COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

Cameron Musco
University of Massachusetts Amherst. Spring 2020.
Lecture 6

LOGISTICS

- Problem Set 1 is due Friday at 8pm in Gradescope.

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- Thanks for your feedback in Piazza on lecture pace.
  - 6% a bit too slow.
  - 22% just right.
  - 63% a bit too fast.
  - 8% way too fast.

Last Class:

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- Distinct Elements via Hashing:
  - Distinct elements algorithm using min-of-hashes approach.
  - Analysis using averaging and the ‘median trick’.
  - Practical implementations (HyperLogLog) and applications.

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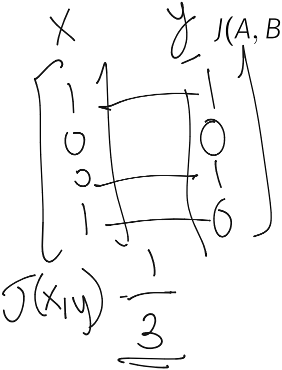
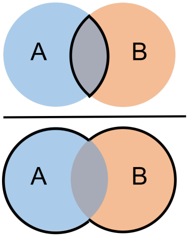
This Class:

- See how a min-of-hashes approach (MinHash) is used to estimate the Jaccard similarity.
- Application of MinHash to fast similarity search.
- Locality sensitive hashing.

Jaccard Index: A similarity measure between two sets.



$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\#\text{ shared elements}}{\#\text{ total elements}}.$$



Natural measure for similarity between bit strings – interpret an n bit string as a set, containing the elements corresponding the positions of its ones. $J(x,y) = \frac{\#\text{ shared ones}}{\text{total ones}}$.

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Want Fast Implementations For:

- **Near Neighbor Search:** Have a database of n sets/bit strings and given a set A, want to find if it has high Jaccard similarity to anything in the database. $\Omega(n)$ time with a linear scan.
- **All-pairs Similarity Search:** Have n different sets/bit strings and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Will speed up via randomized locality sensitive hashing.

SEARCH WITH JACCARD SIMILARITY

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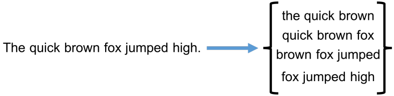
Why can't we just use e.g. binary search or regular hash tables to speed up these search problems?

Document Similarity:

- E.g., to detect plagiarism, copyright infringement, duplicate webpages, spam.
- Use Shingling + Jaccard similarity.



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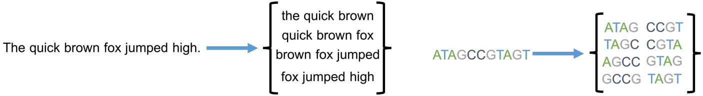

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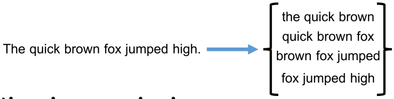
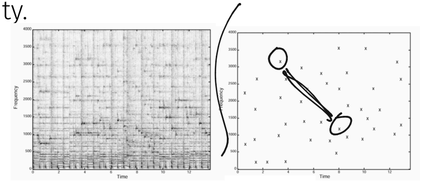


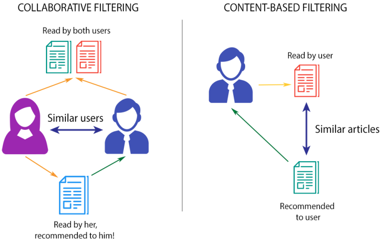
Audio Fingerprinting:

- E.g., in audio search (Shazam), Earthquake detection.
- Represent sound clip via a binary 'fingerprint' then compare with Jaccard similarity.

- not robust to shifting



Online recommendation systems are often based on **collaborative filtering**. Simplest approach: find similar users and make recommendations based on those users.



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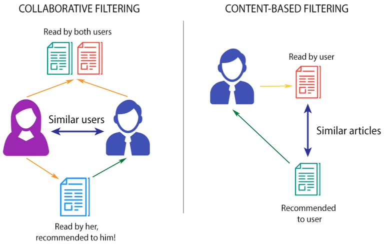

- Twitter: represent a user as the set of accounts they follow. Match similar users based on the Jaccard similarity of these sets. Recommend that you follow accounts followed by similar users.

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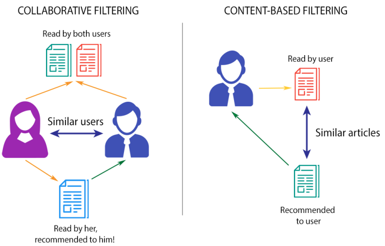

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- Netflix: look at sets of movies watched. Amazon: look at products purchased, etc.

Entity Resolution Problem: Want to combine records from multiple data sources that refer to the same entities.

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- Still want to match records that all refer to the same person using all pairs similarity search.

Customer Records

Name	Phone Number	Account #
...	...	...
...	...	...
...	...	...
Cam Musco	1-401-555-5555	...
...	...	...
⋮	⋮	⋮

resolve ⟷

Public Property Records

Name	Phone Number	Address
...	...	...
...	...	...
...	...	...
Cameron Musco	402-555-5555	...
...	...	...
⋮	⋮	⋮

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- E.g. data on individuals from voting registrations, property records, and social media accounts. Names and addresses may not exactly match, due to typos, nicknames, moves, etc.
- Still want to match records that all refer to the same person using all pairs similarity search.

See Section 3.8.2 of *Mining Massive Datasets* for a discussion of a real world example involving 1 million customers. Naively this would be $\binom{1000000}{2} \approx 500$ billion pairs of customers to check!

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APPLICATION: SPAM AND FRAUD DETECTION

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- **Fake Reviews**: Very common on websites like Amazon. Detection often looks for (near) duplicate reviews on similar products, which have been copied. 'Near duplicate' measured with shingles + Jaccard similarity.
- **Lateral phishing**: Phishing emails sent to addresses at a business coming from a legitimate email address at the same business that has been compromised.
  - One method of detection looks at the recipient list of an email and checks if it has small Jaccard similarity with any previous recipient lists. If not, the email is flagged as possible spam.

Why use Jaccard similarity over other metrics like: Hamming distinct (bit strings), correlation (sound waves, seismograms), edit distance (text, genome sequences, etc.)?

WHY JACCARD SIMILARITY?

[a ~~dog~~ walked]   [the cat walked]
cat

Why use Jaccard similarity over other metrics like: Hamming distinct (bit strings), correlation (sound waves, seismograms), edit distance (text, genome sequences, etc.)?

Two Reasons:

- Depending on the application, often is a very good measure.
- Even when not ideal, very efficient to compute and (as we will see today) implement near neighbor search and all-pairs similarity search with.

Goal: Speed up Jaccard similarity search (near neighbor and all-pairs similarity search).

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Strategy: Use random hashing to map each set to a very compressed representation. Jaccard similarity can be estimated from these representations.

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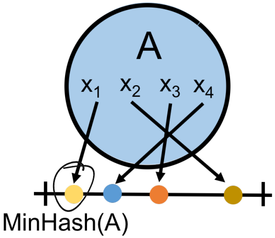

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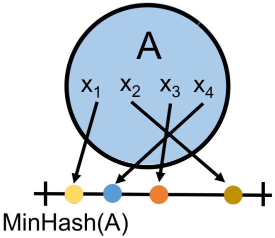

Identical to our distinct elements sketch!

For two sets A and B, what is $\Pr(MinHash(A) = MinHash(B))$?

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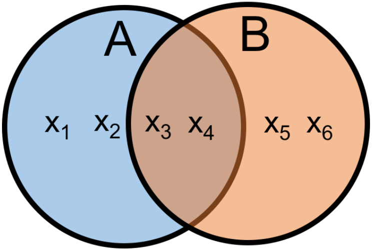
$\frac{2}{6} = \frac{1}{3}$



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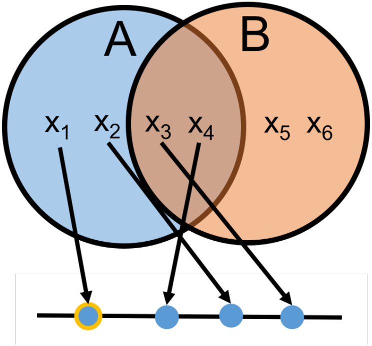

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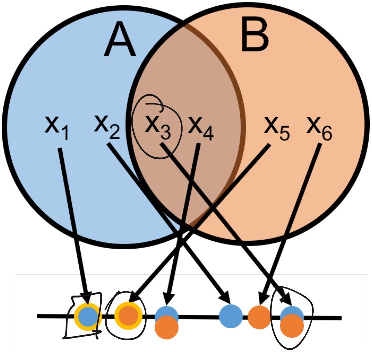

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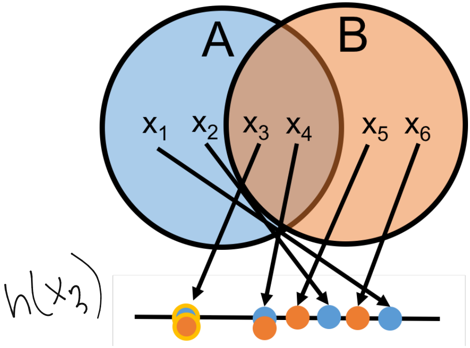

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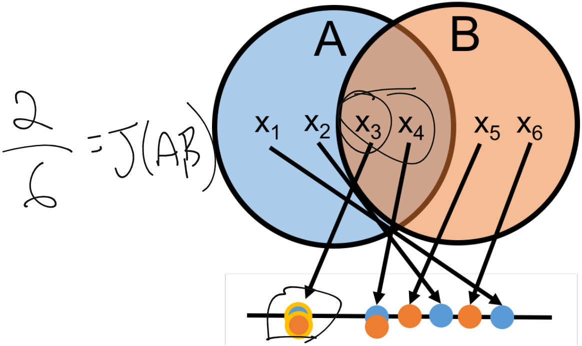

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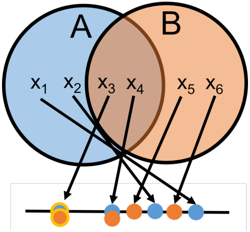

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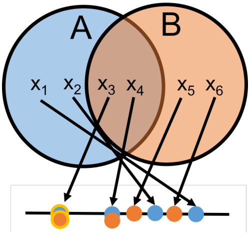

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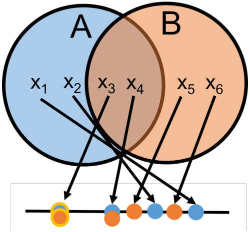

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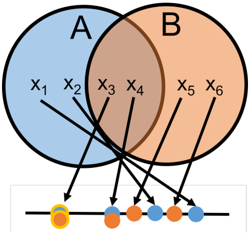

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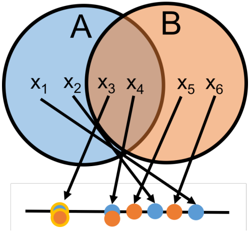

$$\Pr(MinHash(A) = MinHash(B)) = \frac{|A \cap B|}{\text{total \# items hashed}}$$
$$= \frac{|A \cap B|}{|A \cup B|} = J(A, B).$$

Upshot: MinHash reduces estimating the Jaccard similarity to checking equality of a *single number*.

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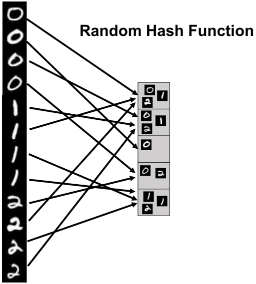

$Pr(h(A) = h(B) = i) = 1/n^2 \rightarrow Pr(h(A) = h(B)) = 1/n$

Upshot: MinHash reduces estimating the Jaccard similarity to checking equality of a *single number*.

$$\Pr(MinHash(A) = MinHash(B)) = J(A, B).$$

Pairwise independent.

is it LSH?

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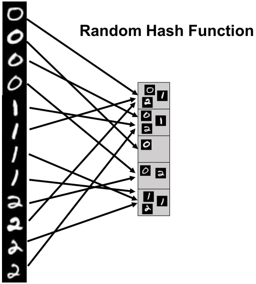
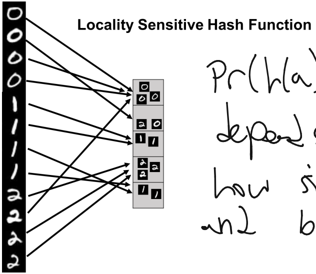
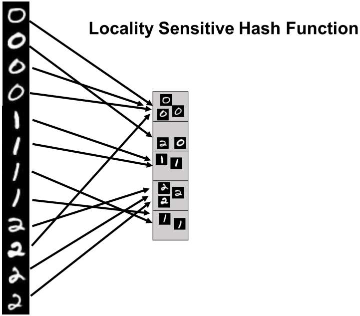




$Pr(h(a) = h(b))$ depends on how similar a and b are

How does locality sensitive hashing (LSH) help with similarity search?



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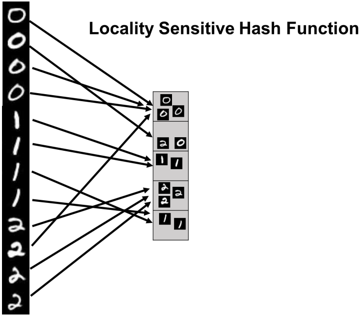

MinHash(x)

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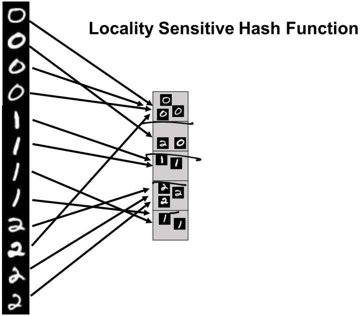

- **Near Neighbor Search:** Given item x, compute $h(x)$. Only search for similar items in the $h(x)$ bucket of the hash table.
- **All-pairs Similarity Search:** Scan through all buckets of the hash table and look for similar pairs within each bucket.

LSH FOR SIMILARITY SEARCH

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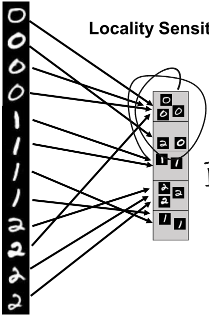

Is h LSH?

$Pr(h(A) = h(B))$

$\simeq Pr(MinHash(A) = MinHash(B))$

- **Near Neighbor Search:** Given item x, compute $\mathbf{h}(x)$. Only search for similar items in the $\mathbf{h}(x)$ bucket of the hash table.
- **All-pairs Similarity Search:** Scan through all buckets of the hash table and look for similar pairs within each bucket.
- We will use $\mathbf{h}(x) = \mathbf{g}(MinHash(x))$ where $\mathbf{g} : [0, 1] \rightarrow [n]$ is a random hash function. Why?

Goal: Given a document y, identify all documents x in a database with Jaccard similarity (of their shingle sets) $J(x, y) \geq 1/2$.

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Our Approach:

- Create a hash table of size m, choose a random hash function $\mathbf{g} : [0, 1] \to [m]$, and insert every item x into bucket $\mathbf{g}(MinHash(x))$. Search for items similar to y in bucket $\mathbf{g}(MinHash(y))$.

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LSH WITH MINHASH

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- For every document x in your database with $J(x, y) \geq 1/2$ what is the probability you will find x in bucket $\mathbf{g}(MinHash(y))$? 1/2

REDUCING FALSE NEGATIVES

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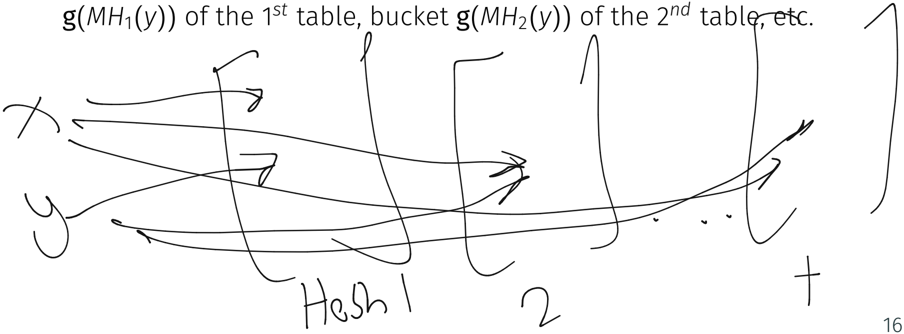

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$g_1, g_2, \ldots, g_t$

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Potential for a lot of false positives! Slows down search time.

BALANCING HIT RATE AND QUERY TIME

We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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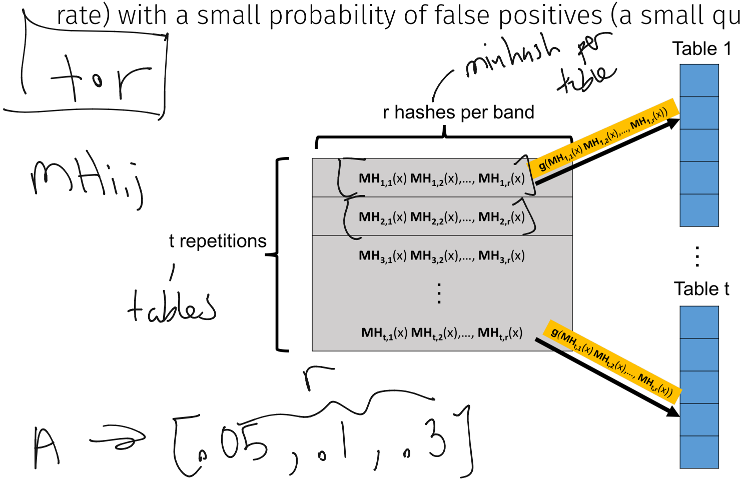

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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[.1, .3, .05]   [.1, .3, .05]

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$$\text{Hit Probability: } 1 - (1 - s^r)^t.$$

THE s-CURVE

Using t repetitions each with a signature of r MinHash values, the probability that x and y with Jaccard similarity $J(x, y) = s$ match in at least one repetition is: $1 - (1 - s^r)^t$.

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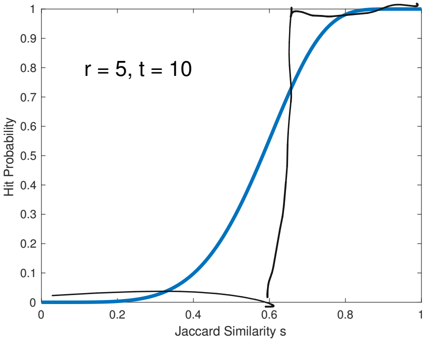

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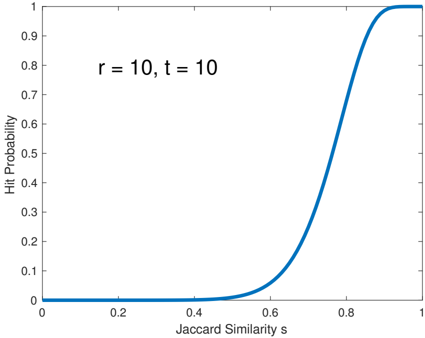

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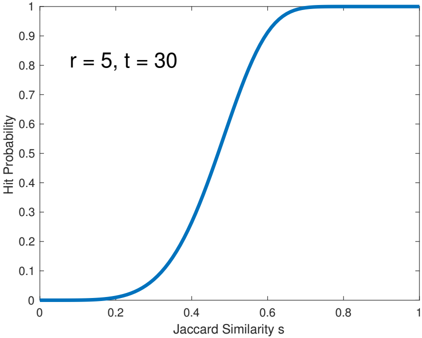

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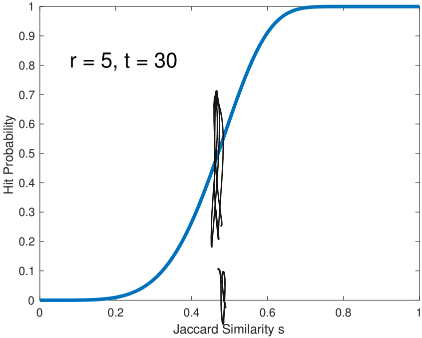

r and t are tuned depending on application. 'Threshold' when hit probability is 1/2 is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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HASHING FOR DUPLICATE DETECTION

	Bloom Filters	Hash Table	MinHash	Distinct Elements
Goal	Check if x is a duplicate of y in database.	Check if x is a duplicate of any y in database and return y.	Check if x is a duplicate of any y in database and return y.	Count # of items, excluding duplicates.
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All different variants of detecting duplicates/finding matches in large datasets. This is an important problem in many contexts!