COMPSCI 514: Algorithms for Data Science

Prof. Cameron Musco

University of Massachusetts Amherst. Fall 2024.

Lecture 1

Motivation For this Class

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- The Vera C. Rubin Observatory in Chile is planned to produce 30 terabytes of data/night.
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- Meta's LLaMA-2 large language model was trained on 2 trillion tokens of data.
  - How is this data collected and cleaned? How is it used to train a language model with tens of billions of parameters?

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- Massive data sets require storage in a distributed manner or processing in a continuous stream.

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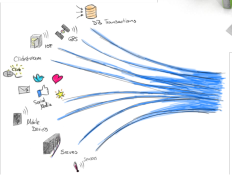

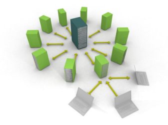

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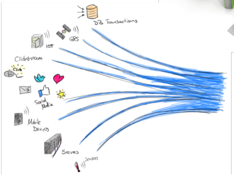

- Even 'simple' problems can become very difficult in this setting.

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- How can Google estimate the number of unique search queries that are made in a given week? Given that no machine can store the full list of queries.
- When you use Shazam to identify a song from a recording or perform a Google reverse image search, how does it provide an answer in < 10 seconds, without scanning over all of the millions or billions of possible images/audio files.

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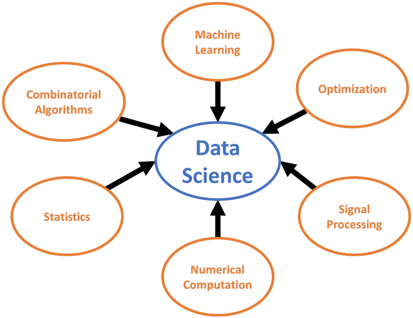

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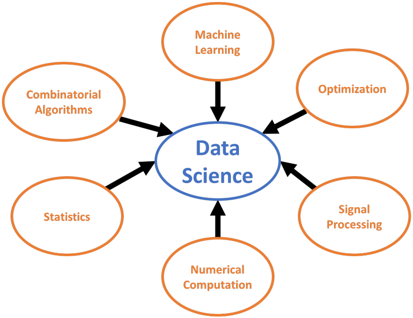

- Many techniques that aren't covered in the traditional CS algorithms curriculum.
- Emphasis on building comfort with mathematical tools that underly data science and machine learning.

What We'll Cover

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How can we efficiently compress large data sets in a way that lets us answer important algorithmic questions rapidly?

- Probability tools and concentration inequalities.
- Randomized hashing for efficient lookup, load balancing, and estimation. Bloom filters.
- Streaming algorithms: identifying frequent items in a data stream, counting distinct items, etc.
- Locality sensitive hashing and nearest neighbor search.
- Random compression of high-dimensional vectors: the Johnson-Lindenstrauss lemma, applications, and connections to the weirdness of high-dimensional geometry.

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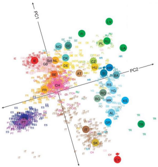

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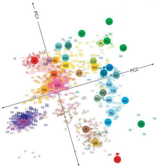

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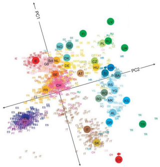

How do we identify the most important features of a dataset using linear algebraic techniques?

- Principal component analysis, low-rank approximation, dimensionality reduction.
- The singular value decomposition (SVD) and its applications to PCA, low-rank approximation, LSI, MDS, …
- Spectral graph theory. Spectral clustering, community detection, network visualization.
- Computing the SVD on large matrices via iterative methods.

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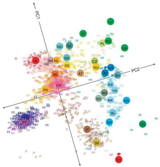

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If you open up the codes that are underneath [most data science applications] this is all linear algebra on arrays.

– Michael Stonebraker

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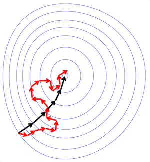
Section 3: Optimization



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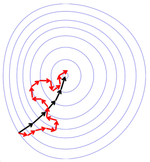

Fundamental continuous optimization approaches that drive methods in machine learning and statistics.

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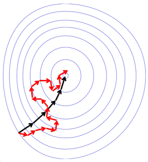

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- Gradient descent. Analysis for convex functions.
- Stochastic and online gradient descent.
- Focus on convergence analysis.

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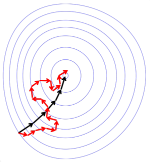

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A small taste of what you can find in COMPSCI 590OP or 651.

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    - E.g., regression methods, kernel methods, random forests, SVM, deep neural networks.

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- Systems/Software Tools.
  - COMPSCI 532: Systems for Data Science
- **Machine Learning/Data Analysis Methods and Models.**
  - E.g., regression methods, kernel methods, random forests, SVM, deep neural networks.
  - COMPSCI 589/689: Machine Learning, other 500/600 level ML courses.

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For example: Baye's rule in conditional probability. What it means for a vector x to be an eigenvector of a matrix A, orthogonal projection, greedy algorithms, divide-and-conquer algorithms.

Course Logistics

See course webpage for logistics, policies, lecture notes, assignments, etc.:

http://people.cs.umass.edu/~cmusco/CS514F24/

See Canvas for this link if you lose it, or search my name and follow the link from my homepage.

Canvas will be used only for weekly quizzes and possibly the posting of grades. Use the course page for mostly everything else.

Personnel

Professor: Cameron Musco

- Email: cmusco@cs.umass.edu
- Office Hours: ~~Over Zoom,~~ Tuesdays, 2:30pm-3:30pm (directly after class) in CS 234 (location subject to change).
- I encourage you to come as regularly as possible to ask questions/work together on practice problems.
- If you need to chat individually, please email meet to set up a time.

TAs:

- Rajarshi Bhattacharjee, Fatemeh Ghaffari, Saishradha Mohanty, Shreyas Chaudhari, Jacob Urisman, Chien Nguyen
- The TAs are shared across this section and Prof. McGregor's online Section. They will hold in person and virtual office hours.
- See website for office hours (some TBD) and contact info.

Piazza and Participation

We will use Piazza for class discussion and questions.

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You may earn up to 5% extra credit for participation.

- Asking good clarifying questions and answering student/instructor questions during the lecture or on Piazza.
- Posting helpful links on Piazza, e.g., resources that cover class material, research articles related to the class, etc.
- It is completely fine to post private questions on Piazza, but these don't count towards participation credit.
- You can post anonymously on Piazza. Instructors will see the author behind all posts, so we can assign participation credit.
- Regularly attending office hours is highly encouraged but doesn't itself count towards participation credit.

Textbooks and Materials

We will use material from three textbooks (links to free online versions on the course webpage): *Foundations of Data Science*, *Mining of Massive Datasets*, and *Probability and Computing*, but will follow none of them closely.

- I will post optional readings before each class.
- Lecture notes will be posted before each class, and annotated notes posted after class.
- Recordings of the live lectures will also be posted on Echo360.
- Sometimes it takes a lecture or two to get the Echo360 set up working properly. And typically at least a few lectures are not properly recorded due to technical issues.

Problem Sets

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- We strongly encourage working in groups, as it will make completing the problem sets easier and more educational.
- Collaboration with students outside your group is limited to discussion at a high level. You may not work through problems in detail or write up solutions together.
- See Piazza for a thread to help you organize groups.
- You can change groups as you like over the course of the semester.

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Problem set submissions will be via Gradescope.

- See website for a link to join and an entry code

Problem Sets

The problem sets will have two components:

- **Core Competency Problems:** Must be completed. ~~Graded numerically~~. Similar in difficulty to exam problems and designed to prepare you for the exams.
- **Challenge Problems:** Designed to strengthen your ability to think creatively about algorithmic problems and push beyond what is taught in class, to design solutions of your own.
  - Will take significantly longer to tackle than the core competency problems.
  - Only some fraction need to be completed on each problem set (e.g., one out of two or three problems).

- No extra credit for extra challenge problems.

Weekly Quizzes

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- Designed as a check-in that you are following the material, and to help me make adjustments as needed.
- Will take around 15-30 minutes per week, open notes.
- Will also include free response check-in questions to get your feedback on how the course is going, what material from the past week you find most confusing, interesting, etc.

Grading

Grade Breakdown:

- Problem Sets (5 total): 40%.
- Weekly Quizzes: 10%, weighted equally.
- Midterm (October 17th, evening exam): 25%.
- Final (December 18th, 10:30am - 12:30pm): 25%.
- Extra Credit: Up to 5% for participation. Potentially more available on problem sets and exams.

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There is no option to take the exams remotely. If you must miss an exam due to sickness or another emergency, we'll schedule an in-person make-up.

Academic Honesty and Exceptions

- No late homework submissions, unless there are extenuating circumstances, approved by the instructor before the deadline.
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Academic Honesty:

- A first violation cheating on a homework, quiz, or other assignment will result in a 0 on that assignment. For problem sets, this will include all components of the assignment – core competency and challenge problems.
- A second violation, or cheating on an exam will result in failing the class.
- For fairness, I adhere very strictly to these policies.

Disability Services and Accomodations

UMass Amherst is committed to making reasonable, effective, and appropriate accommodations to meet the needs to students with disabilities.

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- If your disability requires an accommodation, please email me by next Thursday 9/12 so that we can make arrangements.

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I understand that people have different learning needs, home situations, etc. If something isn't working for you in the class, please reach out and let's try to work it out.

Questions?

Section 1: Randomized Methods & Sketching

Some Probability Review

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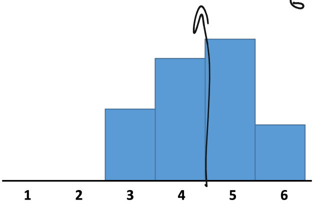
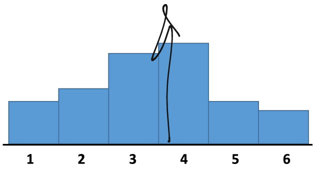
- **Expectation:** $\mathbb{E}[X] = \sum_{s \in S} \Pr(X = s) \cdot s.$

$$= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \ldots \frac{1}{6} \cdot 6 = 3.5$$





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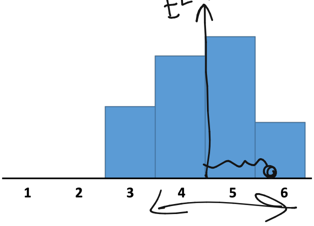
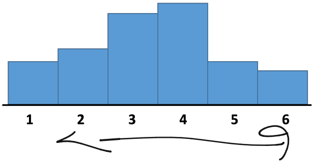
- **Expectation:** $\mathbb{E}[X] = \sum_{s \in S} \Pr(X = s) \cdot s$.
- **Variance:** $\mathrm{Var}[X] = \mathbb{E}[(X - \mathbb{E}[X])^2]$.

E[X]





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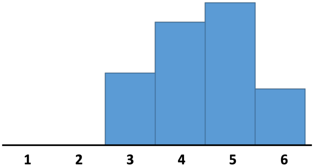

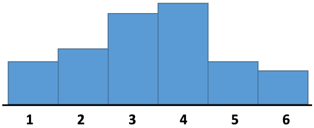

Exercise: Show that for any scalar α, $\mathbb{E}[\alpha \cdot X] = \alpha \cdot \mathbb{E}[X]$ and $\mathrm{Var}[\alpha \cdot X] = \alpha^2 \cdot \mathrm{Var}[X]$.

$\alpha = 2$ $\quad 2 \cdot X$ $\quad \{2, 4, 6 \dots 12\}$

Independence

Consider two random events A and B.

$A \cap B$: event that both events *A and B* happen.

Independence

$\cup \Leftrightarrow$ "OR"

Consider two random events A and B.

- **Conditional Probability:**

AND

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

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Independence

Consider two random events A and B.

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$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

- **Independence:** A and B are independent if:
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- **Independence:** A and B are independent if:

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Using the definition of conditional probability, independence means:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \Pr(A) \implies \Pr(A \cap B) = \Pr(A) \cdot \Pr(B).$$

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Independence

Example 1: What is the probability that for two independent dice rolls the first is a 6 and the second is odd?

$A: D_1 = 6$

$B: D_2 \in \{1, 3, 5\}$

$P(A \cap B) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

$P(A) = \frac{1}{6}$

$P(B) = \frac{1}{2}$

Example 1: What is the probability that for two independent dice rolls the first is a 6 and the second is odd?

Example 2: What is the probability that a random dice roll is a prime number, conditioned on it being even.

$P(A \cap B) = \frac{1}{6}$

$P(B) = \frac{1}{2}$

$Pr(\underset{\text{prime}}{A} | \underset{\text{even}}{B}) = \frac{1/6}{1/2} = 1/3$

Independent Random Variables: Two random variables X, Y are independent if for all s, t, $X = s$ and $Y = t$ are independent events. In other words:

$$\Pr(X = s \cap Y = t) = \Pr(X = s) \cdot \Pr(Y = t).$$

$\frac{1}{6} \cdot \frac{1}{6}$

Linearity of Expectation and Variance

Think-Pair-Share: When are the expectation and variance linear?

I.e., under what conditions on X and Y do we have:

$$\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$$

↳ always true.

and

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Var}[Y].$$

↳ indepent

"uncorrelated"

X, Y: any two random variables.

Linearity of Expectation

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Proof:

$$\mathbb{E}[X + Y] = \sum_{s \in S} \sum_{t \in T} \Pr(X = s \cap Y = t) \cdot (s + t)$$

Linearity of Expectation

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Proof:

$$\begin{aligned}\mathbb{E}[X+Y] &= \sum_{s\in S}\sum_{t\in T}\Pr(X = s \cap Y = t)\cdot(s+t)\\ &= \sum_{s\in S}\sum_{t\in T}\Pr(X = s \cap Y = t)\cdot s + \sum_{t\in T}\sum_{s\in S}\Pr(X = s \cap Y = t)\cdot t\end{aligned}$$

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$$
\begin{aligned}
\mathbb{E}[X+Y] &= \sum_{s\in S}\sum_{t\in T}\Pr(X=s\cap Y=t)\cdot(s+t)\\
&= \sum_{s\in S}\sum_{t\in T}\Pr(X=s\cap Y=t)\cdot s+\sum_{t\in T}\sum_{s\in S}\Pr(X=s\cap Y=t)\cdot t\\
&= \sum_{s\in S}\Pr(X=s)\cdot s+\sum_{t\in T}\Pr(Y=t)\cdot t
\end{aligned}
$$

(law of total probability)

Linearity of Expectation

$\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$ for any random variables X and Y.

Proof:

$$
\begin{aligned}
\mathbb{E}[X+Y] &= \sum_{s \in S} \sum_{t \in T} \Pr(X = s \cap Y = t) \cdot (s+t) \\
&= \sum_{s \in S} \sum_{t \in T} \Pr(X = s \cap Y = t) \cdot s + \sum_{t \in T} \sum_{s \in S} \Pr(X = s \cap Y = t) \cdot t \\
&= \sum_{s \in S} \Pr(X = s) \cdot s + \sum_{t \in T} \Pr(Y = t) \cdot t \\
&\qquad\qquad\qquad\qquad \text{(law of total probability)} \\
&= \mathbb{E}[X] + \mathbb{E}[Y].
\end{aligned}
$$