Log-Linear Models
a.k.a. Logistic Regression, Maximum Entropy Models

Introduction to Natural Language Processing
Computer Science 585—Fall 2009
University of Massachusetts Amherst

David Smith
(some slides from Jason Eisner and Dan Klein)
Probability is Useful
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  - Bayesian smoothing: \( \max p(\theta \mid data) = \max p(\theta, data) = p(\theta)p(data \mid \theta) \)
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  - Syntactic features, morph. Could be stochasticized?
summary of other half of the course (linguistics)

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- But probabilities have wormed their way into most things
- $p(...)$ has to capture our intuitions about the ling. data
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- Old AI hacking technique:
  - Possible parses (or whatever) have scores.
  - Pick the one with the best score.
  - How do you define the score?
    - Completely ad hoc!
    - Throw anything you want into the stew
    - Add a bonus for this, a penalty for that, etc.
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really so alternative?

Exposé at 9

Probabilistic Revolution
Not Really a Revolution, Critics Say

Log-probabilities no more than scores in disguise

“We’re just adding stuff up like the old corrupt regime did,” admits spokesperson
Nuthin’ but adding weights
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- n-grams: ... + log p(w7 | w5,w6) + log(w8 | w6, w7) + ...
Nuthin’ but adding weights

- **n-grams:** ... + \( \log p(w_7 \mid w_5, w_6) + \log(w_8 \mid w_6, w_7) + ... \)

- **PCFG:** \( \log p(NP \ VP \mid S) + \log p(Papa \mid NP) + \log p(VP \ PP \mid VP) \) ...
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- **HMM tagging**: ... + \( \log p(t_7 | t_5, t_6) + \log p(w_7 | t_7) + ... \)
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- **n-grams:** $\ldots + \log p(w_7 \mid w_5, w_6) + \log p(w_8 \mid w_6, w_7) + \ldots$
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- **Noisy channel:** $[\log p(\text{source})] + [\log p(\text{data} \mid \text{source})]$
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- **Cascade of FSTs:**
  $\left[ \log p(A) \right] + \left[ \log p(B | A) \right] + \left[ \log p(C | B) \right] + ...$
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*Note:* Today we’ll use +logprob not –logprob: i.e., bigger weights are better.
Nuthin’ but adding weights

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- **PCFG:** \( \log p(NP \ VP \mid S) + \log p(Papa \mid NP) + \log p(VP \ PP \mid VP) \ldots \)
  - Can regard any linguistic object as a collection of features (here, tree = a collection of context-free rules)
  - Weight of the object = total weight of features
  - Our weights have always been conditional log-probs (\( \leq 0 \))
    - but that is going to change in a few minutes!

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“.2, .4, .6, .8! We’re not gonna take your bait!”
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1. Can estimate our parameters automatically
   - e.g., log p(t7 | t5, t6) (trigram tag probability)
   - from supervised or unsupervised data
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3. Our results can be meaningfully combined $\Rightarrow$ modularity!
   - Multiply indep. conditional probs – normalized, unlike scores
   - $p(\text{English text}) \times p(\text{English phonemes} \mid \text{English text}) \times p(\text{Jap. phonemes} \mid \text{English phonemes}) \times p(\text{Jap. text} \mid \text{Jap. phonemes})$
   - $p(\text{semantics}) \times p(\text{syntax} \mid \text{semantics}) \times p(\text{morphology} \mid \text{syntax}) \times p(\text{phonology} \mid \text{morphology}) \times p(\text{sounds} \mid \text{phonology})$
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  - Buy this supercalifragilistic Ginsu knife set for only $39 today ...
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  - Contains Buy
  - Contains supercalifragilistic
  - Contains a dollar amount under $100
  - Contains an imperative sentence
  - Reading level = 8th grade
  - Mentions money (use word classes and/or regexp to detect this)
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- Naïve Bayes: pick C maximizing $p(C) \times p(\text{feat 1} \mid C) \times \ldots$
- What assumption does Naïve Bayes make? True here?
Probabilists Regret Being Bound by Principle

- Ad-hoc approach does have one advantage
- Consider e.g. Naïve Bayes for text categorization:
  - Buy this supercalifragilistic Ginsu knife set for only $39 today ...
- Some useful features:
  - Contains a dollar amount under $100
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- Naïve Bayes: pick $C$ maximizing $p(C) \times p(\text{feat } 1 \mid C) \times \ldots$
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\[
\begin{array}{ll}
\text{spam} & 0.5 \quad 0.02 \\
\text{ling} & 0.9 \quad 0.1
\end{array}
\]

50% of spam has this – 25x more likely than in ling
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Naïve Bayes claims $0.5 \times 0.9 = 45\%$ of spam has both features – 25x more likely than in ling.

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90% of spam has this – 9x more likely than in ling
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<table>
<thead>
<tr>
<th>spam</th>
<th>ling</th>
</tr>
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<tbody>
<tr>
<td>.5</td>
<td>.02</td>
</tr>
<tr>
<td>.9</td>
<td>.1</td>
</tr>
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</table>

Naïve Bayes claims $.5 \times .9 = 45\%$ of spam has both features – 25x more likely than in ling.

50% of spam has this – 25x more likely than in ling

90% of spam has this – 9x more likely than in ling

but here are the emails with both features – only 25x!
But ad-hoc approach does have one advantage

- Can adjust scores to compensate for feature overlap …
- Some useful features of this message:
  - Contains a dollar amount under $100
  - Mentions money
- Naïve Bayes: pick C maximizing $p(C) \times p(\text{feat 1} | C) \times ...$
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<table>
<thead>
<tr>
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Revolution Corrupted by Bourgeois Values
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- Naïve Bayes needs overlapping but independent features
Revolution Corrupted by Bourgeois Values

- Naïve Bayes needs overlapping but independent features
- But not clear how to restructure these features like that:
  - Contains Buy
  - Contains supercalifragilistic
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  - Contains an imperative sentence
  - Reading level = 7th grade
  - Mentions money (use word classes and/or regexp to detect this)
  - ...

12
Revolution Corrupted by Bourgeois Values

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- Well, maybe we can add up scores and pretend like we got a log probability:
Naïve Bayes needs overlapping but independent features

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+4
+0.2
+1
+2
-3
+5
...

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Revolution Corrupted by Bourgeois Values

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  - +4: Contains Buy
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  - ... (total: 5.77)
- Boy, we’d like to be able to throw all that useful stuff in without worrying about feature overlap/independence.
- Well, maybe we can add up scores and pretend like we got a log probability: $\log p(\text{feats} | \text{spam}) = 5.77$
- Oops, then $p(\text{feats} | \text{spam}) = \exp 5.77 = 320.5$
Renormalize by $1/Z$ to get a

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scale down so
everything $< 1$
and sums to 1!
Renormalize by $1/Z$ to get a

- $p(\text{feats} | \text{spam}) = \exp 5.77 = 320.5$

- $p(m | \text{spam}) = \left(\frac{1}{Z(\lambda)}\right) \exp \Sigma_i \lambda_i f_i(m)$ where
  
  - $m$ is the email message
  - $\lambda_i$ is weight of feature $i$
  - $f_i(m) \in \{0,1\}$ according to whether $m$ has feature $i$

  More generally, allow $f_i(m) = \text{count or strength of feature}$.

  $1/Z(\lambda)$ is a normalizing factor making $\Sigma_m p(m | \text{spam}) = 1$

  (summed over all possible messages $m$! hard to find!)
Renormalize by $1/Z$ to get a

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- $\text{p(}\text{feats} \mid \text{spam}) = \exp 5.77 = 320.5$

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- Why is it called “log-linear”?
Why Bother?
Why Bother?

- Gives us probs, not just scores.
  - Can use ‘em to bet, or combine w/ other probs.
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- Gives us probs, not just scores.
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- We can now learn weights from data!

  - Choose weights $\lambda_i$ that maximize logprob of labeled training data:
    \[ \log \prod_j p(c_j) p(m_j | c_j) \]
    - where $c_j \in \{\text{ling, spam}\}$ is classification of message $m_j$
    - and $p(m_j | c_j)$ is log-linear model from previous slide
  - **Convex** function – easy to maximize! (why?)
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  - Convex function – easy to maximize! (why?)

- But: $p(m_j | c_j)$ for a given $\lambda$ requires $Z(\lambda)$: hard!
Attempt to Cancel out Z
Attempt to Cancel out Z

- Set weights to maximize $\prod_j p(c_j) p(m_j \mid c_j)$
  - where $p(m \mid \text{spam}) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m)$
  - But normalizer $Z(\lambda)$ is awful sum over all possible emails
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- **So instead:** Maximize $\prod_j p(c_j \mid m_j)$
  - Doesn’t model the emails $m_j$, only their classifications $c_j$
  - Makes more sense anyway given our feature set
Attempt to Cancel out $Z$

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- $p(\text{spam} | m) = p(\text{spam})p(m | \text{spam}) / (p(\text{spam})p(m | \text{spam}) + p(\text{ling})p(m | \text{ling}))$
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  - $Z$ appears in both numerator and denominator
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- Z appears in both numerator and denominator
- Alas, doesn’t cancel out because Z differs for the spam and ling models
Attempt to Cancel out Z

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- $Z$ appears in both numerator and denominator
- Alas, doesn’t cancel out because $Z$ differs for the spam and ling models
- But we can fix this ...
So: Modify Setup a Bit
So: Modify Setup a Bit

- Instead of having separate models
  \[ p(m|\text{spam}) \times p(\text{spam}) \quad \text{vs.} \quad p(m|\text{ling}) \times p(\text{ling}) \]
So: Modify Setup a Bit

- Instead of having separate models
  \[ p(m|\text{spam}) \cdot p(\text{spam}) \quad \text{vs.} \quad p(m|\text{ling}) \cdot p(\text{ling}) \]
- Have just one joint model \( p(m,c) \)
  gives us both \( p(m,\text{spam}) \) and \( p(m,\text{ling}) \)
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  which gives us both \( p(m,\text{spam}) \) and \( p(m,\text{ling}) \)
- Equivalent to changing feature set to:
  - spam
  - spam and Contains \textit{Buy}
  - spam and Contains \textit{supercalifragilistic}
  - ...
  - ling
  - ling and Contains \textit{Buy}
  - ling and Contains \textit{supercalifragilistic}
So: Modify Setup a Bit

- Instead of having separate models $p(m|\text{spam}) \times p(\text{spam})$ vs. $p(m|\text{ling}) \times p(\text{ling})$
- Have just one joint model $p(m,c)$
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  - spam
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  - ...
  - ling
  - ling and Contains Buy
  - ling and Contains supercalifragilistic
- No real change, but 2 categories now share single feature set and single value of $Z(\lambda)$
So: Modify Setup a Bit

- Instead of having separate models
  \[\text{p}(m|\text{spam}) \times \text{p}(\text{spam}) \text{ vs. } \text{p}(m|\text{ling}) \times \text{p}(\text{ling})\]
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  gives us both \(\text{p}(m,\text{spam})\) and \(\text{p}(m,\text{ling})\)
- Equivalent to changing feature set to:
  - spam
  - spam and Contains Buy \(\leftarrow\) old spam model’s weight for “contains Buy”
  - spam and Contains supercalifragilistic
  - ...
  - ling
  - ling and Contains Buy \(\leftarrow\) old ling model’s weight for “contains Buy”
  - ling and Contains supercalifragilistic
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  - spam and Contains supercalifragilistic
  - ...
  - ling
    \( \leftarrow \text{weight of this feature is } \log p(\text{ling}) + \text{a constant} \)
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    \( \leftarrow \text{old ling model’s weight for “contains Buy”} \)
  - ling and Contains supercalifragilistic
- No **real** change, but 2 categories now share single feature set and single value of \( Z(\lambda) \)
Now we can cancel out Z
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Now \( p(m,c) = \frac{1}{Z(\lambda)} \exp \sum_i \lambda_i f_i(m,c) \) where \( c \in \{\text{ling, spam}\} \)
Now we can cancel out Z

Now \( p(m,c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m,c) \) where \( c \in \{\text{ling, spam}\} \)

- **Old**: choose weights \( \lambda_i \) that maximize prob of labeled training data = \( \prod_j p(m_j, c_j) \)
Now we can cancel out $Z$

Now $p(m,c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m,c)$ where $c \in \{\text{ling, spam}\}$

- **Old**: choose weights $\lambda_i$ that maximize prob of labeled training data $= \prod_j p(m_j, c_j)$
- **New**: choose weights $\lambda_i$ that maximize prob of labels given messages $= \prod_j p(c_j | m_j)$
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- **New**: choose weights $\lambda_i$ that maximize prob of labels given messages = $\prod_j p(c_j \mid m_j)$

- Now $Z$ cancels out of conditional probability!
Now we can cancel out $Z$

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  - $p(\text{spam} | m) = p(m,\text{spam}) / (p(m,\text{spam}) + p(m,\text{ling}))$
Now we can cancel out Z

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- Now Z cancels out of conditional probability!
  - \( p(\text{spam} | m) = p(m, \text{spam}) / (p(m, \text{spam}) + p(m, \text{ling})) \)
    \[= \exp \sum_i \lambda_i f_i(m, \text{spam}) / (\exp \sum_i \lambda_i f_i(m, \text{spam}) + \exp \sum_i \lambda_i f_i(m, \text{ling}))\]
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  - $p(\text{spam} \mid m) = \frac{p(m, \text{spam})}{p(m, \text{spam}) + p(m, \text{ling})}$
    - $= \frac{\exp \sum_i \lambda_i f_i(m, \text{spam})}{\exp \sum_i \lambda_i f_i(m, \text{spam}) + \exp \sum_i \lambda_i f_i(m, \text{ling})}$
  - Easy to compute now ...
Now we can cancel out $Z$

Now $p(m, c) = (1/Z(\lambda)) \exp \sum_i \lambda_i f_i(m, c)$ where $c \in \{\text{ling, spam}\}$

- **Old**: choose weights $\lambda_i$ that maximize prob of labeled training data $= \prod_j p(m_j, c_j)$

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- Now $Z$ cancels out of conditional probability!
  - $p(\text{spam} | m) = p(m, \text{spam}) / (p(m, \text{spam}) + p(m, \text{ling}))$
    
    $= \exp \sum_i \lambda_i f_i(m, \text{spam}) / (\exp \sum_i \lambda_i f_i(m, \text{spam}) + \exp \sum_i \lambda_i f_i(m, \text{ling}))$

  - Easy to compute now ...

  - $\prod_j p(c_j | m_j)$ is still convex, so easy to maximize too
Generative vs. Conditional

- What is the most likely label for a given input?
- How likely is a given label for a given input?
- What is the most likely input value?
- How likely is a given input value?
- How likely is a given input value with a given label?
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- **Question:** Now what is your guess for $p(C \mid m)$, if $m$ contains `Buy`?
- **OUCH!**
## Maximum Entropy

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- Column A sums to 0.55  ("55% of all messages are in class A")
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- Column A sums to 0.55
- **Row Buy sums to 0.1** (“10% of all messages contain Buy”)
# Maximum Entropy

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- Column A sums to 0.55
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- (Buy, A) and (Buy, C) cells sum to 0.08 (“80% of the 10%”)
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# Maximum Entropy

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\[
\text{Entropy} = -0.051 \log 0.051 - 0.0025 \log 0.0025 - 0.029 \log 0.029 - \ldots
\]
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Largest if probabilities are evenly distributed.
### Maximum Entropy

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- Now \(p(\text{Buy}, C) = 0.029\) and \(p(C | \text{Buy}) = 0.29\)
- We got a compromise: \(p(C | \text{Buy}) < p(A | \text{Buy}) < 0.55\)
Generalizing to More Features

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What we just did
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**Amazing Theorem:** This distribution has the form

$$p(m,c) = \frac{1}{Z(\lambda)} \exp \sum_i \lambda_i f_i(m,c)$$

- So it is log-linear. In fact it is the same log-linear distribution that maximizes $\prod_j p(m_j, c_j)$ as before!
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- Gives another motivation for our log-linear approach.
Log-linear form derivation

• Say we are given some constraints in the form of feature expectations:

\[ \sum_x p(x) f_i(x) = \alpha_i \]

• In general, there may be many distributions \( p(x) \) that satisfy the constraints. Which one to pick?
• The one with maximum entropy (making fewest possible additional assumptions---Occum’s Razor)
• This yields an optimization problem

\[
\max H(p(x)) = - \sum_x p(x) \log p(x) \\
\text{Subject to } \sum_x p(x) f_i(x) = \alpha_i, \forall i \text{ and } \sum_x p(x) = 1
\]
Log-linear form derivation

- To solve the maxent problem, we use Lagrange multipliers:

\[
L = -\sum_x p(x) \log p(x) - \sum_i \theta_i \left( \sum_x p(x)f_i(x) - \alpha_i \right) - \mu \left( \sum_x p(x) - 1 \right)
\]

\[
\frac{\partial L}{\partial p(x)} = 1 + \log p(x) - \sum_i \theta_i f_i(x) - \mu
\]

\[
p^*(x) = e^{\mu - 1} \exp \left\{ \sum_i \theta_i f_i(x) \right\}
\]

\[
Z(\theta) = e^{1-\mu} = \sum_x \exp \left\{ \sum_i \theta_i f_i(x) \right\}
\]

\[
p(x|\theta) = \frac{1}{Z(\theta)} \exp \left\{ \sum_i \theta_i f_i(x) \right\}
\]

- So feature constraints + maxent implies exponential family.

- Problem is convex, so solution is unique.
MaxEnt = Max Likelihood

Define two submanifolds on the probability simplex $p(x)$.

The first is $\mathcal{E}$, the set of all exponential family distributions based on a particular set of features $f_i(x)$.

The second is $\mathcal{M}$, the set of all distributions that satisfy the feature expectation constraints.

They intersect at a single distribution $p_M$, the maxent, maximum likelihood
Exponential Model Likelihood

- Maximum Likelihood (Conditional) Models:
  - Given a model form, choose values of parameters to maximize the (conditional) likelihood of the data.

- Exponential model form, for a data set (C,D):

\[
\log P(C \mid D, \lambda) = \sum_{(c,d) \in (C,D)} \log P(c \mid d, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}
\]
Building a Maxent Model

- Define features (indicator functions) over data points.
  - Features represent sets of data points which are distinctive enough to deserve model parameters.
  - Usually features are added incrementally to "target" errors.

- For any given feature weights, we want to be able to calculate:
  - Data (conditional) likelihood
  - Derivative of the likelihood wrt each feature weight
    - Use expectations of each feature according to the model

- Find the optimum feature weights (next part).
The Likelihood Value

- The (log) conditional likelihood is a function of the iid data \((C,D)\) and the parameters \(\lambda\):

\[
\log P(C \mid D, \lambda) = \log \prod_{(c,d) \in (C,D)} P(c \mid d, \lambda) = \sum_{(c,d) \in (C,D)} \log P(c \mid d, \lambda)
\]

- If there aren’t many values of \(c\), it’s easy to calculate:

\[
\log P(C \mid D, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c,d)}
\]

- We can separate this into two components:

\[
\log P(C \mid D, \lambda) = \sum_{(c,d) \in (C,D)} \log \exp \sum_i \lambda_i f_i(c,d) - \sum_{(c,d) \in (C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c',d)
\]

\[
\log P(C \mid D, \lambda) = N(\lambda) - M(\lambda)
\]

- The derivative is the difference between the derivatives of each component
\[
\frac{\partial N(\lambda)}{\partial \lambda_i} = \frac{\partial}{(c,d) \in (C,D)} \log \exp \sum_i \lambda_{ci} f_i(c,d) \frac{\partial}{\partial \lambda_i} \sum \lambda_i f_i(c,d) = \frac{\partial}{(c,d) \in (C,D)} \sum_i \lambda_i f_i(c,d)
\]

= \sum_{(c,d) \in (C,D)} \frac{\partial}{\partial \lambda_i} \sum_i \lambda_i f_i(c,d)

= \sum_{(c,d) \in (C,D)} f_i(c,d)

Derivative of the numerator is: the empirical count(\(f_i, c\))
The Derivative II: Denominator

$$\frac{\partial M(\lambda)}{\partial \lambda_i} = \frac{\partial}{\partial \lambda_i} \sum_{(c,d) \in (C,D)} \log \sum_{c'} \exp \sum_i \lambda_i f_i(c', d)$$

$$= \sum_{(c,d) \in (C,D)} \frac{1}{\sum \exp \sum_i \lambda_i f_i(c''', d)} \frac{\partial \sum \exp \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i}$$

$$= \sum_{(c,d) \in (C,D)} \frac{1}{\sum \exp \sum_i \lambda_i f_i(c''', d)} \sum_{c'} \exp \sum_i \lambda_i f_i(c', d) \frac{\partial \sum \exp \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i}$$

$$= \sum_{(c,d) \in (C,D)} \frac{\exp \sum_i \lambda_i f_i(c', d)}{\sum \exp \sum_i \lambda_i f_i(c''', d)} \sum_{c'} \frac{\partial \sum \exp \sum_i \lambda_i f_i(c', d)}{\partial \lambda_i}$$

$$= \sum_{(c,d) \in (C,D)} \sum_{c'} P(c' \mid d, \lambda) f_i(c', d) = \text{predicted count}(f_i, \lambda)$$
The Derivative III

$$\frac{\partial \log P(C | D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda)$$

- The optimum parameters are the ones for which each feature’s predicted expectation equals its empirical expectation. The optimum distribution is:
  - Always unique (but parameters may not be unique)
  - Always exists (if features counts are from actual data).

- Features can have high model expectations (predicted counts) either because they have large weights or because they occur with other features which have large weights.
Summary

- We have a function to optimize:
  \[ \log P(C | D, \lambda) = \sum_{(c,d) \in (C,D)} \log \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c,d)} \]

- We know the function’s derivatives:
  \[ \frac{\partial \log P(C | D, \lambda)}{\partial \lambda_i} = \text{actual count}(f_i, C) - \text{predicted count}(f_i, \lambda) \]

- Perfect situation for general optimization (Part II)

  By gradient ascent or conjugate gradient.
Comparison to Naïve-Bayes

- Naïve-Bayes is another tool for classification:
  - We have a bunch of random variables (data features) which we would like to use to predict another variable (the class):

- The Naïve-Bayes likelihood over classes is:

\[
P(c \mid d, \lambda) = \frac{P(c) \prod_i P(\phi_i \mid c)}{\sum_{c'} P(c') \prod_i P(\phi_i \mid c')} \quad \text{exp} \left[ \log P(c) + \sum_i \log P(\phi_i \mid c) \right]
\]

Naïve-Bayes is just an exponential model.
## Comparison to Naïve-Bayes

The primary differences between Naïve-Bayes and maxent models are:

<table>
<thead>
<tr>
<th>Naïve-Bayes</th>
<th>Maxent</th>
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<tbody>
<tr>
<td>Trained to maximize joint likelihood of data and classes.</td>
<td>Trained to maximize the conditional likelihood of classes.</td>
</tr>
<tr>
<td>Features assumed to supply independent evidence.</td>
<td>Features weights take feature dependence into account.</td>
</tr>
<tr>
<td>Feature weights can be set independently.</td>
<td>Feature weights must be mutually estimated.</td>
</tr>
<tr>
<td>Features must be of the conjunctive $\Phi(d) \land c = c_i$ form.</td>
<td>Features need not be of the conjunctive form (but usually are).</td>
</tr>
</tbody>
</table>
Overfitting

- If we have too many features, we can choose weights to model the training data perfectly.

- If we have a feature that only appears in spam training, not ling training, it will get weight $\infty$ to maximize $p(\text{spam} \mid \text{feature})$ at 1.

- These behaviors overfit the training data.
- Will probably do poorly on test data.
Solutions to Overfitting
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1. Throw out rare features.
   - Require every feature to occur > 4 times, and > 0 times with ling, and > 0 times with spam.
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3. Smooth the observed feature counts.

4. Smooth the weights by using a prior.
   - max p(λ|data) = max p(λ, data) = p(λ)p(data|λ)
   - decree p(λ) to be high when most weights close to 0
Smoothing: Priors (MAP)

- What if we had a prior expectation that parameter values wouldn’t be very large?
- We could then balance evidence suggesting large parameters (or infinite) against our prior.
- The evidence would never totally defeat the prior, and parameters would be smoothed (and kept finite!).
- We can do this explicitly by changing the optimization objective to maximum posterior likelihood:

$$\log P(C, \lambda | D) = \log P(\lambda) + \log P(C | D, \lambda)$$

Posterior Prior Evidence
Smoothing: Priors

- Gaussian, or quadratic, priors:
  - Intuition: parameters shouldn’t be large.
  - Formalization: prior expectation that each parameter will be distributed according to a gaussian with mean $\mu$ and variance $\sigma^2$.

$$P(\lambda_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(\lambda_i - \mu_i)^2}{2\sigma_i^2}\right)$$

- Penalizes parameters for drifting too far from their mean prior value (usually $\mu = 0$).
- $2\sigma^2 = 1$ works surprisingly well.
Recipe for a Conditional MaxEnt Classifier

1. Gather \textit{constraints} from training data:
   \[
   \alpha_{iy} = \tilde{E}[f_{iy}] = \sum_{x_j, y_j \in D} f_{iy}(x_j, y_j)
   \]

2. Initialize all parameters to zero.

3. Classify training data with current parameters. Calculate \textit{expectations}.
   \[
   E_{\Theta}[f_{iy}] = \sum_{x_j \in D} \sum_{y'} p_{\Theta}(y' | x_j) f_{iy}(x_j, y')
   \]

4. Gradient is \[
   \tilde{E}[f_{iy}] - E_{\Theta}[f_{iy}]
   \]

5. Take a step in the direction of the gradient

6. Until convergence, return to step 3.