COMPSCI 514: Algorithms for Data Science

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University of Massachusetts Amherst. Fall 2024.
Lecture 10

Logistics

- Problem Set 2 is due next Friday 10/11 at 11:59pm.
- The midterm is 7-9pm on Thursday 10/17. Midterm study guide will be posted shortly.
- We have a quiz this week, but not next two weeks (due to the problem set and midterm).
- Problem Set 1 Grades released later today

Summary

Last Class:

- Analysis of idealized MinHash-based distinct elements algorithm.
- Median trick for boosting success probability.
- Discussion of practical algorithms for distinct elements (LogLog/HyperLogLog).

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This Class:

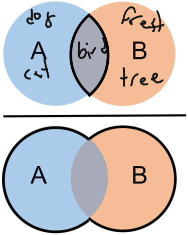
- The similarity search problem.
- Locality sensitive hashing for fast similarity search.
- MinHash as a locality sensitive hash function for Jaccard similarity
- Balancing false positives and negatives with LSH signatures and repeated hash tables.

Another Fundamental Problem

Jaccard Index: A similarity measure between two sets.

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\#\text{ shared elements}}{\#\text{ total elements}}. \quad \frac{1}{5}$$

$\in [0, 1]$



Natural measure for similarity between bit strings – interpret an n bit string as a set, containing the elements corresponding the positions of its ones. $J(x, y) = \frac{\#\text{ shared ones}}{\text{total ones}}$.

$[0, 1, 0, 1] \rightarrow \{2, 4\}$

Search with Jaccard Similarity

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Want Fast Implementations For:

- **Near Neighbor Search:** Have a database of n sets/bit strings and given a set A, want to find if it has high Jaccard similarity to anything in the database. $\Omega(n)$ time with a linear scan.
- **All-pairs Similarity Search:** Have n different sets/bit strings and want to find all pairs with high Jaccard similarity. $\Omega(n^2)$ time if we check all pairs explicitly.

Will speed up via randomized locality sensitive hashing.

Application: Document Similarity

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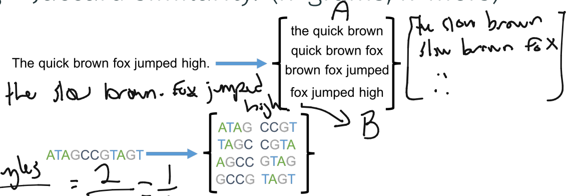
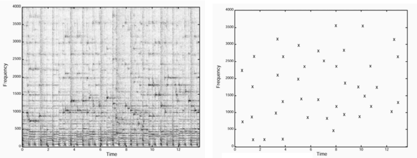


$J(A,B) = \frac{\#\text{ shared shingles}}{\text{total }\#} = \frac{2}{6} = \frac{1}{3}$

Application: Audio Search

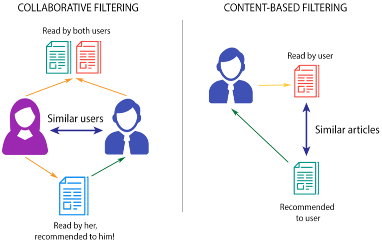
Audio Fingerprinting:

- E.g., in audio search (Shazam), Earthquake detection.
- Represent sound clip via a binary 'fingerprint' then compare with Jaccard similarity.



Application: Collaborative Filtering

Online recommendation systems are often based on **collaborative filtering**. Simplest approach: find similar users and make recommendations based on those users.



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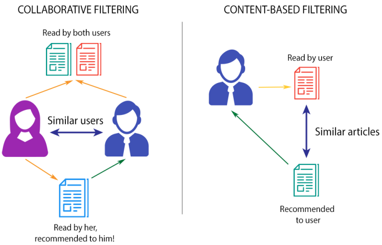

- Twitter: represent a user as the set of accounts they follow. Match users based on the Jaccard similarity of these sets. Recommend that you follow accounts followed by similar users.

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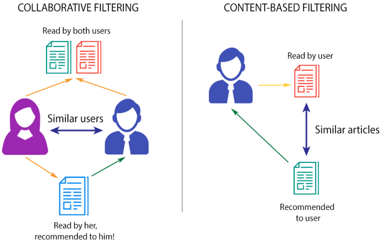

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- Netflix: look at sets of movies watched. Amazon: look at products purchased, etc.

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Entity Resolution Problem: Want to combine records from multiple data sources that refer to the same entities.

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- Still want to match records that all refer to the same person using all pairs similarity search.

Customer Records

Name	Phone Number	Account #
...	...	...
...	...	...
...	...	...
Cam Musco	1-401-555-5555	...
...	...	...
⋮	⋮	⋮

resolve ⟷

Public Property Records

Name	Phone Number	Address
...	...	...
...	...	...
...	...	...
Cameron Musco	402-555-5555	...
...	...	...
⋮	⋮	⋮

all-pairs search problem

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See Section 3.8.2 of *Mining Massive Datasets* for a discussion of a real world example involving 1 million customers. Naively this would be $\binom{1000000}{2} \approx 500$ billion pairs of customers to check!

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- **Fake Reviews**: Very common on websites like Amazon. Detection often looks for (near) duplicate reviews on similar products, which have been copied. 'Near duplicate' measured with shingles + Jaccard similarity.
- **Lateral phishing**: Phishing emails sent to addresses at a business coming from a legitimate email address at the same business that has been compromised.
    - One method of detection looks at the recipient list of an email and checks if it has small Jaccard similarity with any previous recipient lists. If not, the email is flagged as possible spam.

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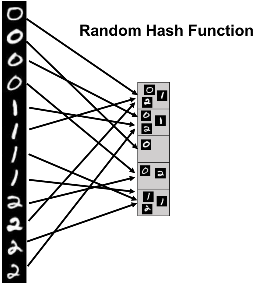
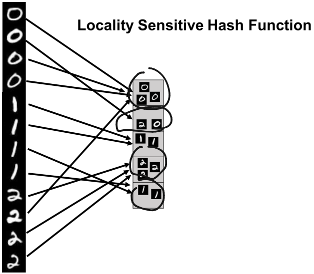
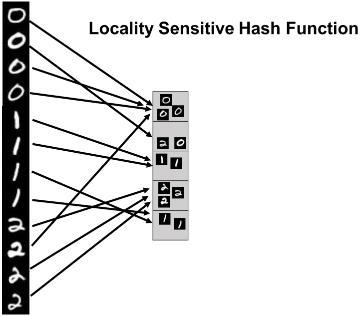




LSH For Similarity Search

How does locality sensitive hashing (LSH) help with similarity search?



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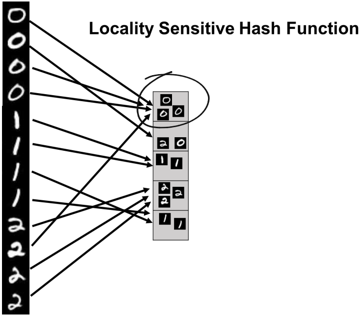

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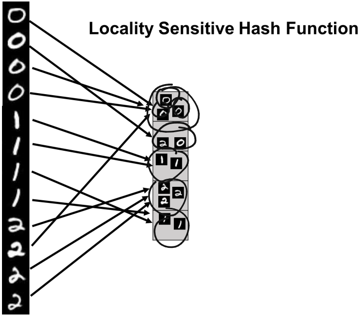

- **Near Neighbor Search:** Given item x, compute $h(x)$. Only search for similar items in the $h(x)$ bucket of the hash table.
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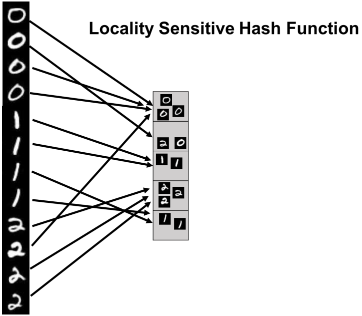

- **Near Neighbor Search:** Given item x, compute $\mathbf{h}(x)$. Only search for similar items in the $\mathbf{h}(x)$ bucket of the hash table.
- **All-pairs Similarity Search:** Scan through all buckets of the hash table and look for similar pairs within each bucket.
- We will use $\mathbf{h}(x) = \mathbf{g}(MinHash(x))$ where $\mathbf{g} : [0, 1] \rightarrow [n]$ is a random hash function. Why?

MinHashing

An Example: Locality sensitive hashing for Jaccard similarity.

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Strategy: Use random hashing to map each set to a single hash value. The probably that two sets have colliding hash values will be proportional to their Jaccard similarity.

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- Let $\mathsf{h} : U \to [0, 1]$ be a random hash function
- $\mathsf{s} := 1$
- For $x_1, \ldots, x_{|A|} \in A$
  - $\mathsf{s} := \min(\mathsf{s}, \mathsf{h}(x_k))$
- Return s

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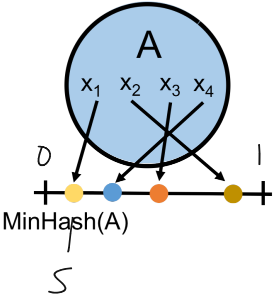

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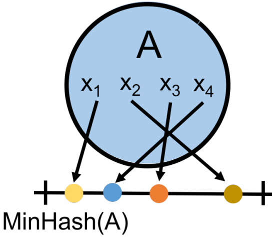

Identical to our distinct elements sketch!

MinHash Analysis

For two sets A and B, what is $\Pr(MinHash(A) = MinHash(B))$?

$$\Pr\left(\underbrace{\min_{x \in A} h(x)}_{} = \underbrace{\min_{y \in B} h(y)}_{}\right) = ?$$

mH(A) = $\min_{x \in A} h(x)$, mH(B) = $\min_{y \in B} h(y)$

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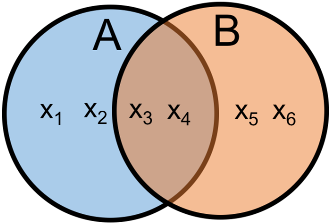

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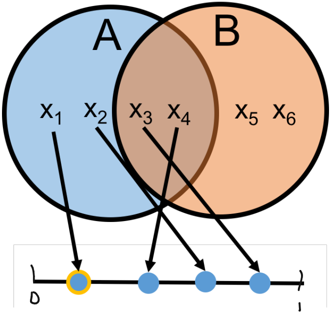

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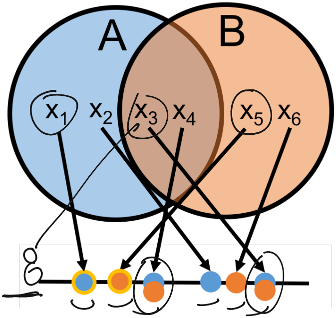
$mH(A) \neq mH(B)$



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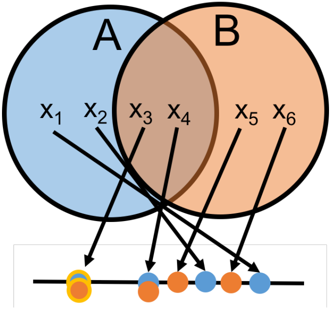

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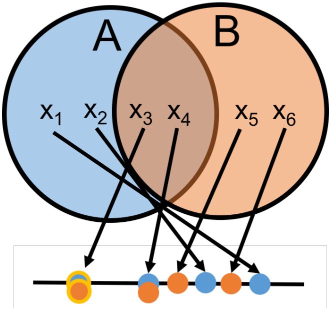

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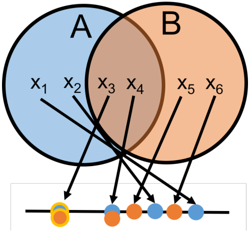

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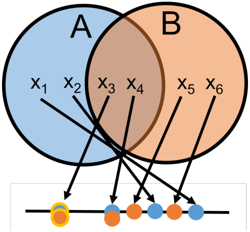

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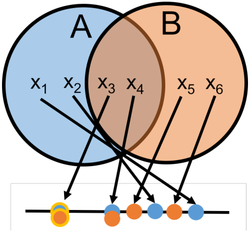

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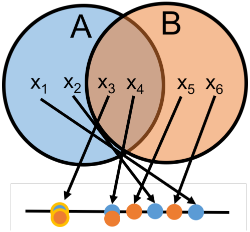

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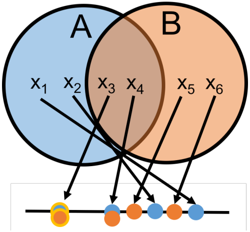

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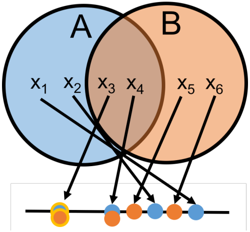

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Locality sensitive: the higher $J(A, B)$ is, the more likely $MinHash(A), MinHash(B)$ are to collide.

Similarity Search with MinHash

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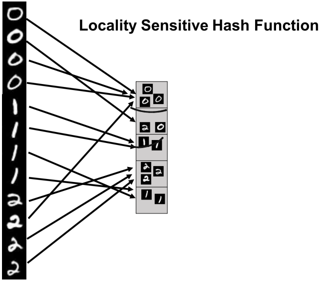
Our Approach:

- Create a hash table of size m, choose a random hash function $g : [0, 1] \rightarrow [m]$, and insert every item x into bucket $g(MinHash(x))$. Search for items similar to y in bucket $g(MinHash(y))$.



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- What is $\Pr[\mathbf{g}(MinHash(x)) = \mathbf{g}(MinHash(y))]$ assuming $J(x, y) = 1/2$ and $\mathbf{g}$ is collision free? $= 1/2$

$$\Pr(mH(x) = mH(y)) = J(x,y) = 1/2$$

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- What is $\Pr[\mathbf{g}(MinHash(x)) = \mathbf{g}(MinHash(y))]$ assuming $J(x, y) = 1/2$ and $\mathbf{g}$ is collision free?
- For every document x in your database with $J(x, y) \geq 1/2$ what is the probability you will find x in bucket $\mathbf{g}(MinHash(y))$? ≥1/2

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With a simple use of MinHash, we miss a match x with $J(x, y) = 1/2$ with probability $1/2$. How can we reduce this false negative rate?

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- To search for items similar to y, look at all items in bucket $\mathbf{g}(MH_1(y))$ of the 1^{st} table, bucket $\mathbf{g}(MH_2(y))$ of the 2^{nd} table, etc.

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Potential for a lot of false positives! Slows down search time.

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We want to balance a small probability of false negatives (a high hit rate) with a small probability of false positives (a small query time.)

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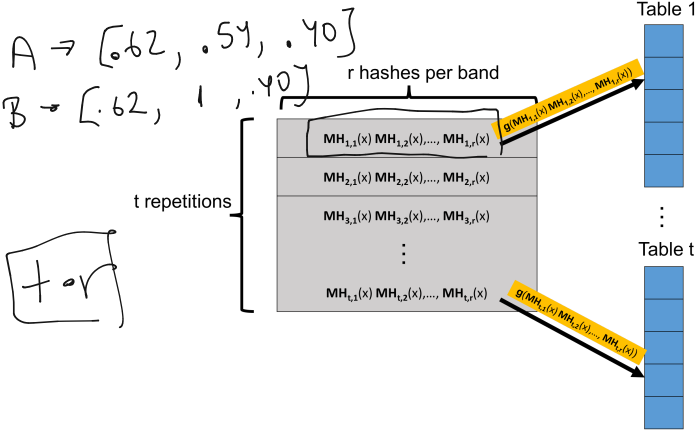

Create t hash tables. Each is indexed into not with a single MinHash value, but with r values, appended together. A length r signature.

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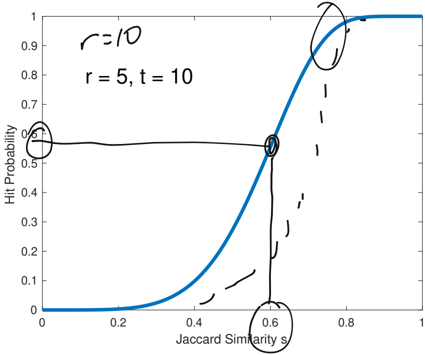

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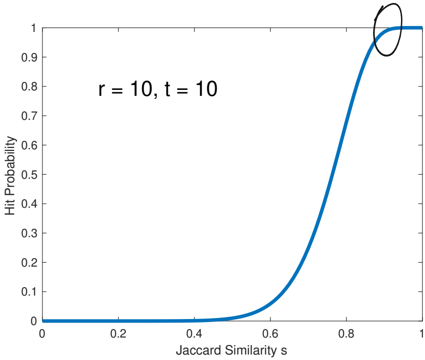

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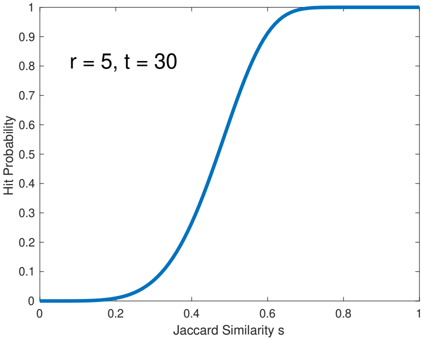

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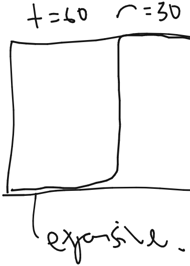

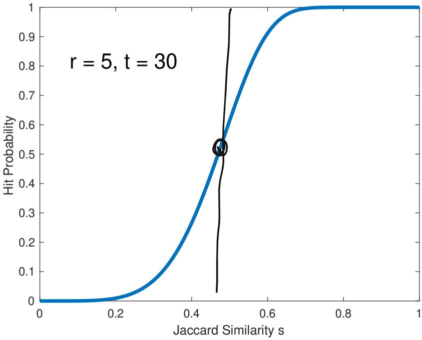

r and t are tuned depending on application. 'Threshold' when hit probability is 1/2 is $\approx (1/t)^{1/r}$. E.g., $\approx (1/30)^{1/5} = .51$ in this case.

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