COMPSCI 514: ALGORITHMS FOR DATA SCIENCE

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University of Massachusetts Amherst. Fall 2020.
Lecture 9

LOGISTICS

- Problem Set 2 is due this upcoming Monday. Get an early start on it.
- Problem Set 1 grades have been released. Mean: 34/41, Median 36/41.
- If you are unhappy with your grade, ping me and let's chat about strategies going forward. If you believe there is a grading error, send a private message to the instructors on Piazza or ask during office hours.
- The midterm will be any 2 hour slot on 10/8-10/9. We won't have class on 10/8.
- Study guide/practice questions will be released this week.

Last Class:

- MinHash as a locality sensitive hash function for Jaccard similarity
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A locality sensitive hash function can be: (check all that apply)

Select one or more:

- [ ] a. Randomized
- [ ] b. Pairwise-Independent
- [ ] c. Sensitive to Jaccard Similarity
- [ ] d. Have the distribution of h(x) independent of x.

SUMMARY

$A = \{a_1, a_2 \ldots a_n\}$

Last Class: $MH(A) = \min_i h(a_i)$

a_i would be 1 shingle

- MinHash as a locality sensitive hash function for Jaccard similarity
- Near neighbor search with LSH signatures and repeated hash tables..
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This Class: Frequent Items Estimation

- Count-min sketch (random hashing for frequent element estimation).

UPCOMING

Next Few Classes:

- Random compression methods for high dimensional vectors. The Johnson-Lindenstrauss lemma.
- Connections to the weird geometry of high-dimensional space.

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- Spectral clustering and spectral graph theory.

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Will use a lot of linear algebra. May be helpful to refresh.

- Vector dot product, addition, Euclidean norm. Matrix vector multiplication.
- Linear independence, column span, orthogonal bases, rank.
- Orthogonal projection, eigendecomposition, linear systems.

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k = 3

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- What is the maximum number of items that ~~must~~ can be returned?

  a) n b) k c) n/k d) $\log n$

$\#HHs \leq k$

each heavy hitter appears $\frac{n}{k}$ times

so total frequency is $\frac{n}{k} \cdot \# HHs \leq n$

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- What is the maximum number of items that must be returned? a) n b) k c) n/k d) $\log n$
- Trivial with $O(n)$ space – store the count for each item and return the one that appears $\geq n/k$ times.
- Can we do it with less space? I.e., without storing all n items?

Applications of Frequent Items:

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Generally want very fast detection, without having to scan through database/logs. I.e., want to maintain a running list of frequent items that appear in a stream.

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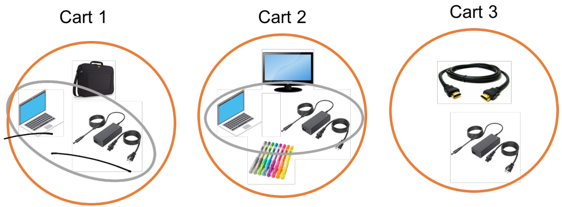

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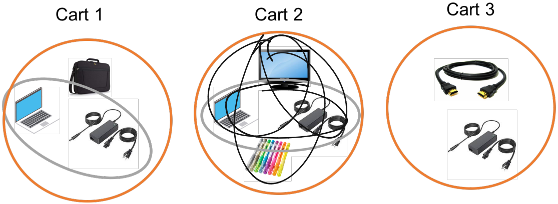

- Identified via frequent itemset counting. Find all sets of k items that appear many times in the same basket.
- Frequency of an itemset is known as its support.
- A single basket includes many different itemsets, and with many different baskets an efficient approach is critical. E.g., baskets are Twitter users and itemsets are subsets of who they follow.

Issue: No algorithm using $o(n)$ space can output just the items with frequency $\geq n/k$. Hard to tell between an item with frequency n/k (should be output) and $n/k - 1$ (should not be output).

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(ϵ, k)-Frequent Items Problem: Consider a stream of n items $x_1, \ldots, x_n$. Return a set F of items, including all items that appear at least $\frac{n}{k}$ times and only items that appear at least $(1-\epsilon) \cdot \frac{n}{k}$ times.

$\epsilon = .1 \quad k = 100$

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- An example of relaxing to a 'promise problem': for items with frequencies in $[(1 - \epsilon) \cdot \frac{n}{k}, \frac{n}{k}]$ no output guarantee.

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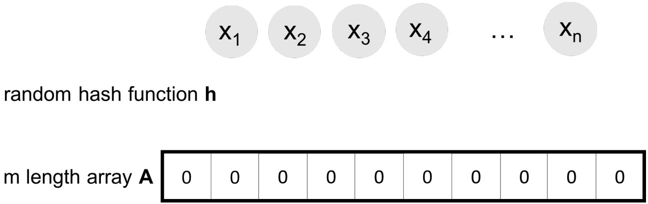

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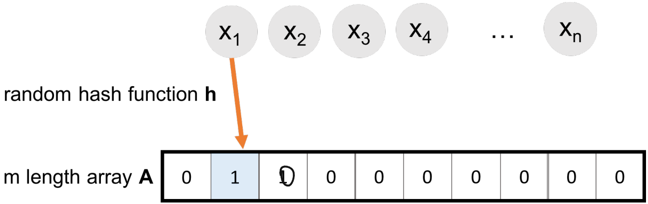

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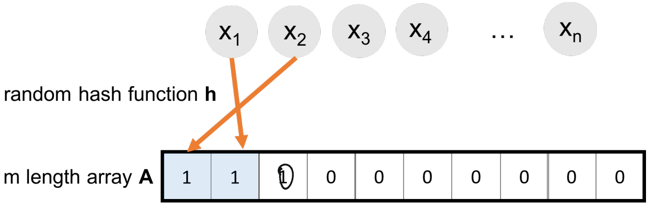

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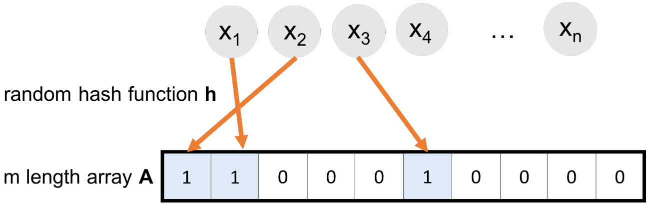

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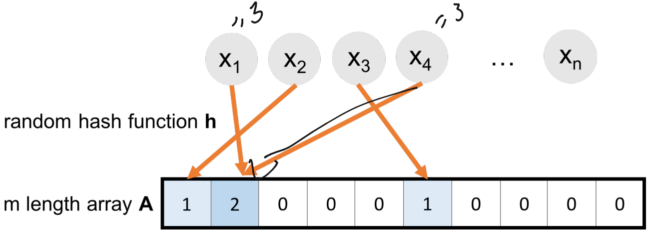

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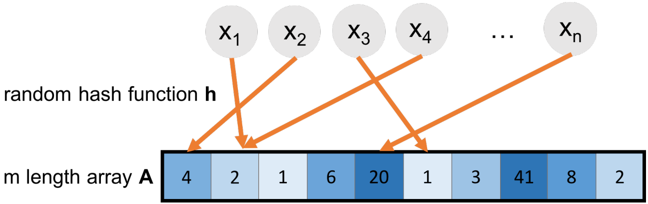

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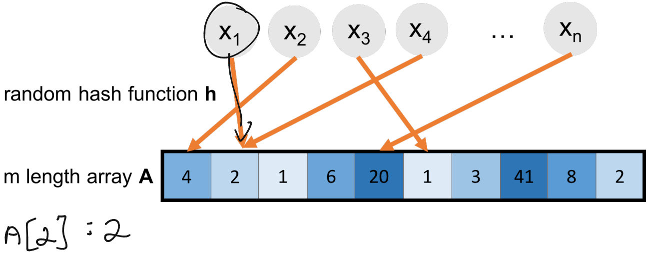
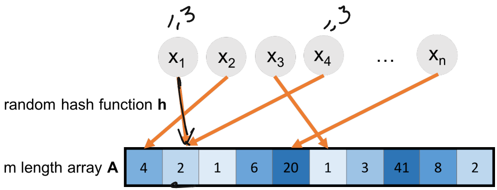


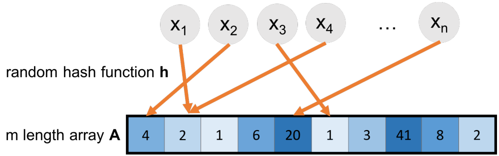
Will use $A[\mathbf{h}(x)]$ to estimate $f(x)$, the frequency of x in the stream. I.e., $|\{x_i : x_i = x\}|$.



Use $A[\mathbf{h}(x)]$ to estimate $f(x)$.

Claim 1: We always have $A[\mathbf{h}(x)] \geq f(x)$. Why?

$f(x)$: frequency of x in the stream (i.e., number of items equal to x). **h**: random hash function. m: size of Count-min sketch array.



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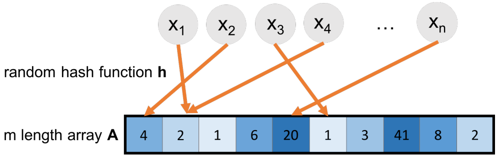

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- $A[\mathbf{h}(x)]$ counts the number of occurrences of any y with $\mathbf{h}(y) = \mathbf{h}(x)$, including x itself.
- $A[\mathbf{h}(x)] = f(x) + \sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)$.

$f(x)$: frequency of x in the stream (i.e., number of items equal to x). $\mathbf{h}$: random hash function. m: size of Count-min sketch array.

$$A[\mathbf{h}(x)] = f(x) + \underbrace{\sum_{y \neq x : \mathbf{h}(y) = \mathbf{h}(x)} f(y)}_{\text{error in frequency estimate}} .$$

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COUNT-MIN SKETCH ACCURACY

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m buckets
counters

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$$\frac{1}{m} \sum_{y \neq x} f(y)$$

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$\alpha \cdot \mathbb{E}(\text{error})$

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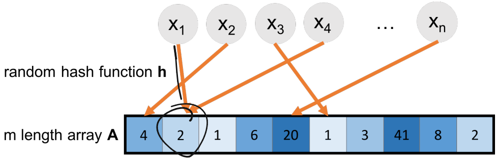

Claim: For any x, with probability at least 1/2,

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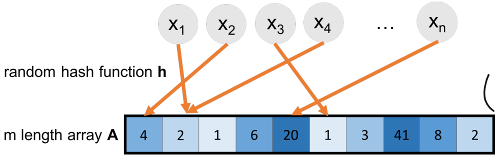

$$\frac{n}{k} \geq (1-\varepsilon)\frac{n}{k}$$

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$$\frac{2n}{m} = \frac{2n}{2k/\varepsilon} = \frac{n}{k} \cdot \varepsilon$$

To solve the (ϵ, k)-Frequent elements problem, set $m = \left\lceil \frac{2k}{\epsilon} \right\rceil$.

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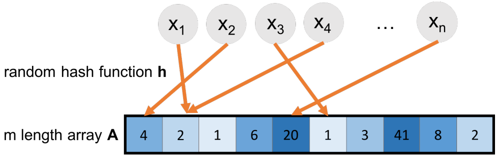

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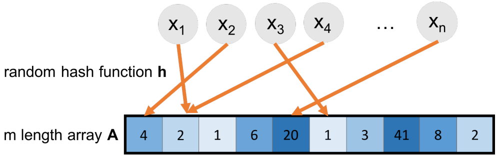

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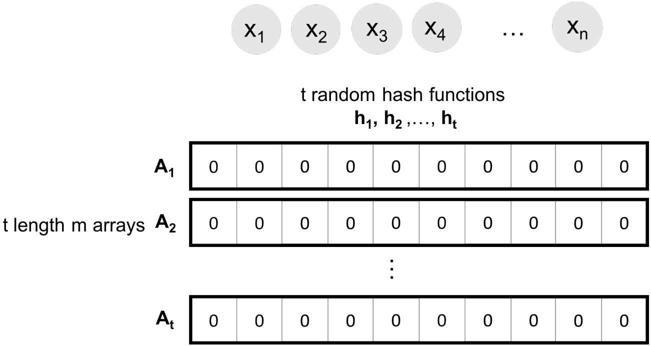
COUNT-MIN SKETCH ACCURACY



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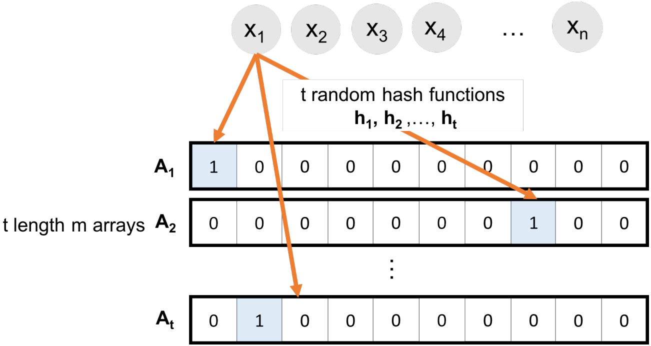

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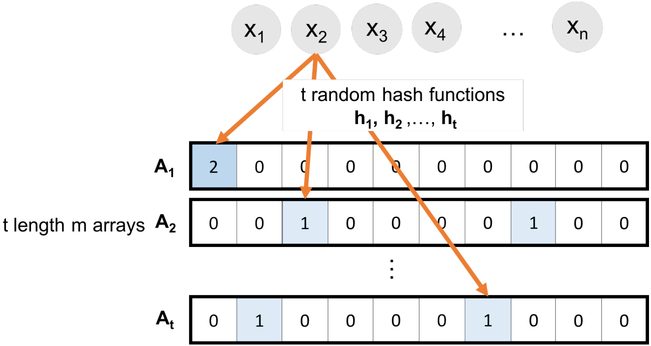
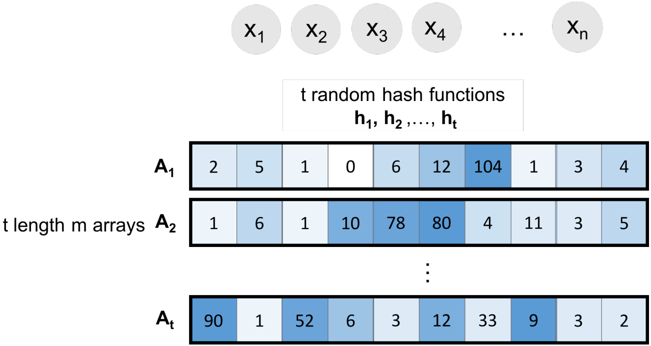
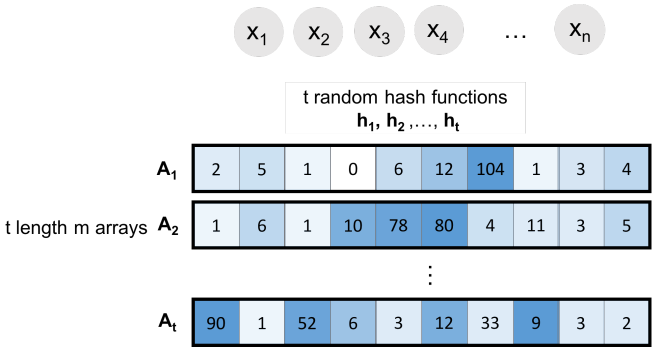


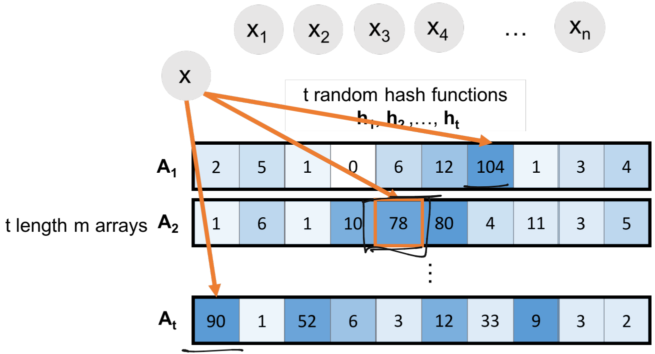
COUNT-MIN SKETCH ACCURACY



COUNT-MIN SKETCH ACCURACY



Estimate $f(x)$ with $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$. (count-min sketch)



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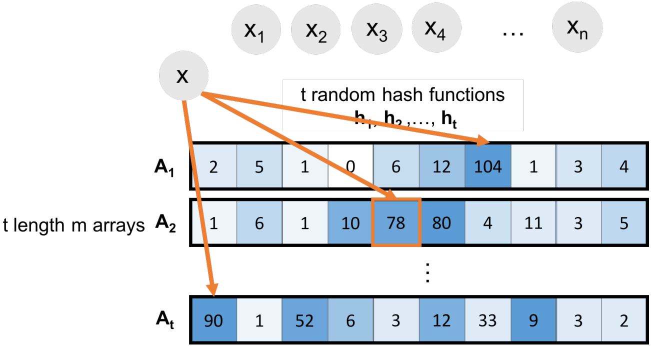

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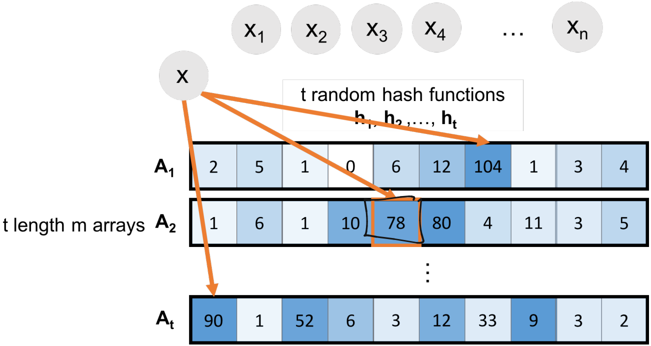
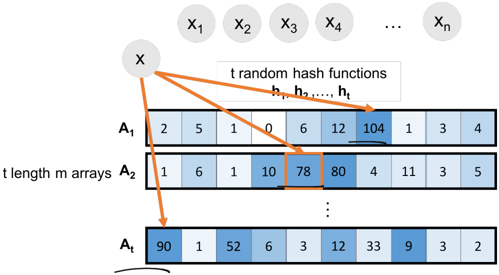
Why min instead of mean or median?



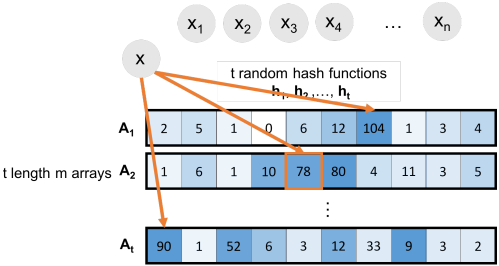
Estimate $f(x)$ with $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$. (count-min sketch)

Why min instead of mean or median? The minimum estimate is always the most accurate since they are all overestimates of the true frequency!



Estimate $f(x)$ by $\tilde{f}(x) = \min_{i \in [t]} A_i[\mathbf{h}_i(x)]$

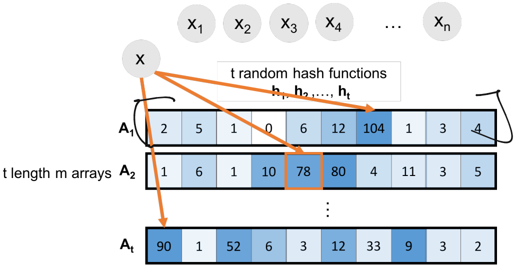
COUNT-MIN SKETCH ANALYSIS



Estimate $f(x)$ by $\tilde{f}(x) = \min_{i\in[t]} A_i[\mathbf{h}_i(x)]$

- For every x and $i \in [t]$, we know that for $m = \frac{2k}{\epsilon}$, with probability $\geq 1/2$:

$$f(x) \leq A_i[\mathbf{h}_i(x)] \leq f(x) + \frac{\epsilon n}{k}.$$



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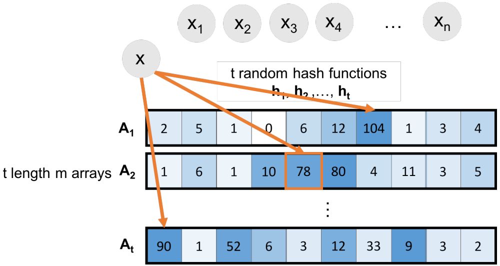
- What is $\Pr[f(x) \leq \tilde{f}(x) \leq f(x) + \frac{\epsilon n}{k}]$?



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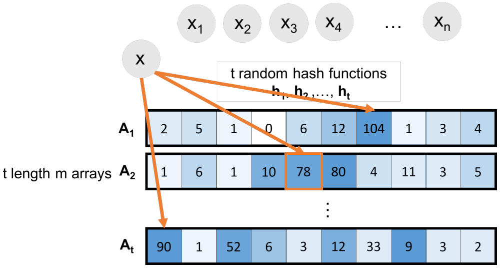
$$f(x) \leq A_i[\mathbf{h}_i(x)] \leq f(x) + \frac{\epsilon n}{k}.$$

- What is $\Pr[f(x) \leq \tilde{f}(x) \leq f(x) + \frac{\epsilon n}{k}]$? $1 - 1/2^t$.

$\Pr(\tilde{f}(x) \notin [f(x), f(x) + \frac{\epsilon n}{k}]) = \frac{1}{2^t}$



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$$f(x) \leq A_i[\mathbf{h}_i(x)] \leq f(x) + \frac{\epsilon n}{k}.$$

- What is $\Pr[f(x) \leq \tilde{f}(x) \leq f(x) + \frac{\epsilon n}{k}]$? $1 - 1/2^t$.
- To get a good estimate with probability $\geq 1 - \delta$, set $t = \log(1/\delta)$.

Upshot: Count-min sketch lets us estimate the frequency of every item in a stream up to error $\frac{\epsilon n}{k}$ with probability $\geq 1 - \delta$ in $O\left(\log(1/\delta) \cdot k/\epsilon\right)$ space.

COUNT-MIN SKETCH

Upshot: Count-min sketch lets us estimate the frequency of every item in a stream up to error $\frac{\epsilon n}{k}$ with probability $\geq 1 - \delta$ in $O\left(\log(1/\delta) \cdot k/\epsilon\right)$ space.

- Accurate enough to solve the (ϵ, k)-Frequent elements problem – distinquish between items with frequency $\frac{n}{k}$ and those with frequency $(1 - \epsilon)\frac{n}{k}$. $\frac{\epsilon n}{k}$

COUNT-MIN SKETCH

set $\delta = \frac{.01}{n}$

$t = \log(1/\delta) = O(\log n)$

apply union bound.

$f(x_i) \le \tilde{f}(x_i) \le f(x_i) + \frac{\epsilon n}{k}$ w.p. $1-\delta$

for all i at once, $f(x_i) \le \tilde{f}(x_i) \le f(x_i) + \frac{\epsilon n}{k}$ w.p. $1 - n\delta$

Upshot: Count-min sketch lets us estimate the frequency of every item in a stream up to error $\frac{\epsilon n}{k}$ with probability $\geq 1 - \delta$ in $O\left(\log(1/\delta) \cdot k/\epsilon\right)$ space.

$O\left(\log(n) \cdot \frac{k}{\epsilon}\right)$

- Accurate enough to solve the (ϵ, k)-Frequent elements problem – distinquish between items with frequency $\frac{n}{k}$ and those with frequency $(1-\epsilon)\frac{n}{k}$.
- How should we set δ if we want a good estimate for all items at once, with 99% probability?

n possible items, we want to estimate all frequencies.

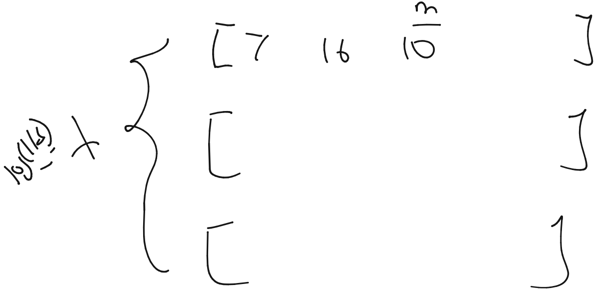
let E_i be the event that I fail to estimate $f(x_i)$ well

$\Pr(E_i) \le \delta$ $\quad \Pr(E_1 \cup E_2 \cup \ldots E_n) \le \sum_{i=1}^{n} \Pr(E_i) \le n\delta$

$O\left(\log n \cdot \frac{k}{\varepsilon}\right)$ space. $\frac{n}{k}$ $(1-\varepsilon)\frac{n}{k}$

Count-min sketch gives an accurate frequency estimate for every item in the stream. But how do we identify the frequent items without having to store/look up the estimated frequency for all elements in the stream?

$m = \frac{k}{\varepsilon}$ $\frac{\varepsilon n}{k}$



total space =

$t \cdot m = \log(1/\delta) \cdot m$

$= \log(1/\delta) \cdot \frac{k}{\varepsilon}$

δ = failure prob. (ε, k) are parameters of freq items prob.

Count-min sketch gives an accurate frequency estimate for every item in the stream. But how do we identify the frequent items without having to store/look up the estimated frequency for all elements in the stream?

One approach: $x_1, x_2 \ldots x_i \ldots x_n$

- When a new item comes in at step i, check if its estimated frequency is $\geq i/k$ and store it if so.
- At step i remove any stored items whose estimated frequency drops below i/k.
- Store at most $O(k)$ items at once and have all items with frequency $\geq n/k$ stored at the end of the stream.

Time complexity: $O(k)$ time to hash x_i an increment buckets

$O(k)$ time to check if any of stored items have freq $\leq \frac{1}{k}$

Questions on Frequent Elements?

$O(t \cdot \frac{k}{\varepsilon})$

$+ t \cdot$ size hash function.

- Defined frequent items
- Relaxed it to the (ε, k)-freq. item
- Used count-min sketch (variant on Bloom filter) to solve this problem in $O(\log n \cdot \frac{k}{\varepsilon})$ space $<< O(n)$