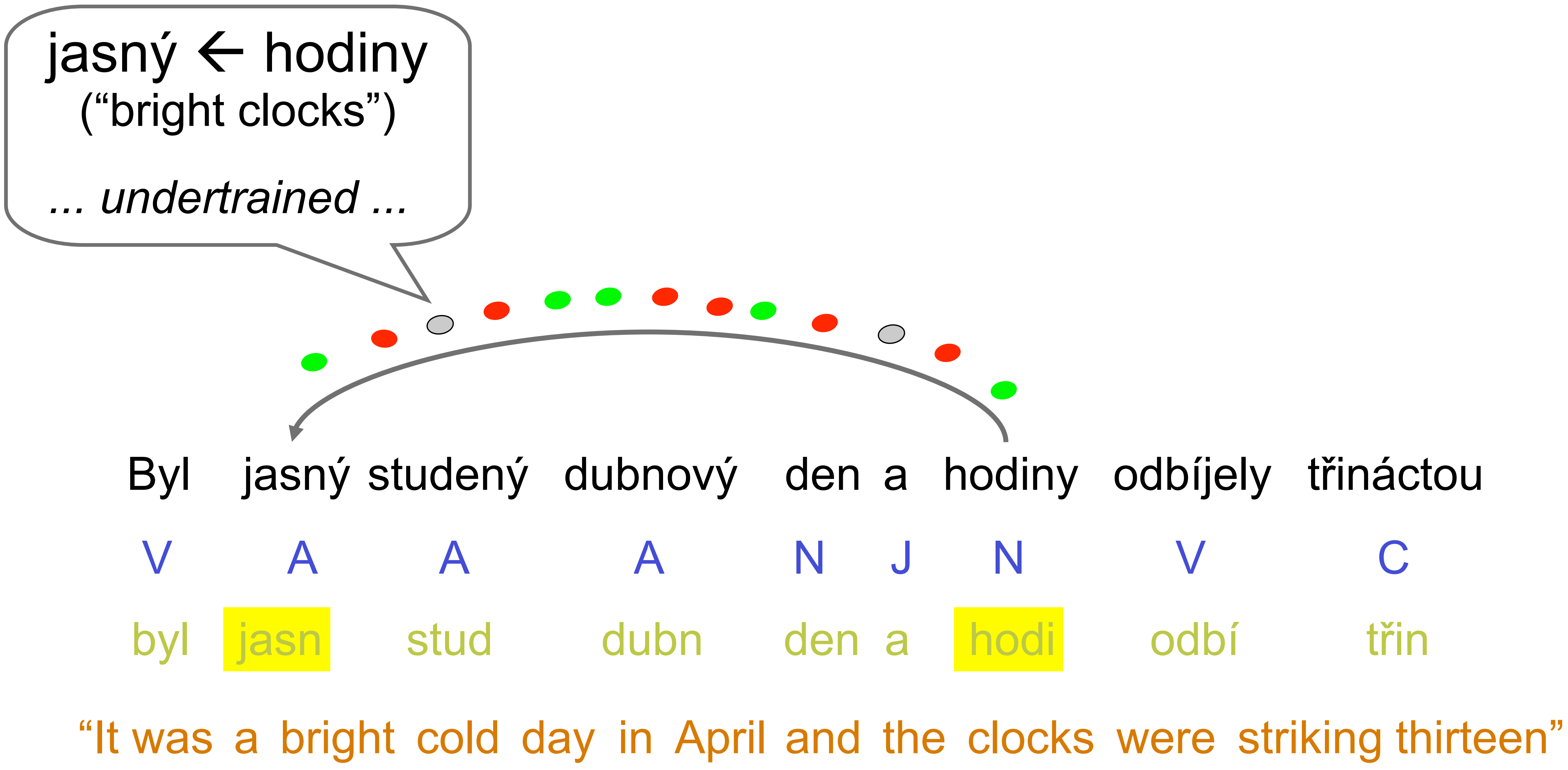
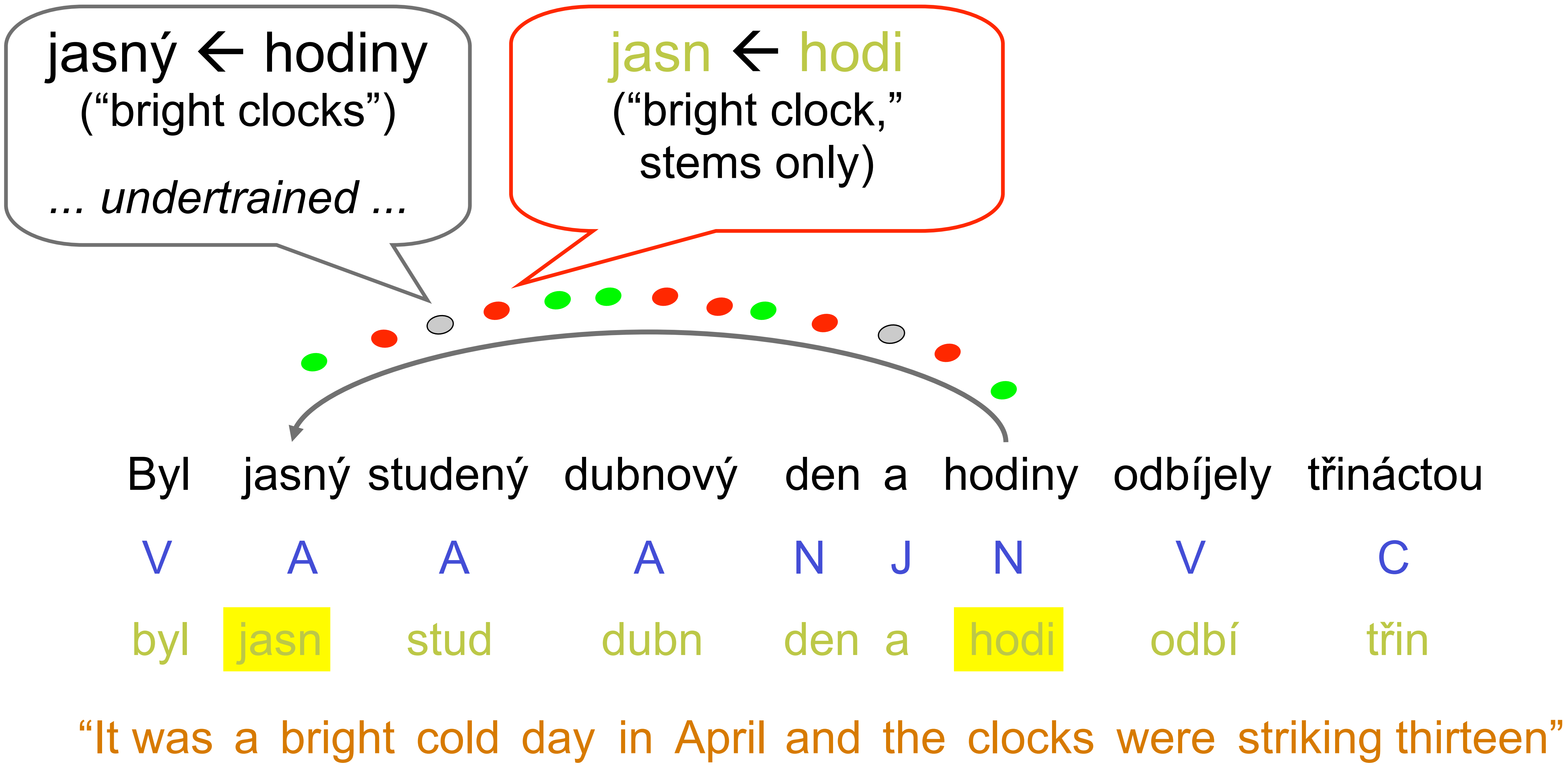

jasný ← hodiny
("bright clocks")
... *undertrained* ...
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C
byl jasn stud dubn den a hodi odbí třin
"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

- How about this competing edge?



jasný ← hodiny
(“bright clocks”)
... undertrained ...
jasn ← hodi
(“bright clock,” stems only)
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C
byl jasn stud dubn den a hodi odbí třin
“It was a bright cold day in April and the clocks were striking thirteen”

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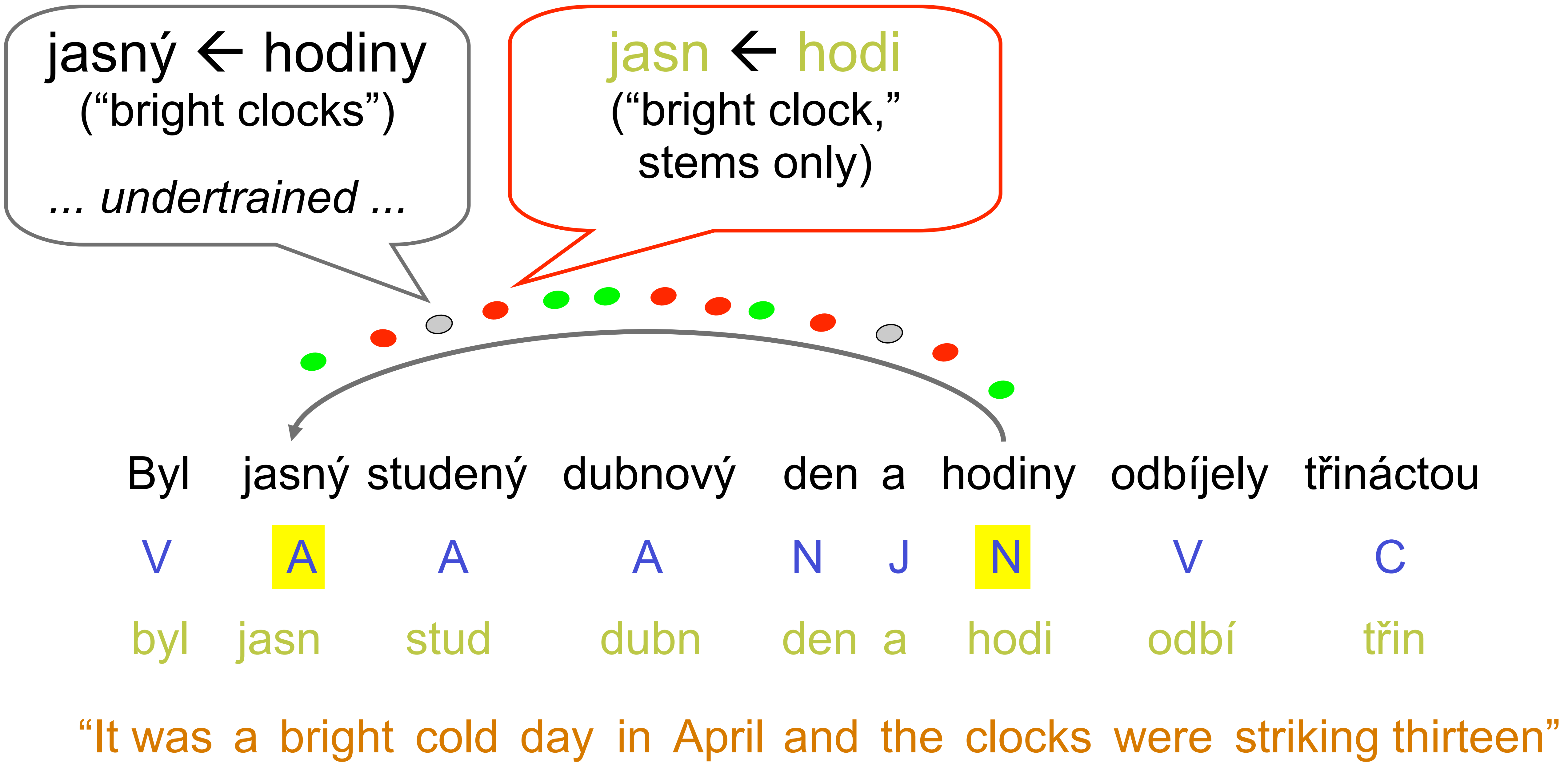
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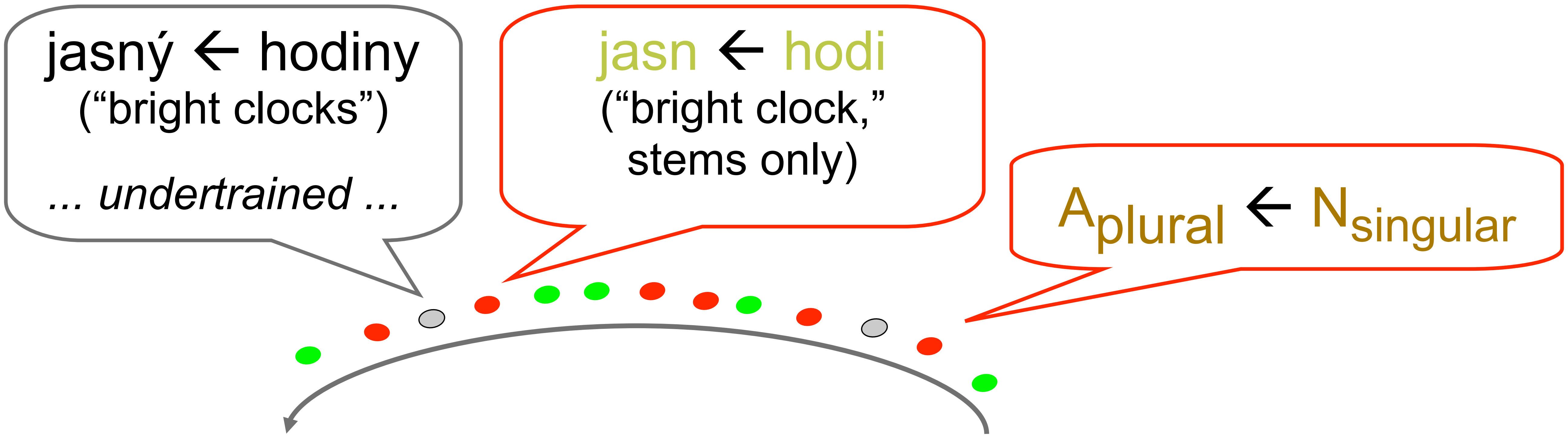
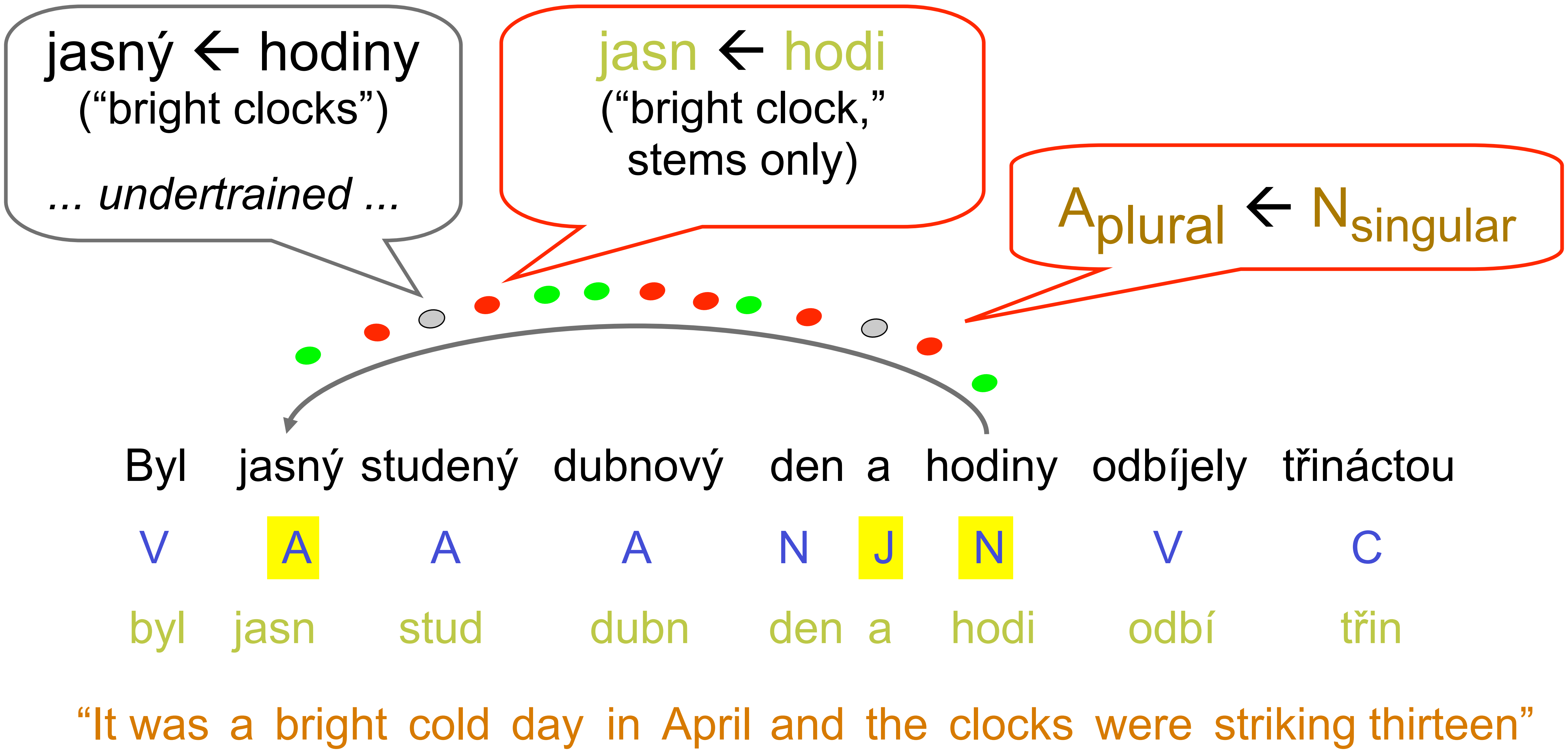

jasný ← hodiny
("bright clocks")
... *undertrained* ...
jasn ← hodi
("bright clock," stems only)
$A_{plural} \leftarrow N_{singular}$

Byl	jasný	studený	dubnový	den	a	hodiny	odbíjely	třináctou
V	A	A	A	N	J	N	V	C
byl	jasn	stud	dubn	den	a	hodi	odbí	třin

"It was a bright cold day in April and the clocks were striking thirteen"

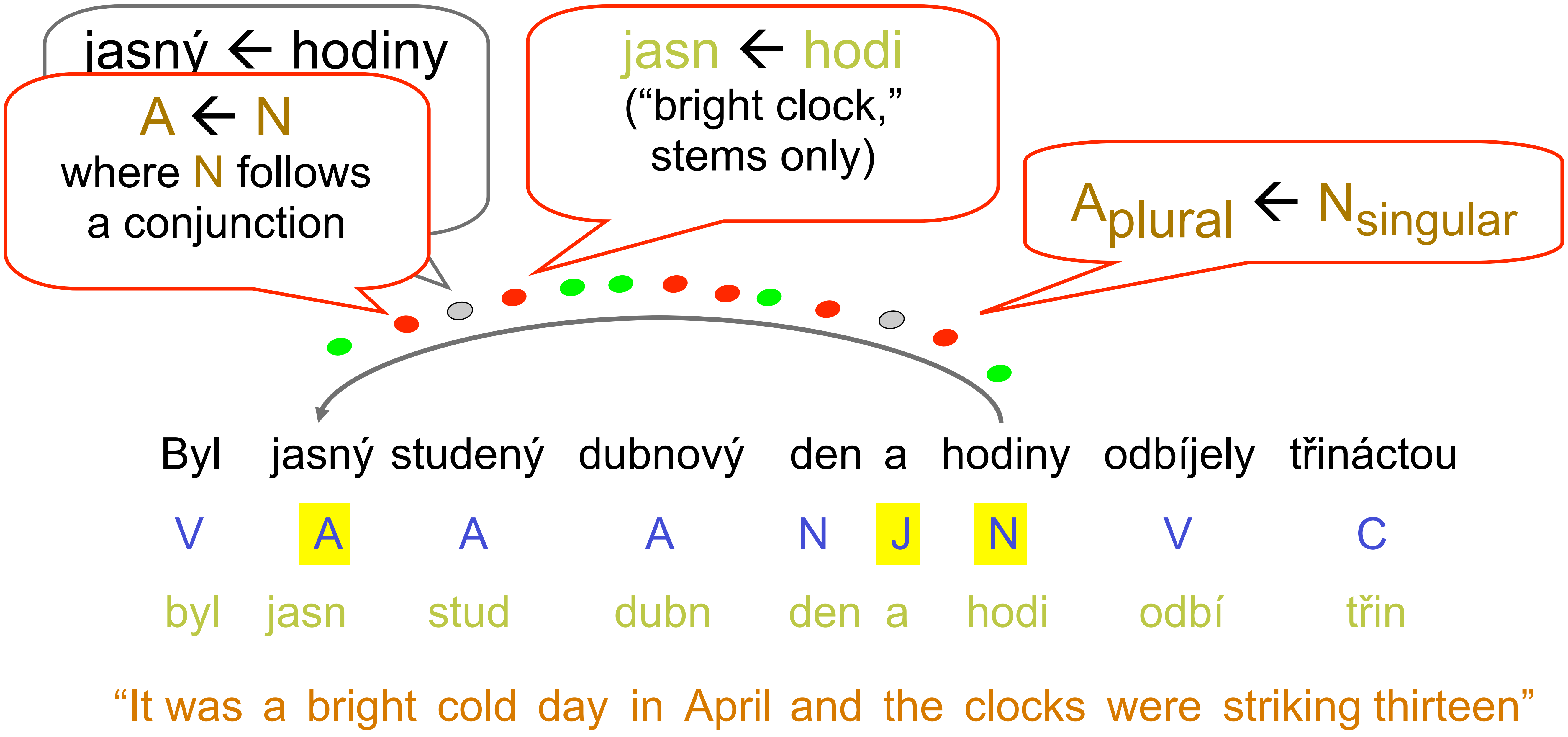
Edge-Factored Parsers (McDonald et al. 2005)

- How about this competing edge?


jasný ← hodiny
("bright clocks")
... *undertrained* ...
jasn ← hodi
("bright clock," stems only)
A_{plural} ← $N_{singular}$
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C
byl jasn stud dubn den a hodi odbí třin
"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

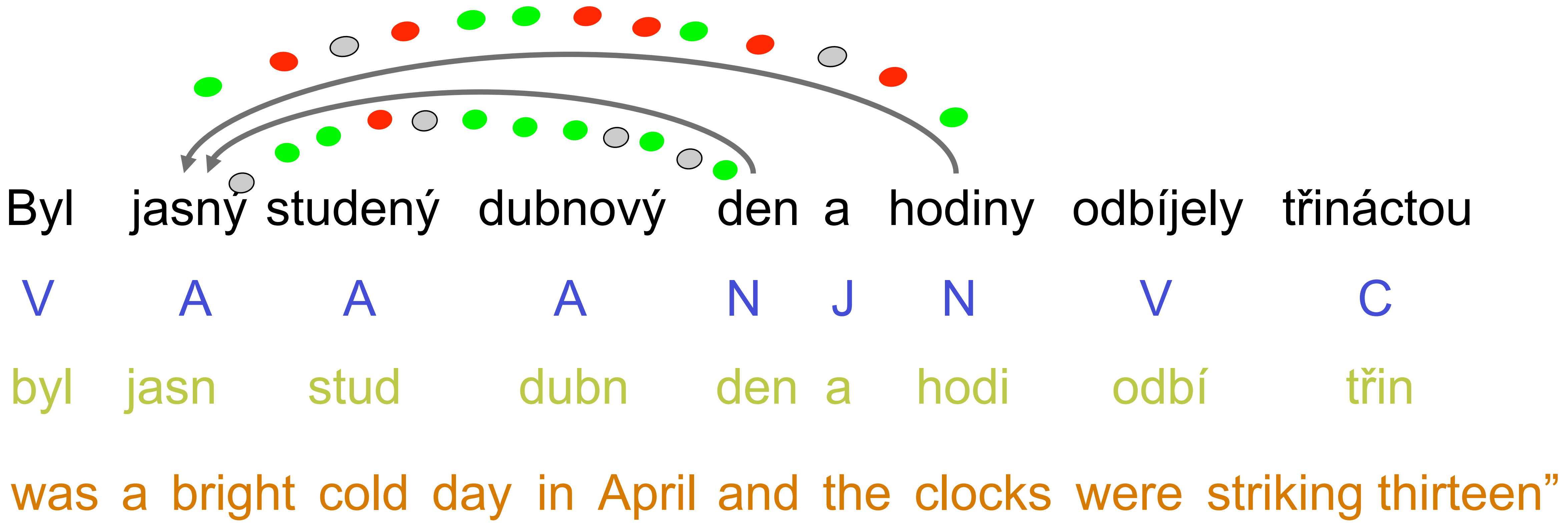
- How about this competing edge?



jasný ← hodiny
A ← N where N follows a conjunction
jasn ← hodi ("bright clock," stems only)
$A_{plural} \leftarrow N_{singular}$
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C
byl jasn stud dubn den a hodi odbí třin
"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

- Which edge is better?
  - "bright day" or "bright clocks"?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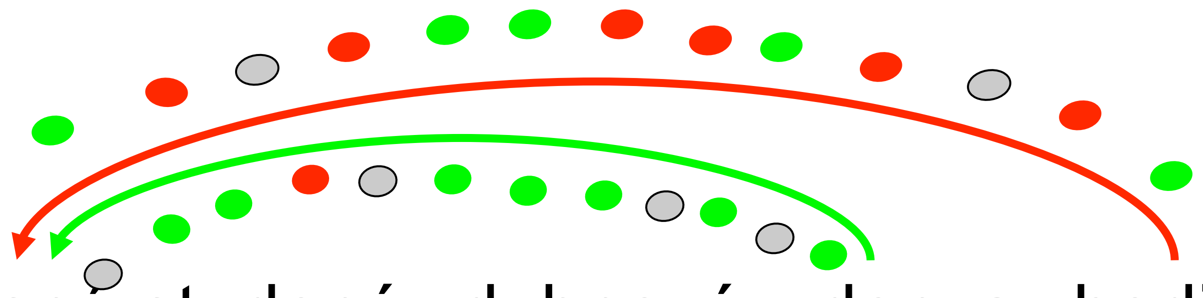

Byl	jasný	studený	dubnový	den	a	hodiny	odbíjely	třináctou
V	A	A	A	N	J	N	V	C
byl	jasn	stud	dubn	den	a	hodi	odbí	třin

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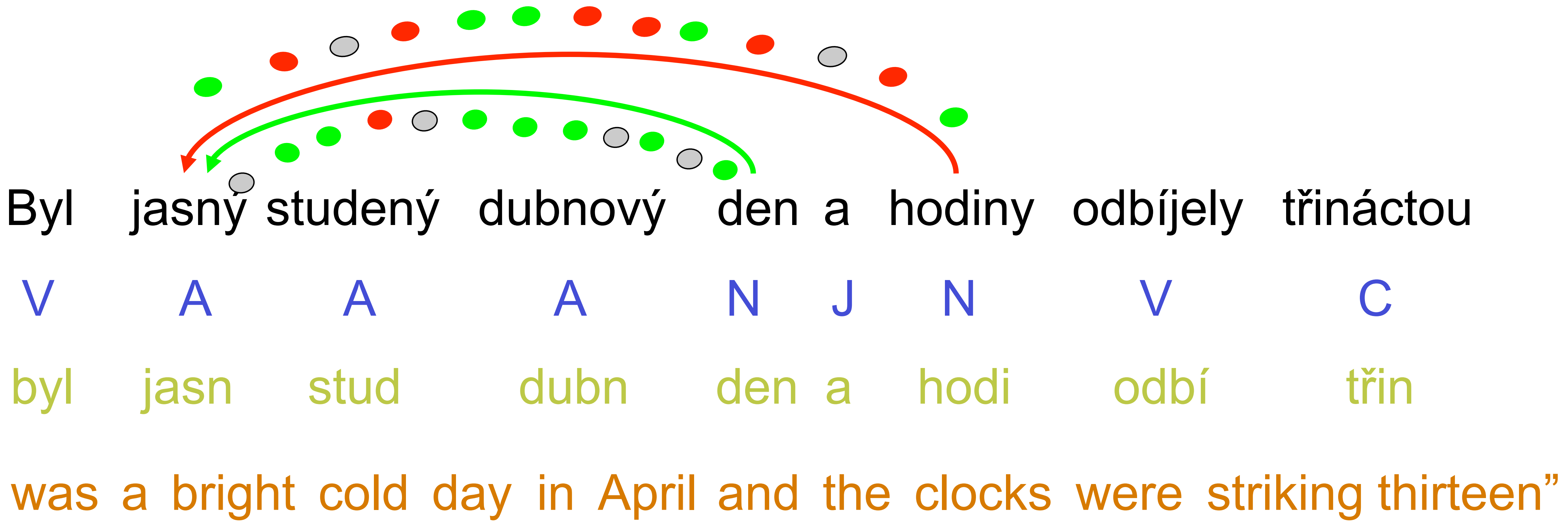
- Which edge is better?



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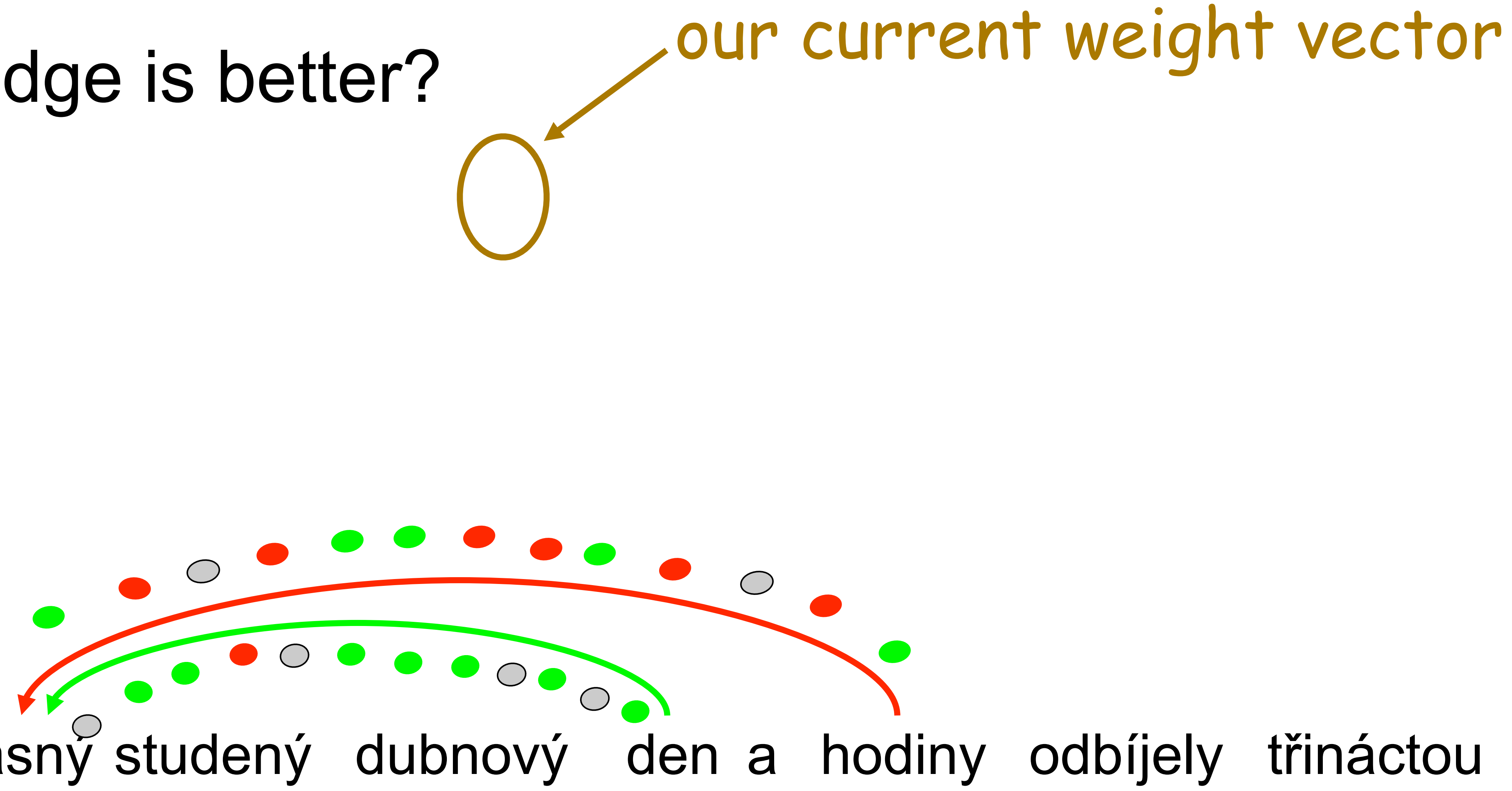


Byl jasný studený dubnový den a hodiny odbíjely třináctou

V A A A N J N V C

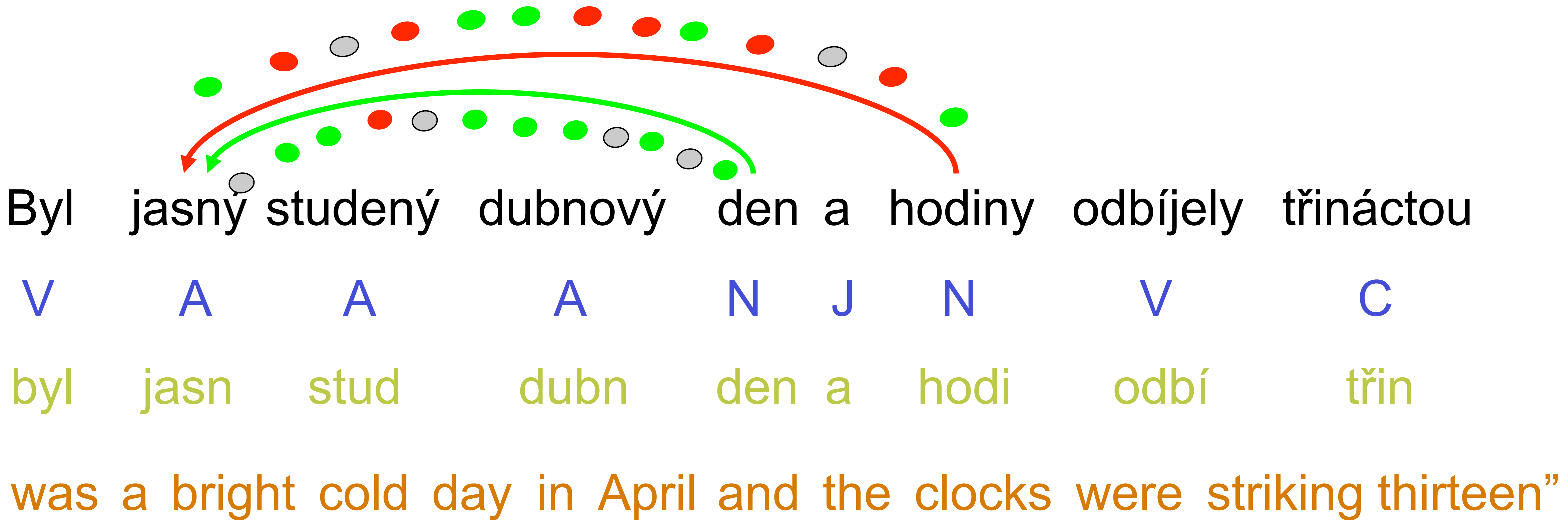
byl jasn stud dubn den a hodi odbí třin

"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

- Which edge is better?
- Score of an edge e = θ · **features**(e)

our current weight vector



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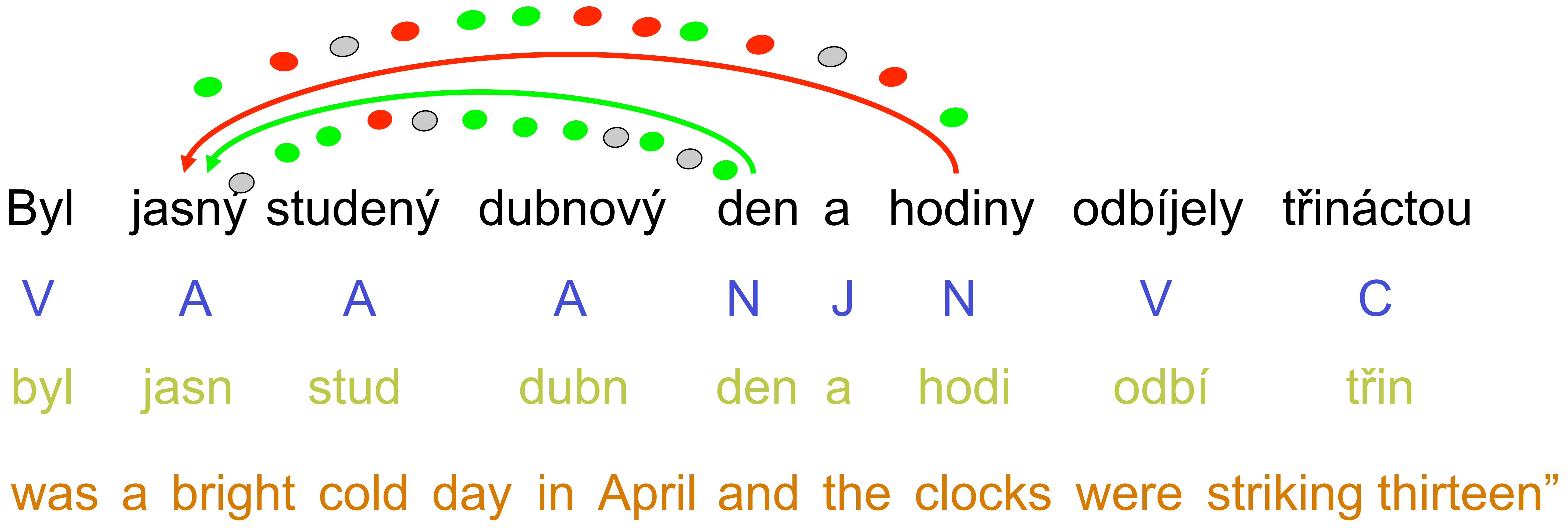
Byl jasný studený dubnový den a hodiny odbíjely třináctou
V A A A N J N V C
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"It was a bright cold day in April and the clocks were striking thirteen"

Edge-Factored Parsers (McDonald et al. 2005)

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- Which edge is better?
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Edge-Factored Parsers (McDonald et al. 2005)

our current weight vector

- Which edge is better?
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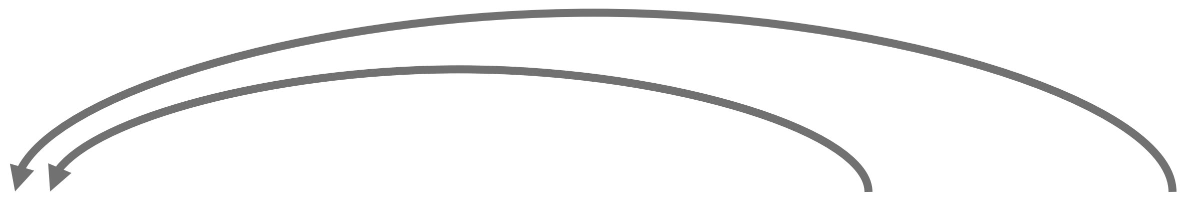
can't have both

(one parent per word)

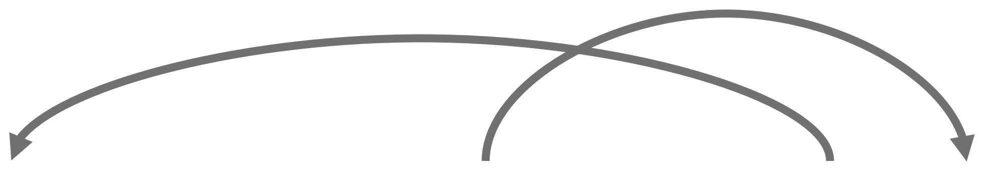
Edge-Factored Parsers (McDonald et al. 2005)

our current weight vector

- Which edge is better?
- Score of an edge e = θ · **features**(e)
- Standard algos ➔ valid parse with max total score



can't have both
(one parent per word)



can't have both
(no crossing links)

Edge-Factored Parsers (McDonald et al. 2005)

our current weight vector

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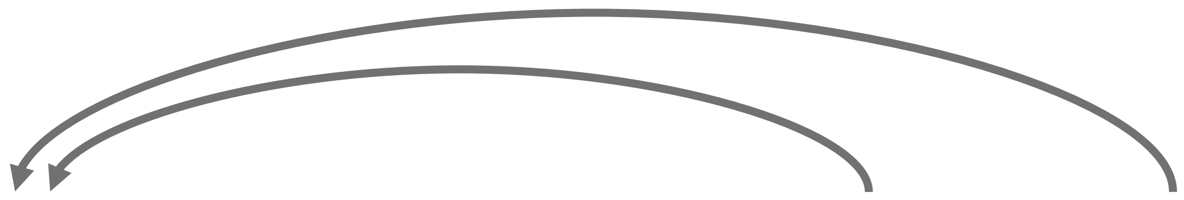

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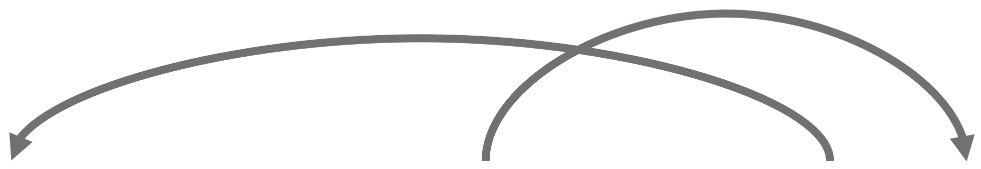

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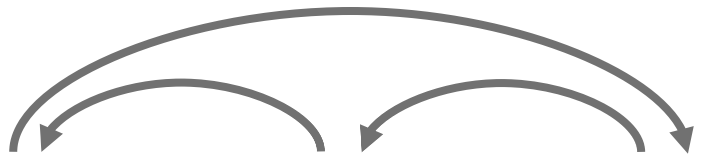

Can't have all three

(no cycles)

Edge-Factored Parsers (McDonald et al. 2005)

our current weight vector

- Which edge is better?
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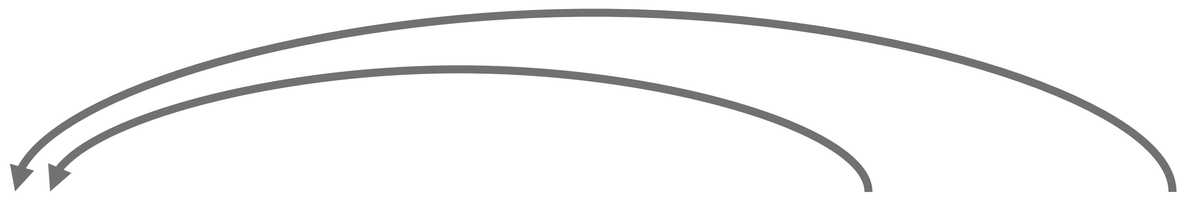

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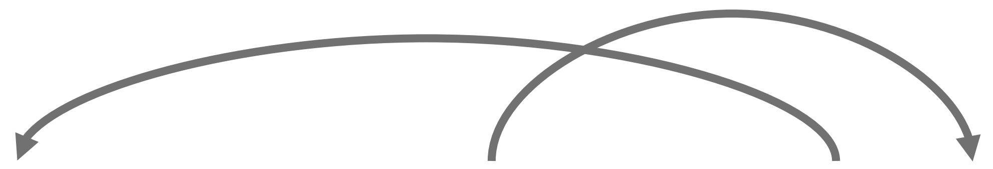

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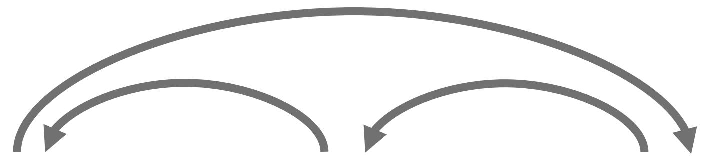


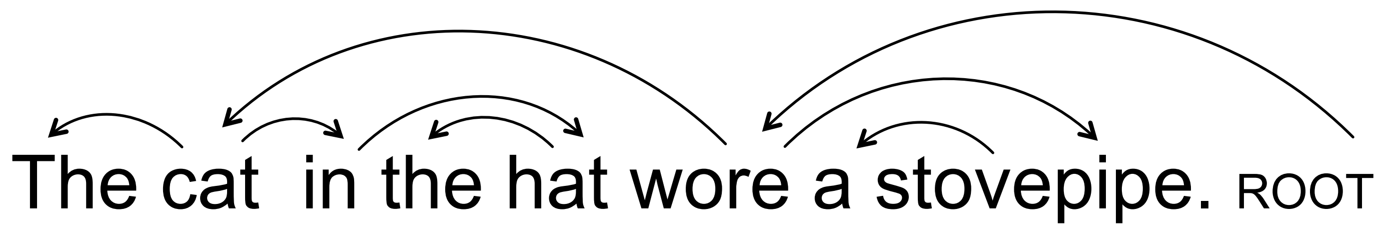
Can't have all three

(no cycles)

Thus, an edge may lose (or win) because of a consensus of **other** edges.

Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
- Then use dynamic programming



Finding Highest-Scoring Parse

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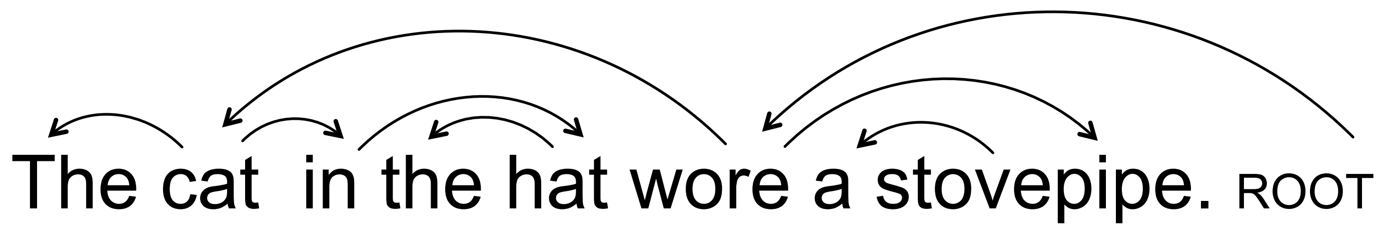

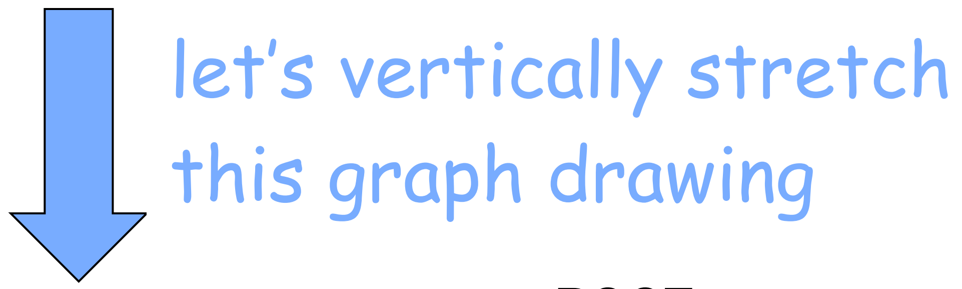

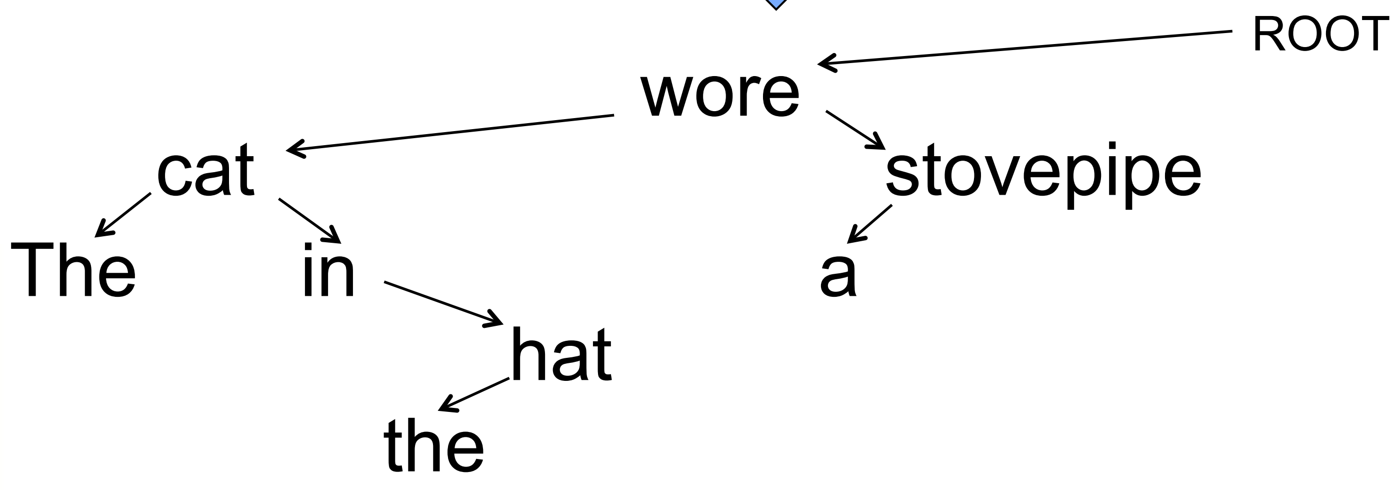

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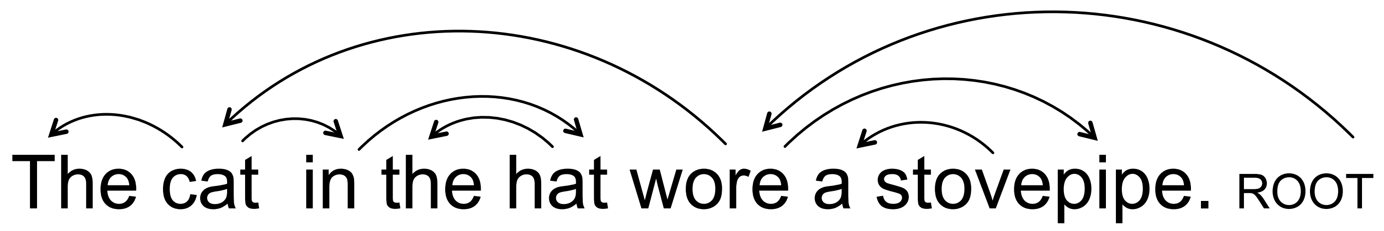
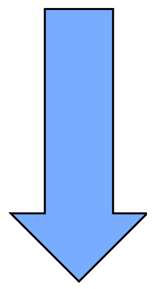
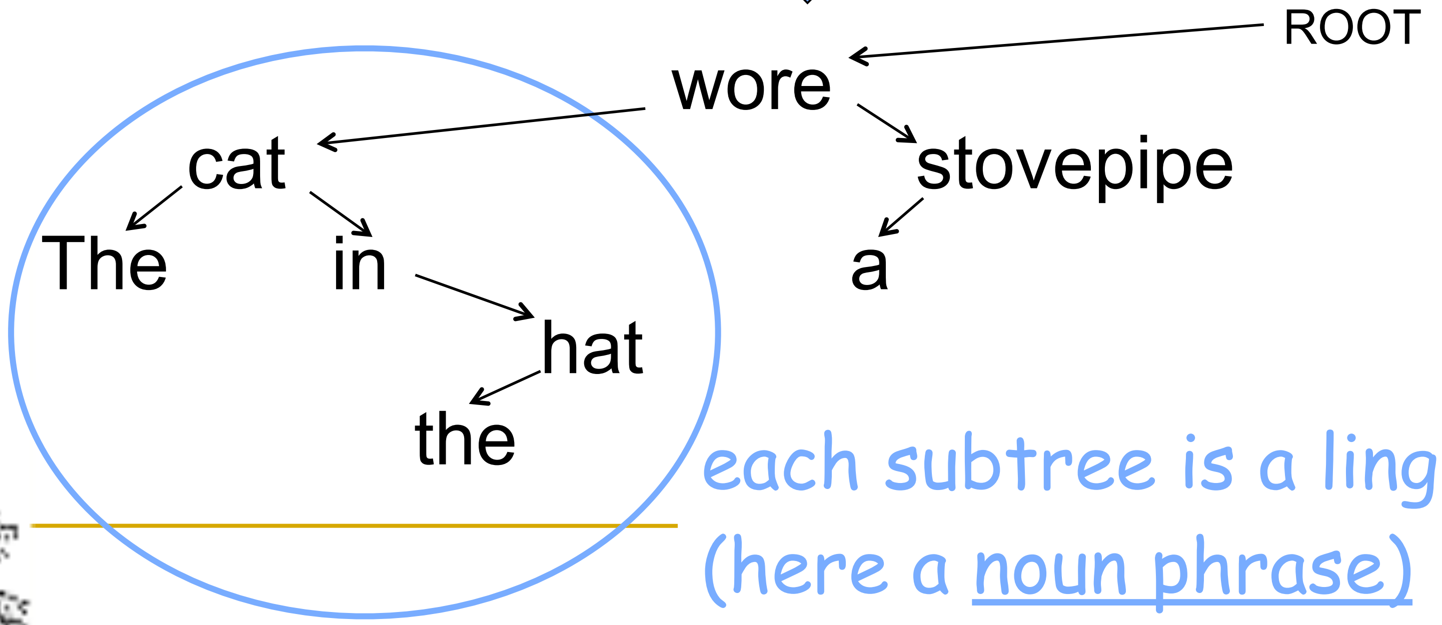




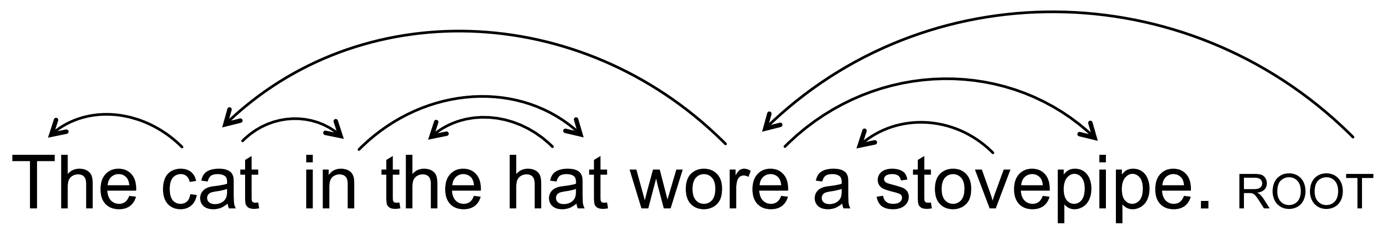
let's vertically stretch this graph drawing

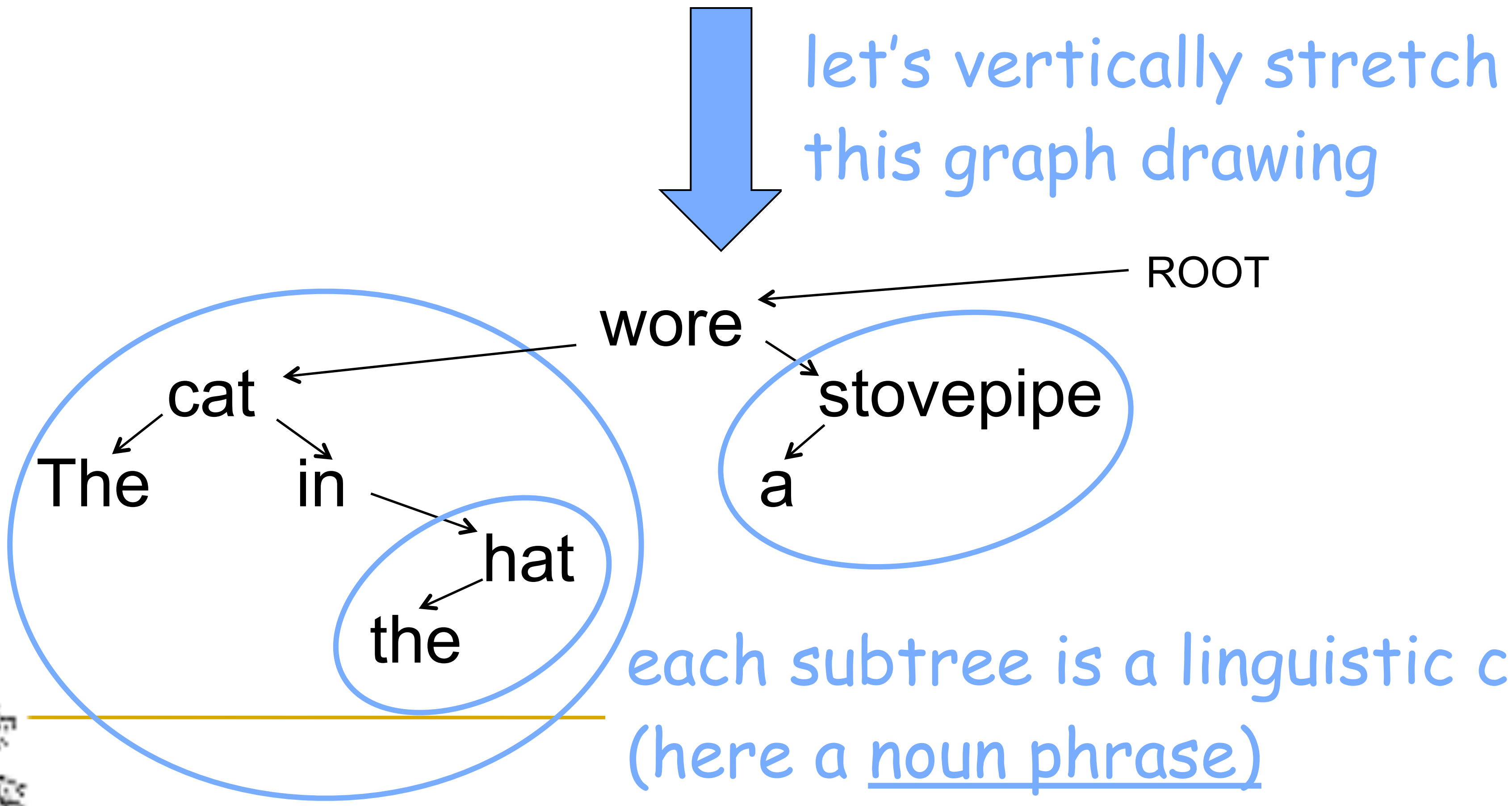


each subtree is a linguistic constituent (here a noun phrase)

Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
- Then use dynamic programming





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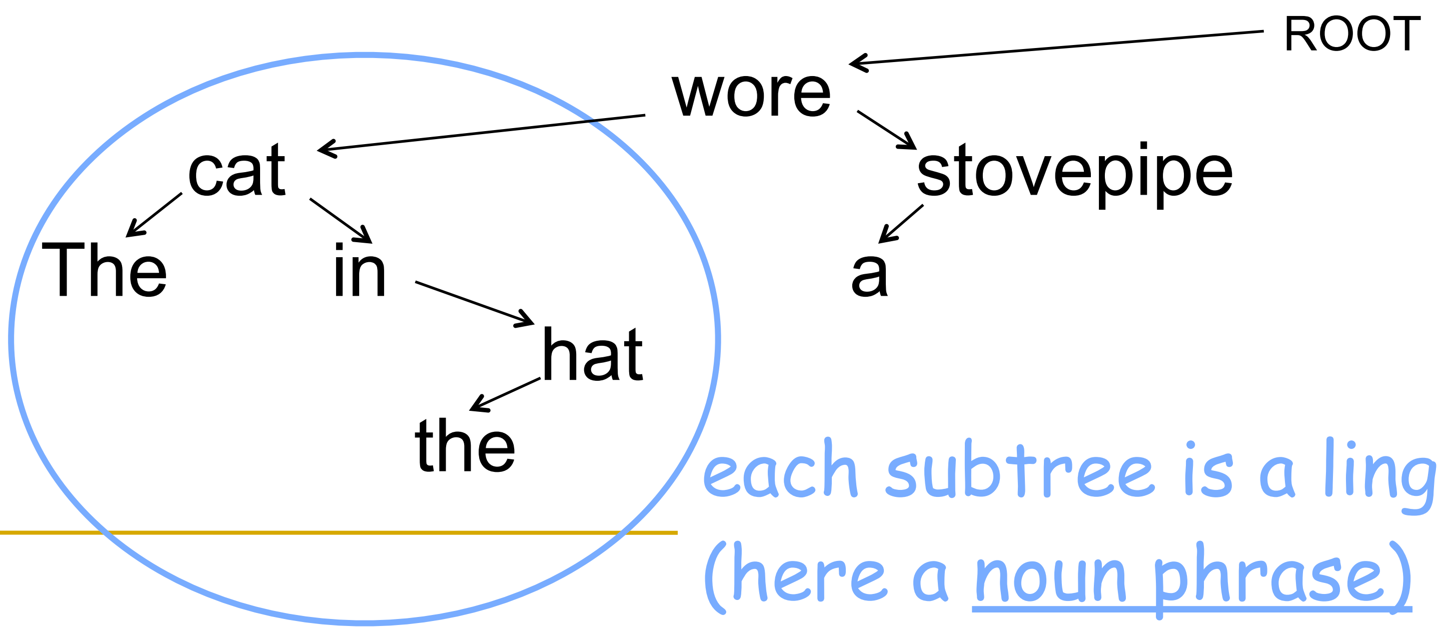
Finding Highest-Scoring Parse



each subtree is a linguistic constituent
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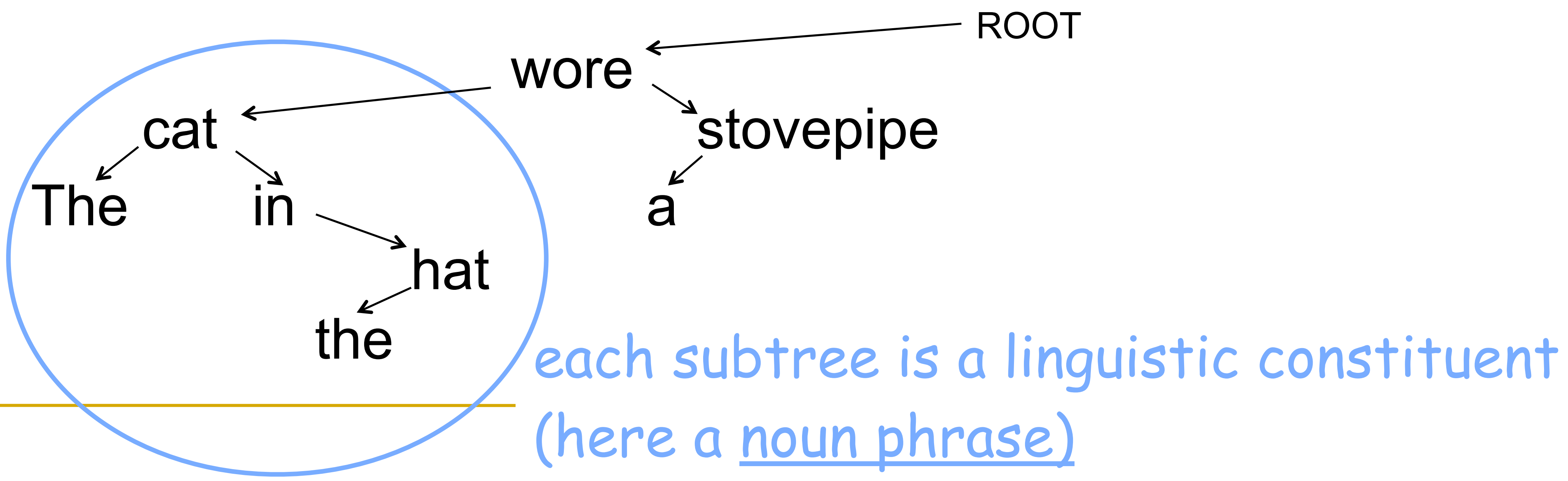
Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
- Then use dynamic programming
  - CKY algorithm for CFG parsing is $O(n^3)$



Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
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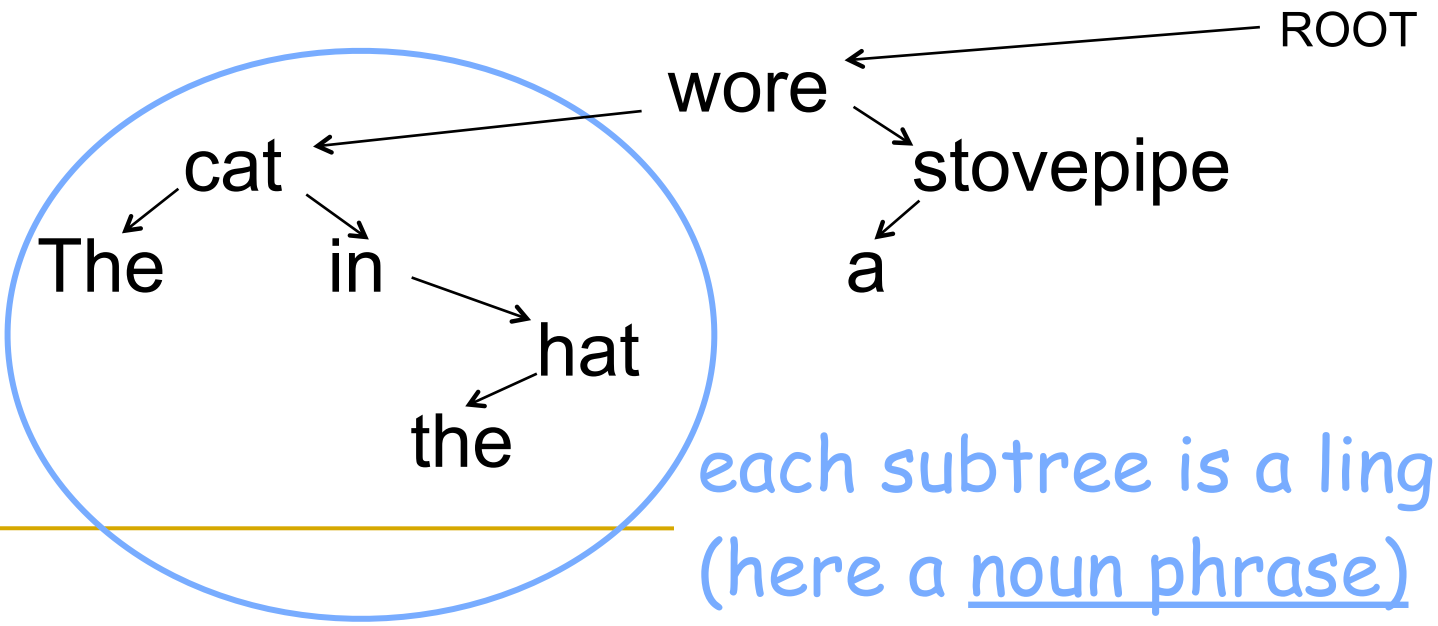
  - CKY algorithm for CFG parsing is $O(n^3)$
  - Unfortunately, $O(n^5)$ in this case



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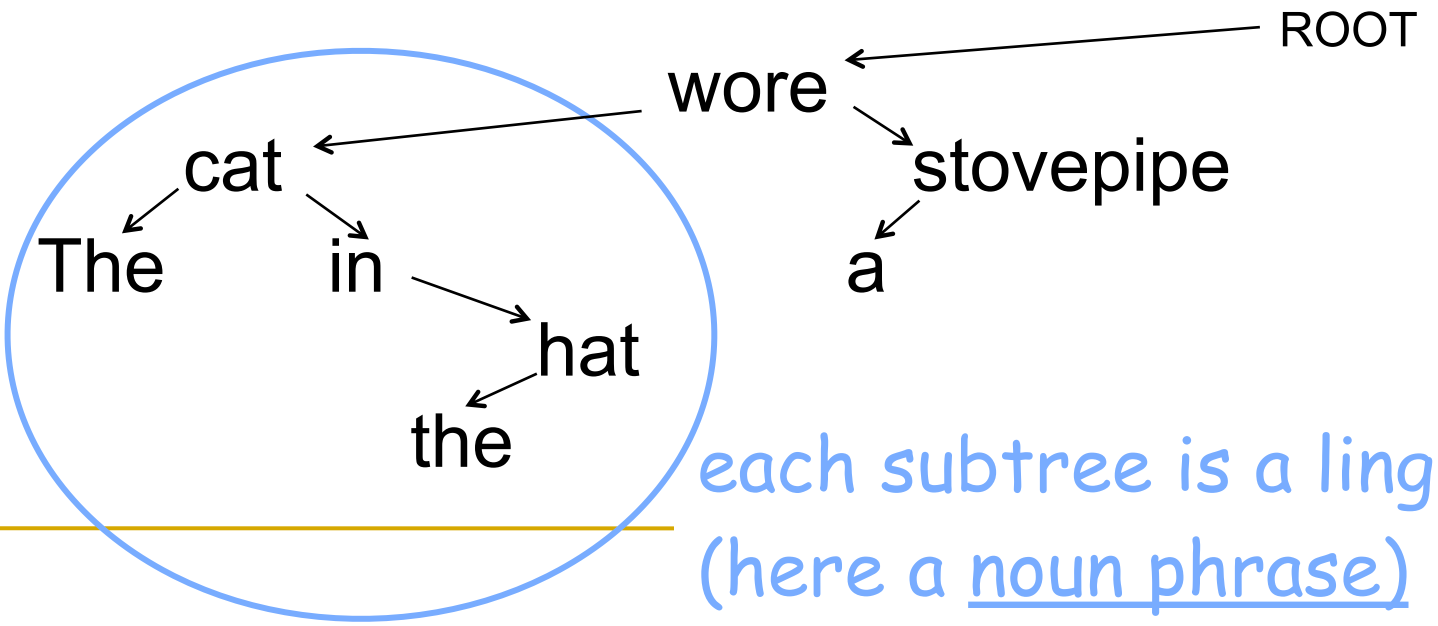
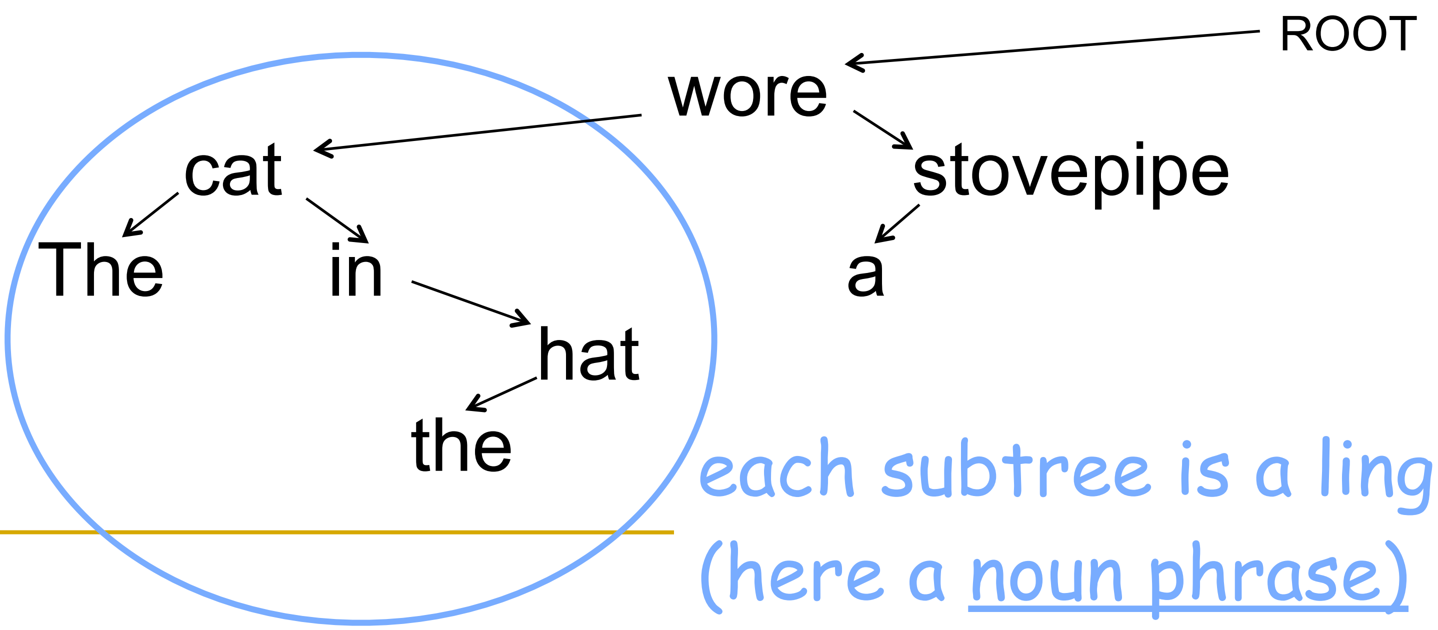
  - CKY algorithm for CFG parsing is $O(n^3)$
  - Unfortunately, $O(n^5)$ in this case
    - to score "cat ← wore" link, not enough to know this is NP



each subtree is a linguistic constituent (here a noun phrase)

Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
- Then use dynamic programming
  - CKY algorithm for CFG parsing is $O(n^3)$
  - Unfortunately, $O(n^5)$ in this case
    - to score "cat ← wore" link, not enough to know this is NP
    - must know it's rooted at "cat"



each subtree is a linguistic constituent (here a noun phrase)

Finding Highest-Scoring Parse

- Convert to context-free grammar (CFG)
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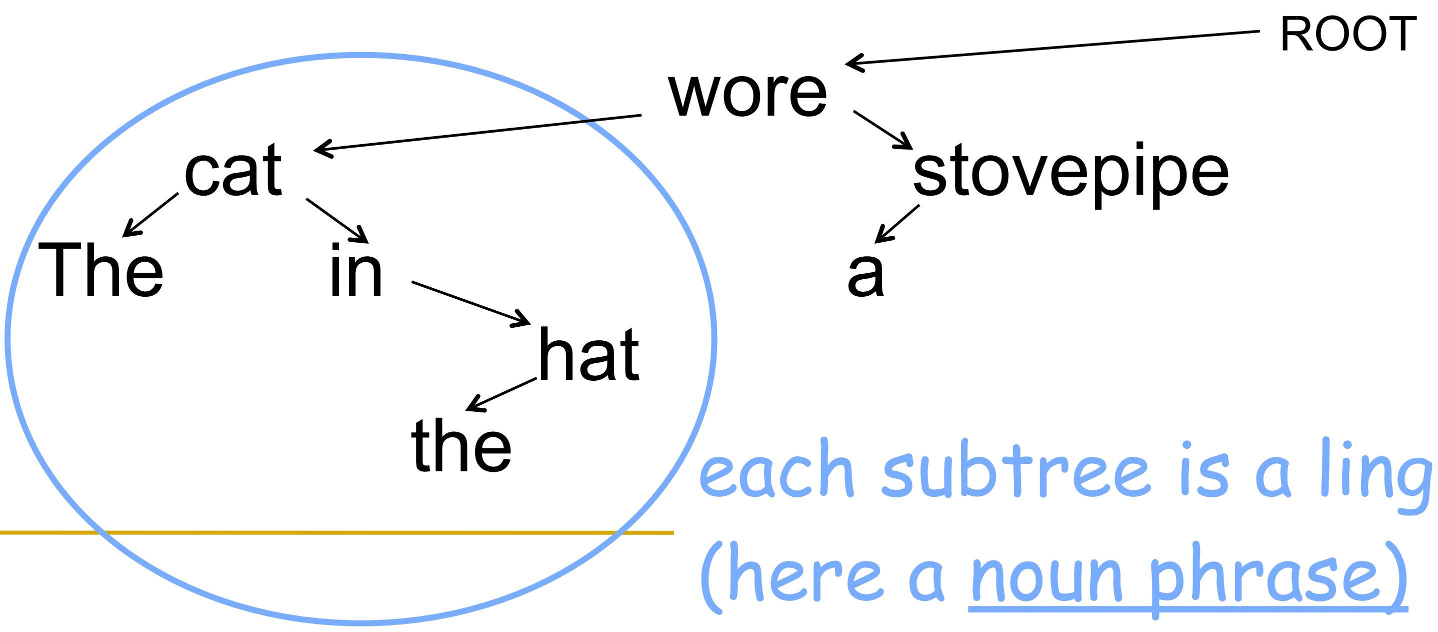

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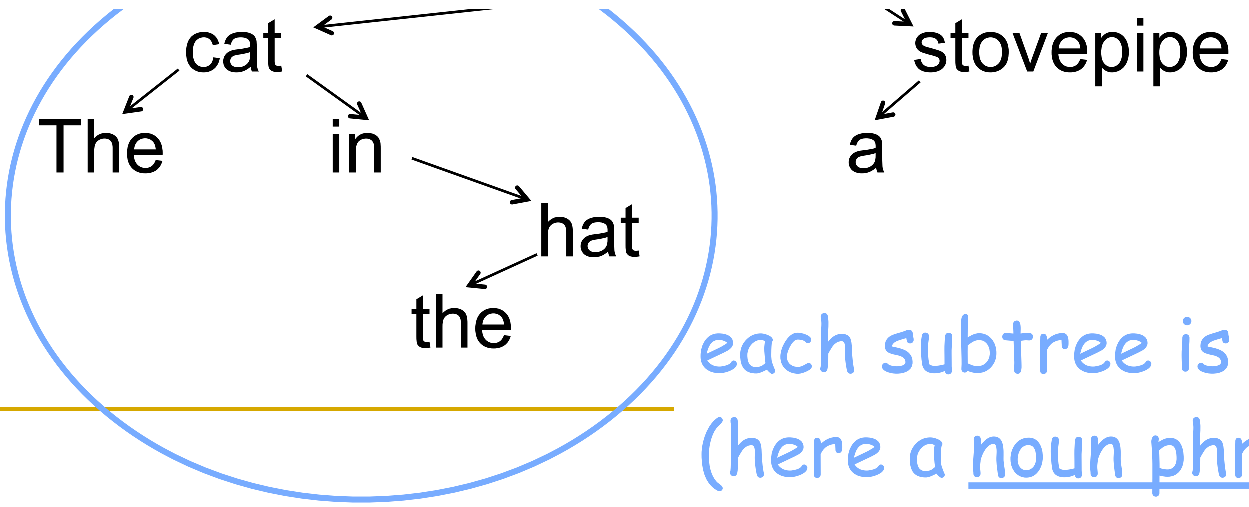
    - so CKY's "grammar constant" is no longer constant ☹



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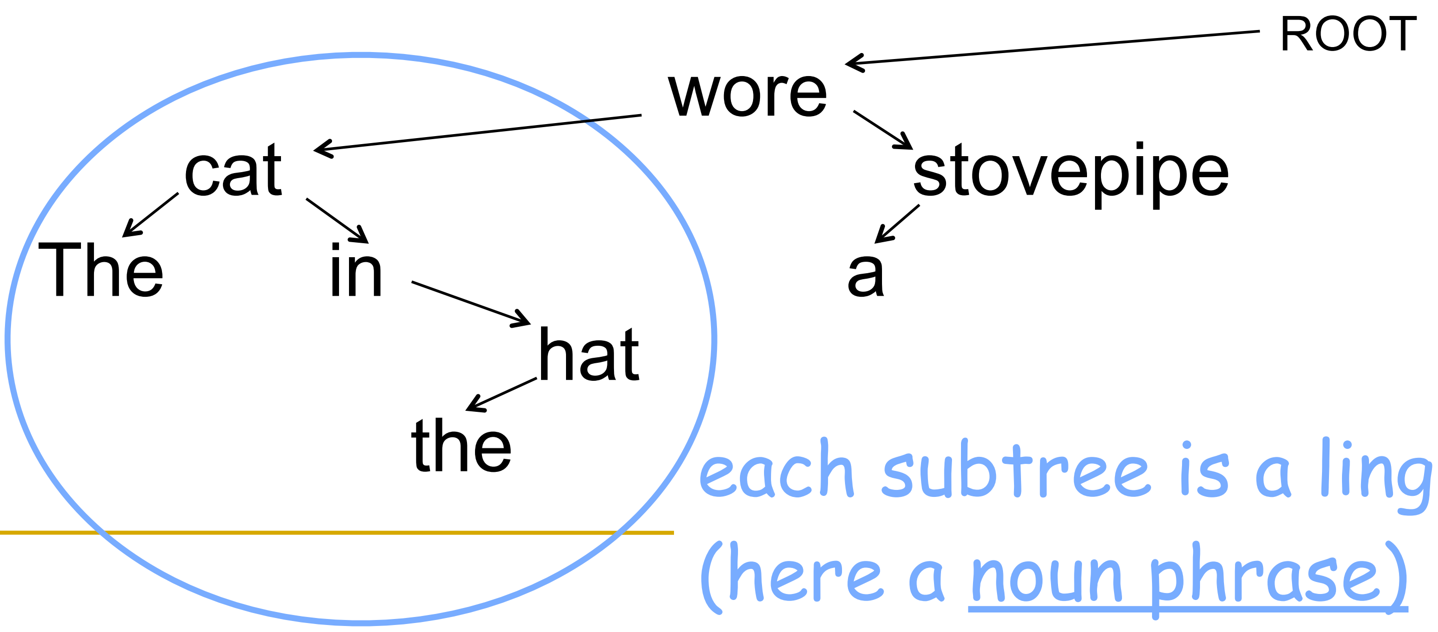

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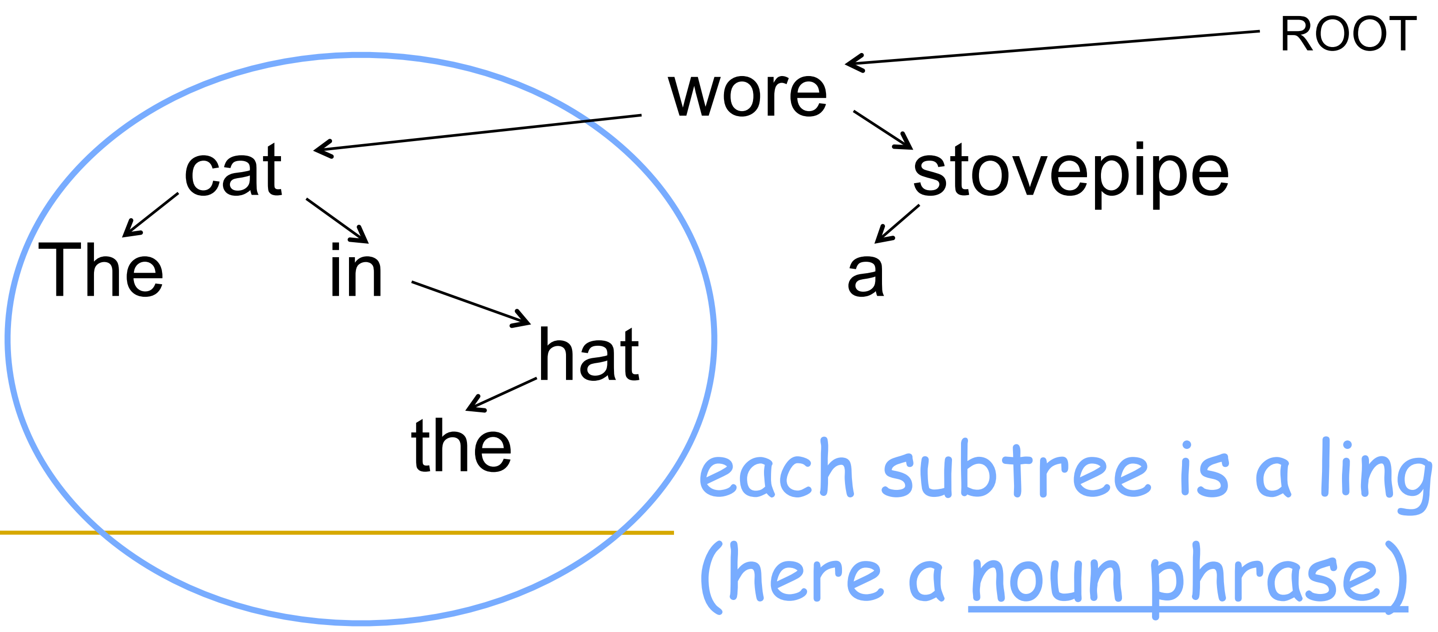
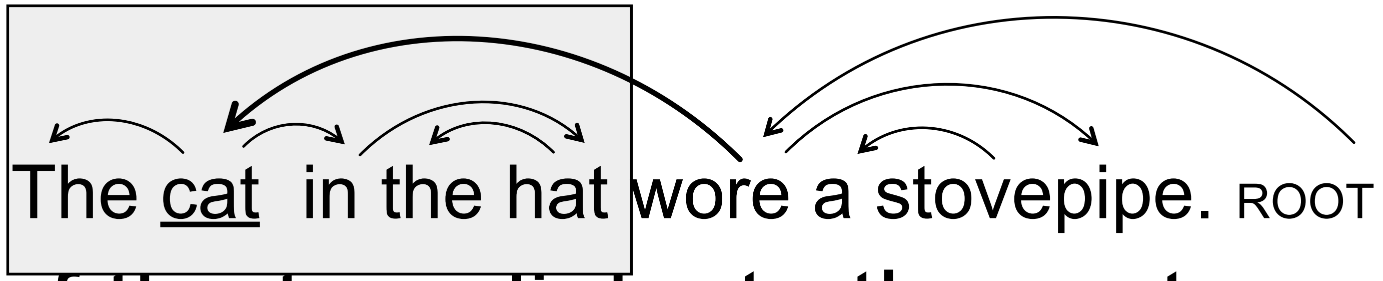


each subtree is a linguistic constituent (here a noun phrase)

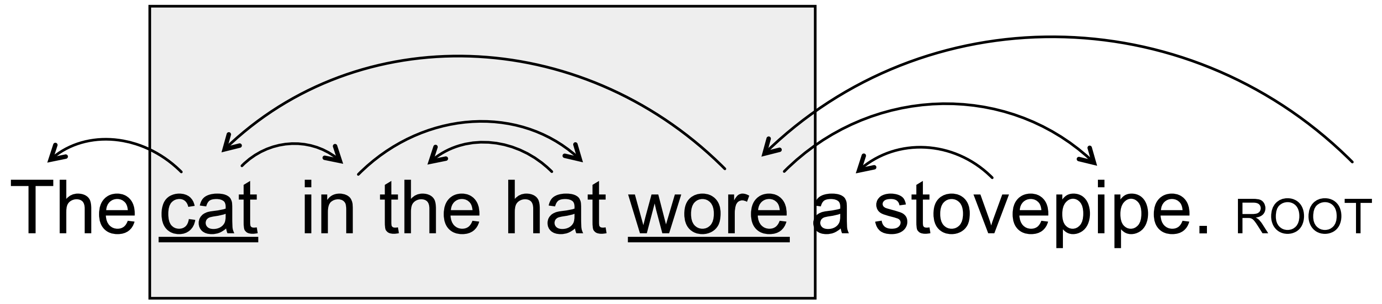
Spans vs. constituents

Two kinds of substring.

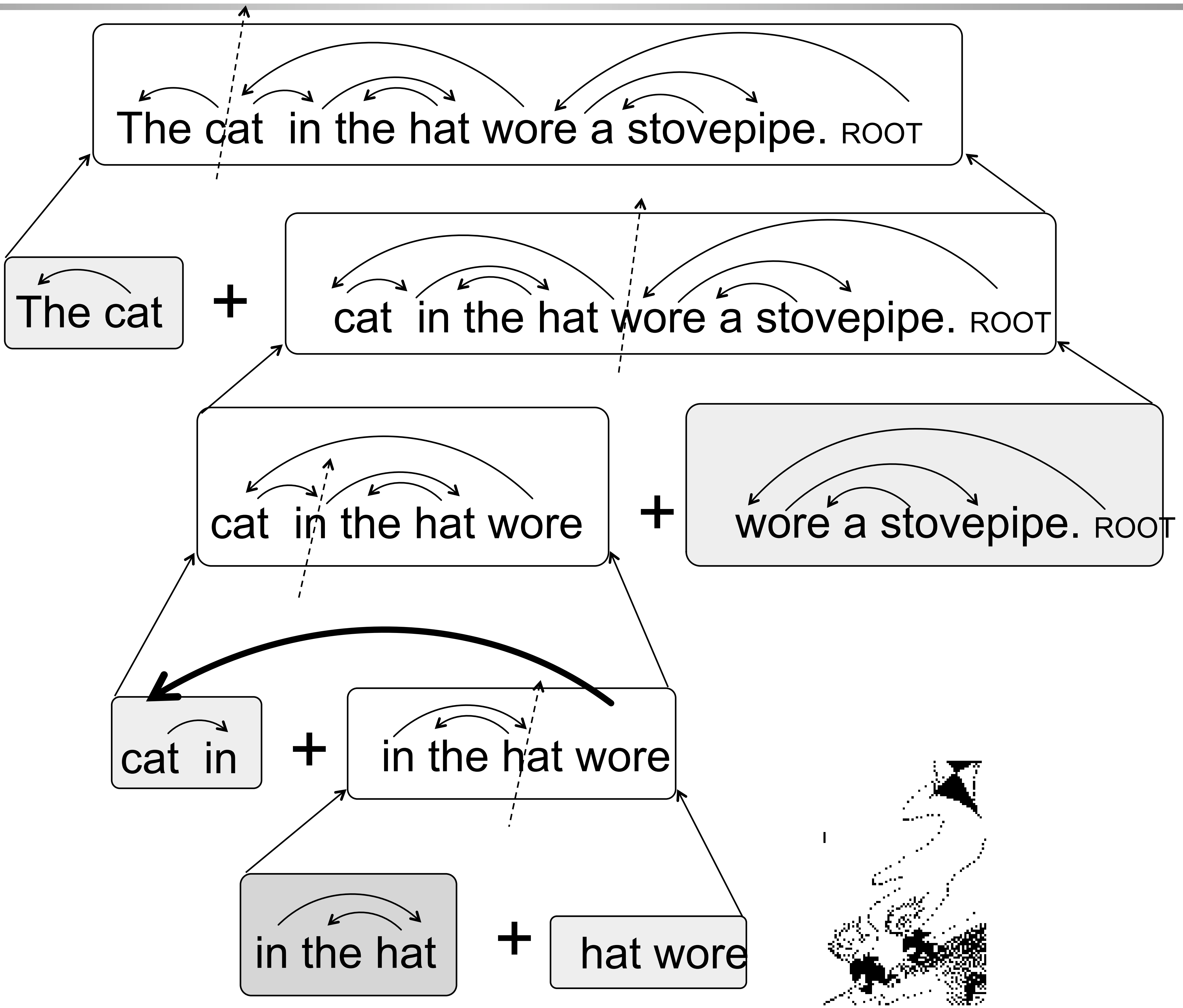
» **<u>Constituent</u>** of the tree: links to the rest only through its <u>headword</u> (root).



» **<u>Span</u>** of the tree: links to the rest only through its <u>endwords</u>.



Decomposing a tree into spans



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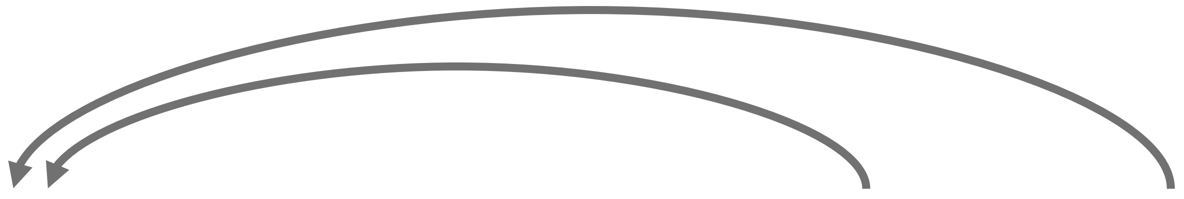
} require "outside" probabilities of constituents, spans, or links

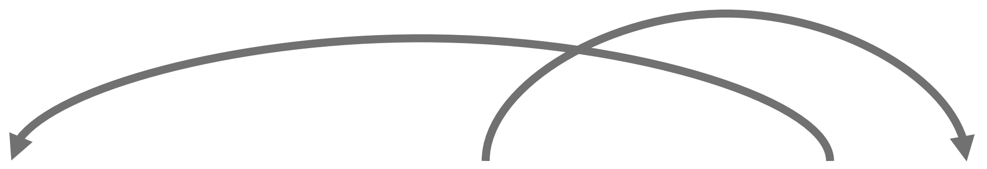
Hard Constraints on Valid Trees

our current weight vector

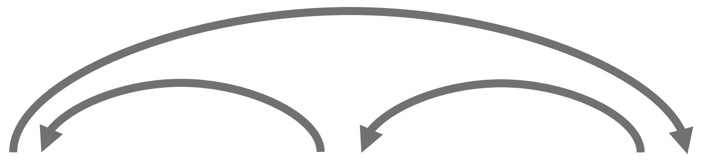
- Score of an edge e = $\theta \cdot$ **features**(e)
- Standard algos ➔ valid parse with max <u>total</u> score



can't have both
(one parent per word)



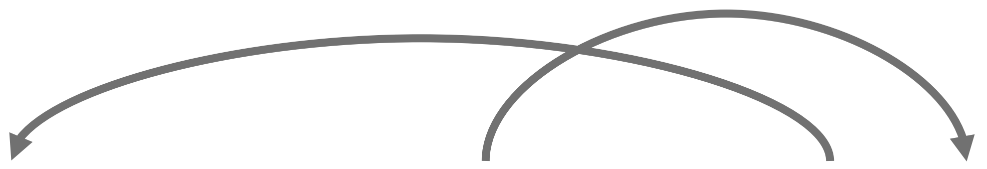
can't have both
(no crossing links)



Can't have all three
(no cycles)

Thus, an edge may lose (or win) because of a consensus of **<u>other</u>** edges.

Hard Constraints on Valid Trees



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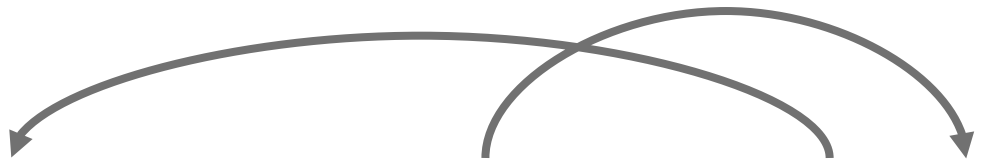
Non-Projective Parses



can't have both

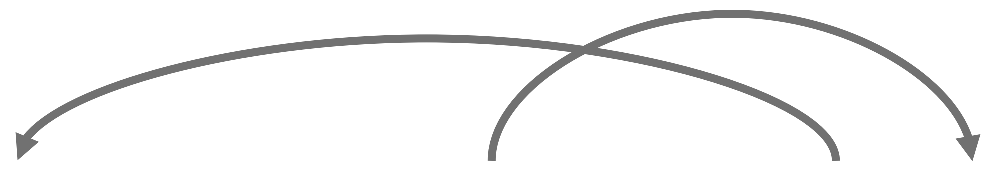
(no crossing links)

The "projectivity" restriction.

Do we really want it?

Non-Projective Parses

ROOT I 'll give a talk tomorrow on bootstrapping



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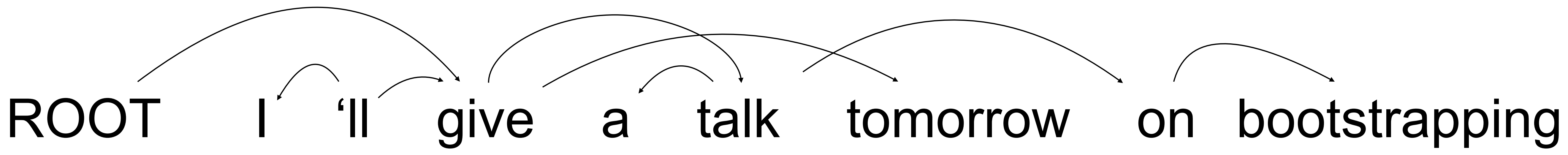
Non-Projective Parses



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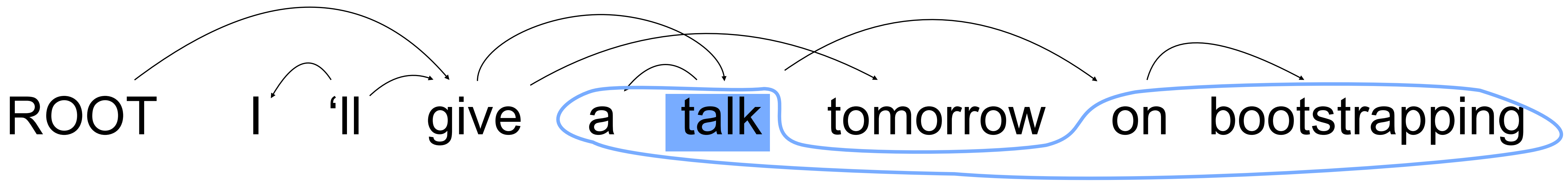
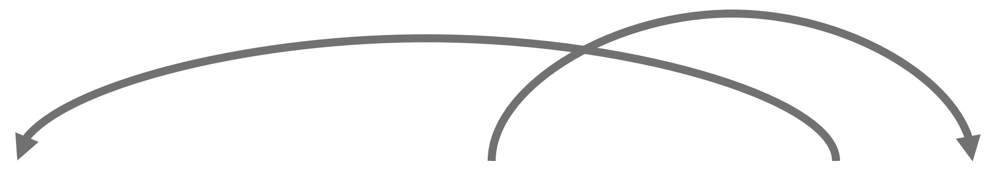


subtree rooted at "talk"
is a **discontiguous** noun phrase



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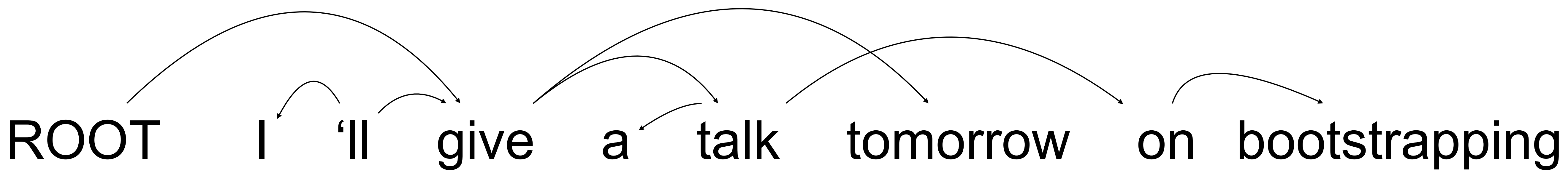

occasional non-projectivity in English

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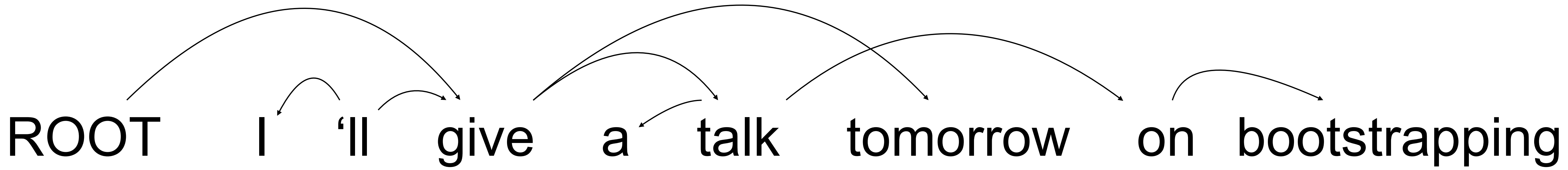

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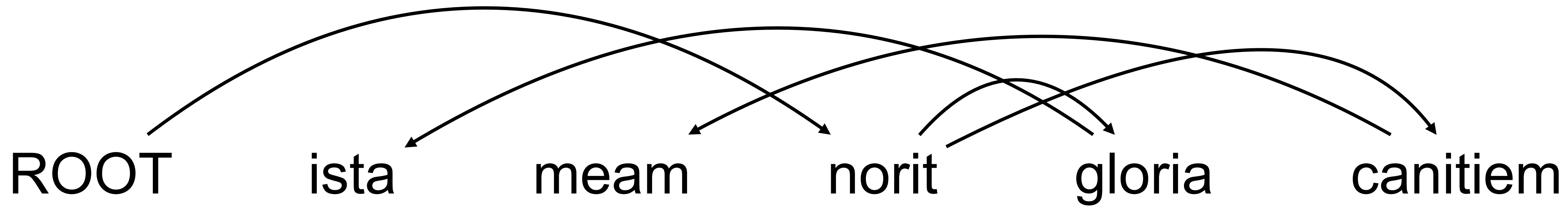

frequent non-projectivity in Latin, etc.

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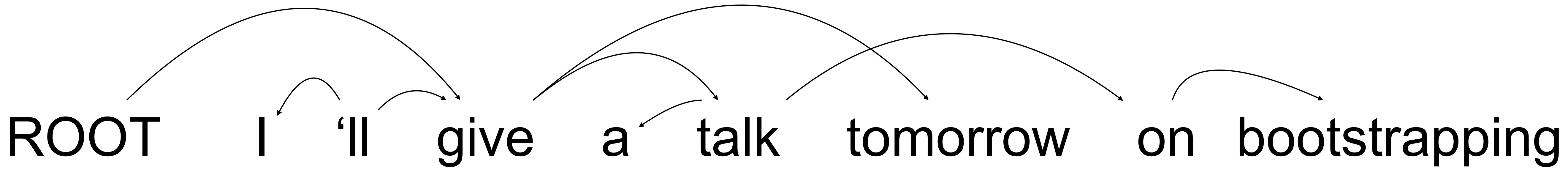

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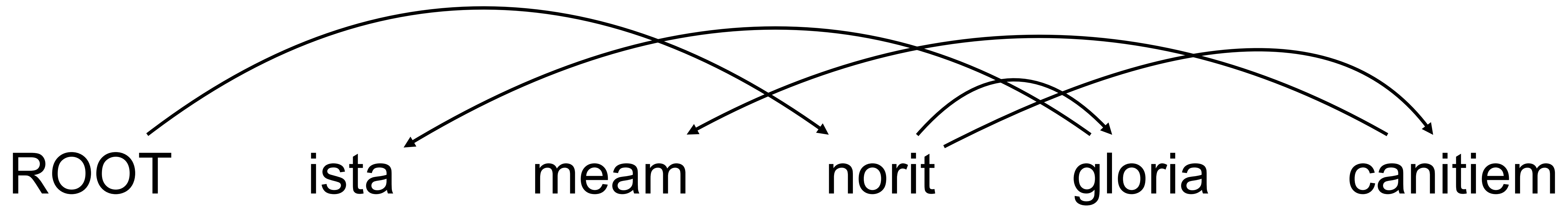

That glory may-know my going-gray
(i.e., it shall last till I go gray)

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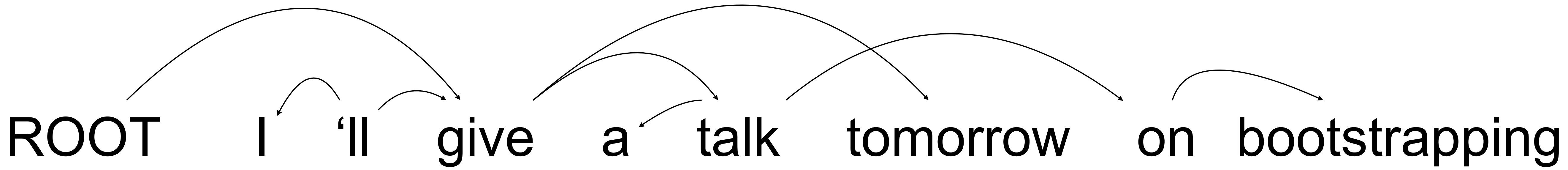

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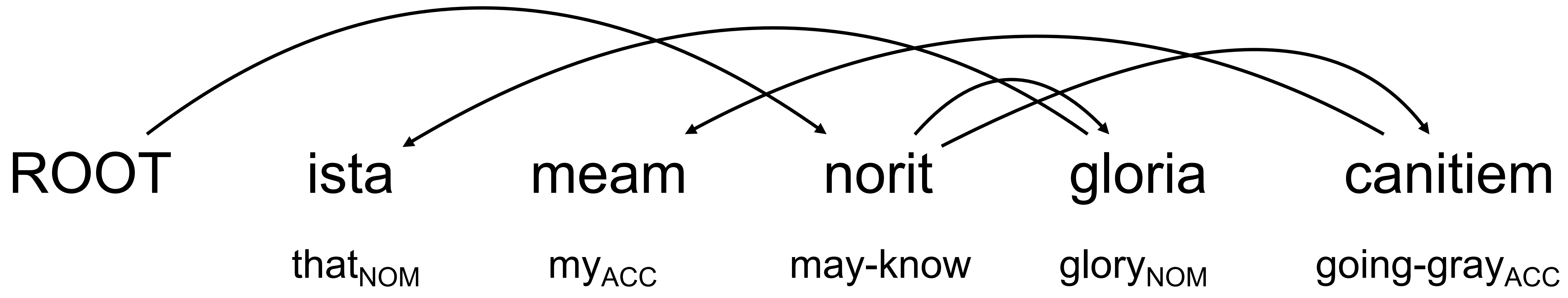

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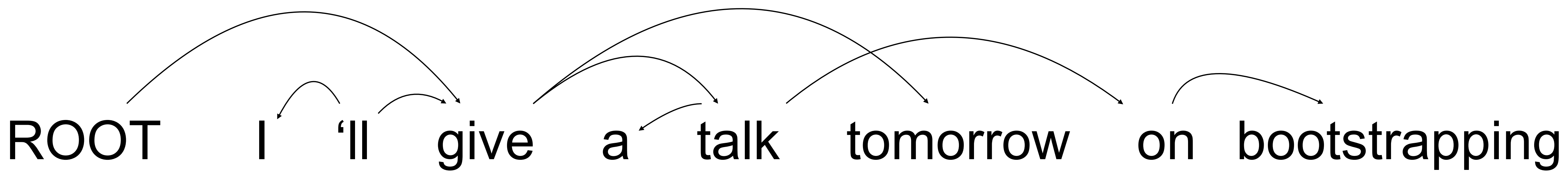

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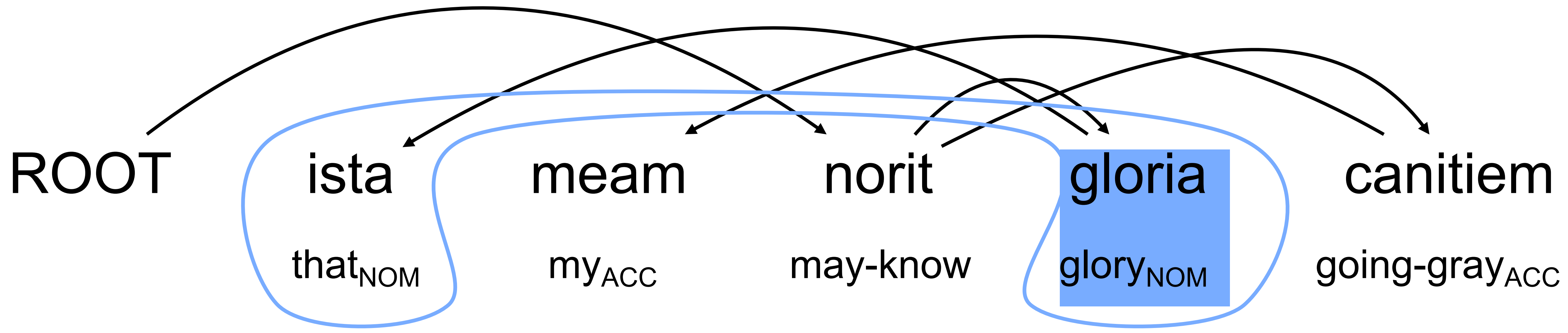

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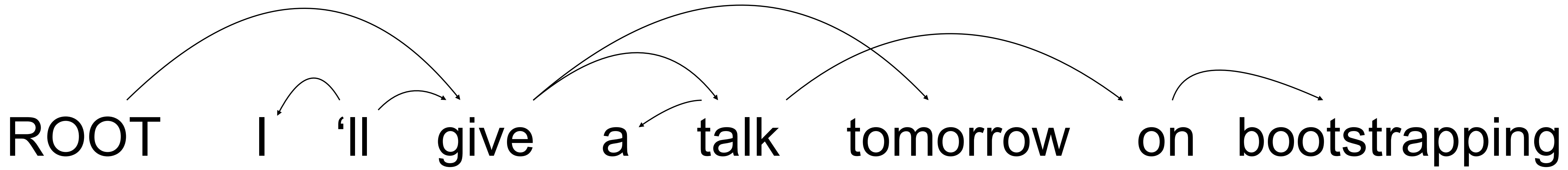

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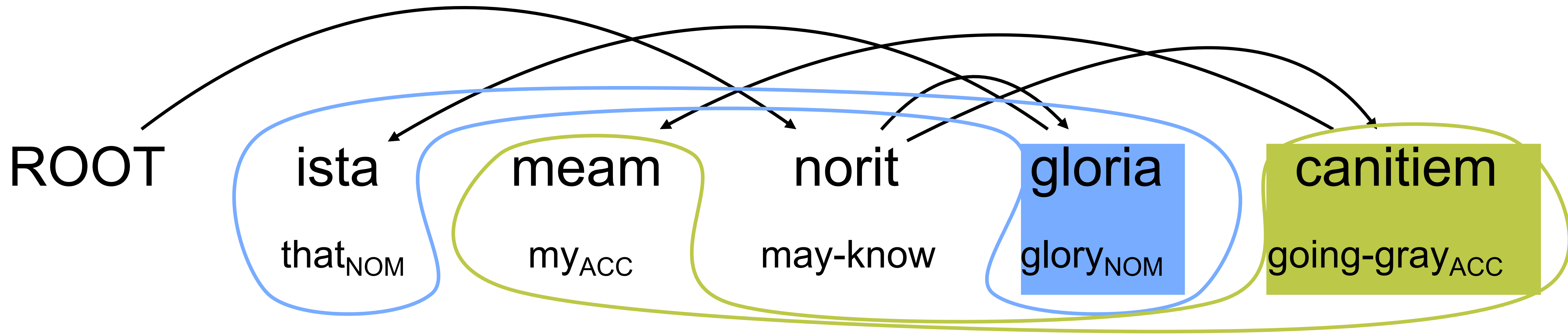

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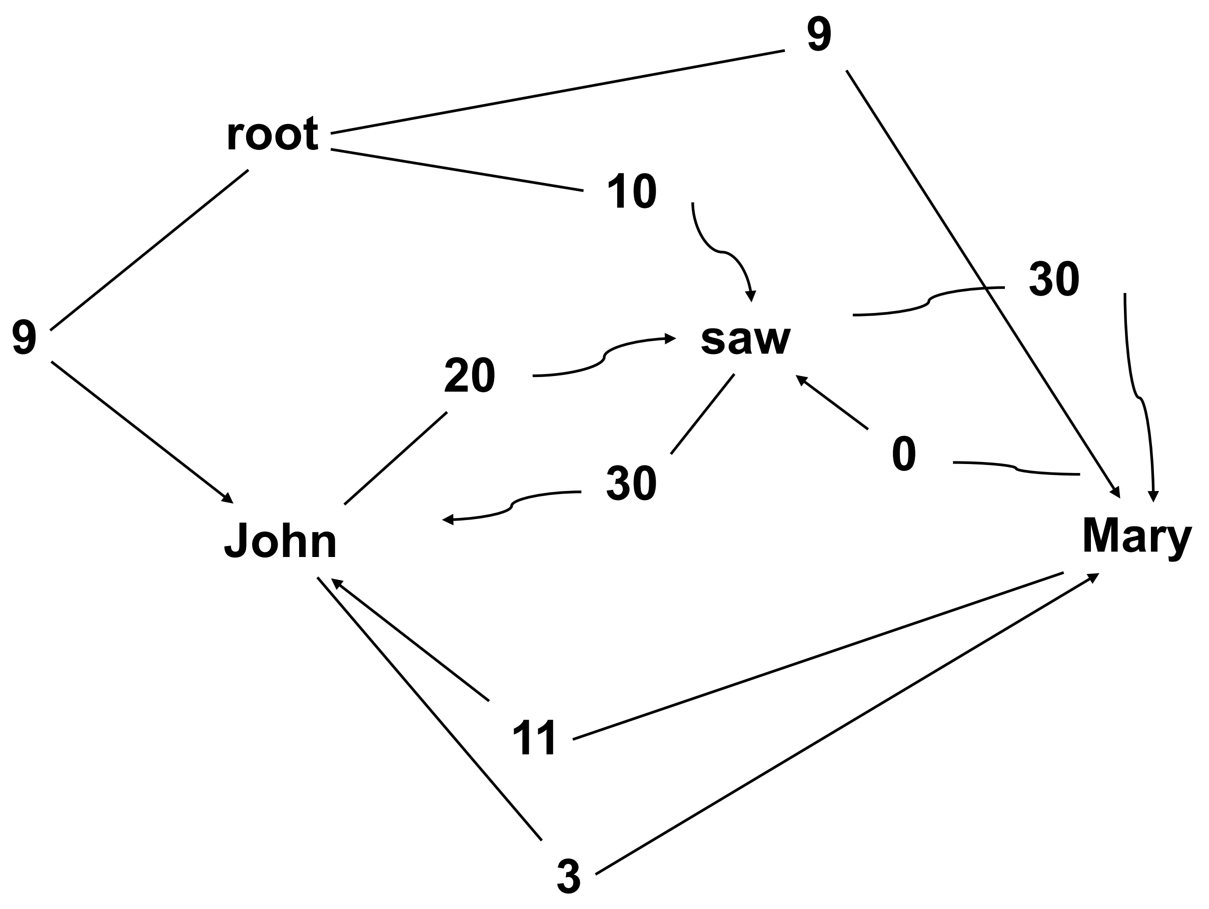
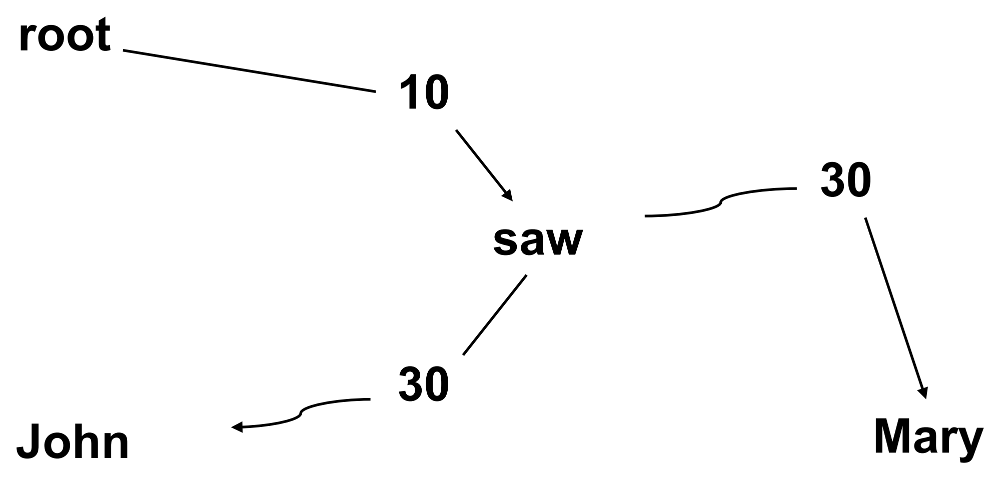
Finding highest-scoring non-projective tree

- Consider the sentence “John saw Mary” (left).
- The Chu-Liu-Edmonds algorithm finds the maximum-weight spanning tree (right) – may be non-projective.
- Can be found in time O(n^2).





Every node selects best parent
If cycles, contract them and repeat

slide thanks to Dragomir Radev

Summing over all non-projective trees
~~Finding highest-scoring non-projective tree~~

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- How about total weight Z of all trees?
- How about outside probabilities or gradients?
- Can be found in time $O(n^3)$ by matrix determinants and inverses (Smith & Smith, 2007).

slide thanks to Dragomir Radev

Graph Theory to the Rescue!

Tutte's **Matrix-Tree Theorem** (1948)

The **determinant** of the Kirchoff (aka Laplacian) adjacency matrix of directed graph *G* without row and column *r* is equal to the **sum of scores of all directed spanning trees** of *G* rooted at node *r*.

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Building the Kirchoff (Laplacian) Matrix

$$\begin{bmatrix} 0 & -s(1,0) & -s(2,0) & \mathrm{L} & -s(n,0) \\ 0 & 0 & -s(2,1) & \mathrm{L} & -\ s(n,1) \\ 0 & -s(1,2) & 0 & \mathrm{L} & -s(n,2) \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{O} & \mathrm{M} \\ 0 & -s(1,n) & -s(2,n) & \mathrm{L} & 0 \end{bmatrix}$$

- Negate edge scores
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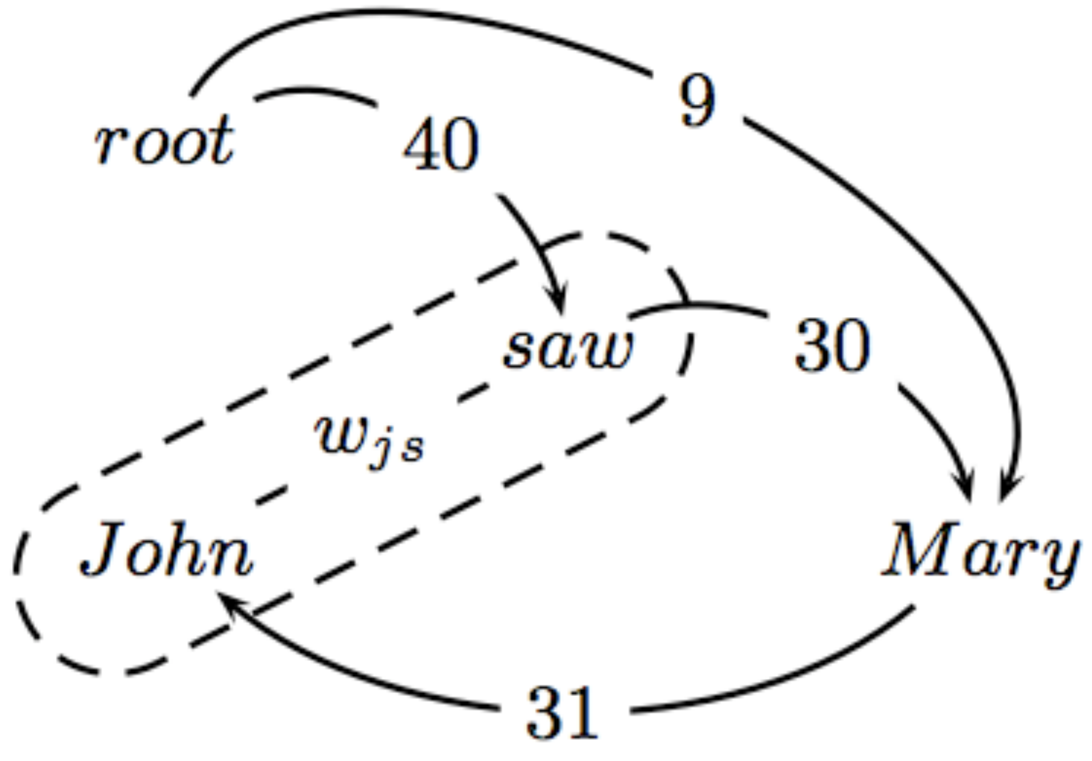
N.B.: This allows multiple children of root, but see Koo et al. 2007.

Why Should This Work?

Clear for 1x1 matrix; use induction

Chu-Liu-Edmonds analogy:
Every node selects best parent
If cycles, contract and recur



$K' \equiv K$ with contracted edge 1,2

$K'' \equiv K(\{1,2\} \mid \{1,2\})$

$|K| = s(1,2)|K'| + |K''|$

Undirected case; special root cases for directed

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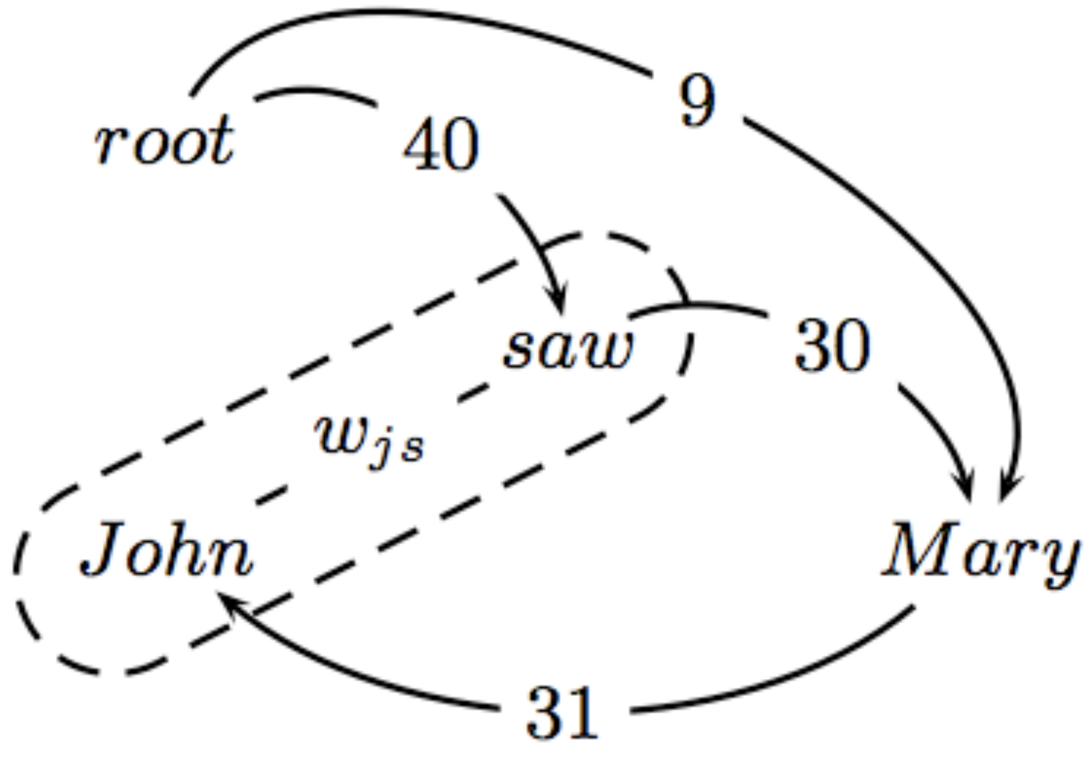

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