PProofster: Automated Formal Verification

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Abstract—Formal verification is an effective but extremely work-intensive method of improving software quality. Verifying the correctness of software systems often requires significantly more effort than implementing them in the first place, despite the existence of proof assistants, such as Coq, aiding the process. Recent work has aimed to fully automate the synthesis of formal verification proofs, but little tool support exists for practitioners. This paper presents PProofster, a web-based tool aimed at assisting developers with the formal verification process via proof synthesis. PProofster inputs a Coq theorem specifying a property of a software system and attempts to automatically synthesize a formal proof of the correctness of that property. When it is unable to produce a proof, PProofster outputs the proof-space search tree and displays the Coq proof script, verifying that the existence of proof assistants, such as Coq, aiding the process. Proof synthesis. For example, CoqHammer [10] uses a set of precomputed mathematical facts to attempt to “hammer” out a proof. Meanwhile, ASTactic [54], Proverbot9001 [45], TacTok [13], Diva [12], and Passport [46] learn a predictive model from a corpus of existing proofs and use that model to guide a meta-heuristic search to synthesize a proof from scratch.

Unfortunately, relatively little tool support exists for practitioners to use these Coq proof-synthesis tools. For example, of the above-mentioned search-based tools, all but one have neither been integrated into IDEs nor built as stand-alone, graphical interfaces, making adoption difficult. Only Tactician [5] has a usable interface, by way of a plugin for Coq that can be integrated into Coq IDEs. But even then, the interface does not expose the features that help the user understand what the tool is doing under the hood, making debugging and explainability difficult.

In this paper, we present PProofster, a new graphical frontend for search-based proof-synthesis techniques that emphasizes explainability. Conceptually, PProofster can be straightforwardly extended to work with any proof-synthesis backend tool, and implements special features to support explainability for search-based backends. Here, we demonstrate PProofster with Proverbot9001 [45] as its backend.

PProofster’s main contributions support the developer in two ways:

1) The developer can enter a theorem describing a software property they want proven, and PProofster uses its underlying backend to attempt to generate a proof. If successful, PProofster displays the Coq proof script, verifying that the
property is correct. Proofster uses the Alectryon library to render literate Coq code [39], which is interactive and easy to read, even when one does not have immediate access to a proof assistant to step through the synthesized proof. The developer can explore the context throughout the proof to better understand why the property is verifiably correct.

2) If the synthesis is unsuccessful, Proofster uses the D3.js library [6] to allow the developer to interactively explore the search tree it used in trying to synthesize a proof, and understand the relevant context. The developer can then identify the most promising search-path, augment it, and have Proofster attempt to synthesize a proof again, using that information.

A live Proofster deployment is available at https://proofster.cs.umass.edu/.

II. Proofster

Proofster is a frontend tool that interfaces with Coq-based proof synthesis tools. Section II-A discusses how proof engineers interactively write proofs in Coq and how machine-learning-guided proof synthesis tools automatically generate proofs. Section II-B then describes the Proofster implementation and Section II-C illustrates, with examples, how a proof engineer can use Proofster to construct proofs.

A. Proofs and proof synthesis in Coq

When using the Coq proof assistant, a developer begins by specifying a theorem to prove. This theorem is a type definition in Coq’s internal language, Gallina. A proof of that theorem is a term of that type. However, writing that proof term directly is difficult, and so Coq provides an interactive environment for reasoning through a proof at a higher level, via a proof script. The developer can use Coq’s Ltac language to construct a proof script, a sequence of tactics which Coq uses to guide its internal search for a Gallina-based proof term. The theorem prover is called interactive, because the developer can specify a tactic to try, have the theorem prover execute the tactic to update the proof state (the set of goals that need to be proven, and the known facts), and use that proof state to decide on the next tactic. This interactive process continues until no goals remain, meaning the theorem is proven.

The burden is on the developer to come up with the sequence of tactics. To ease this burden, recent work has created search-based, machine-learning-guided proof-synthesis tools that perform automatic proof-script generation. Most of these tools train a predictive model on a corpus of human-written proof scripts. This model uses a partially written proof script and the theorem being proven to predict a ranked list of the most likely next tactics that should come in the proof script.

The tools differ in how they model the proof scripts when making predictions. For example, ASTactic considers only the current proof state (and ignores the current, partial proof script) [54]. TacTok is a collection of two models — Tac and Tok — both of which encode both the proof state and the partial proof script. Tac works at the tactic granularity, whereas Tok works at the token granularity; the two prove complementary sets of theorems [13]. These tools model abstract syntax trees using TreeLSTM [50] and proof-script sequences using bidirectional LSTM [38], whereas Proverbot9001, which also models proof state and partial proof script, uses a sequence model [45]. Passport further enhances the model by encoding identifier information for the names of theorems, datatypes, functions, type constructors, and local variables [46]. GamePad, meanwhile, uses its own RNN-based tree encoder and targets only synthetic lemmas [23]. Finally, Diva observes that the variability inherent in machine learning — small perturbations in the learning process, such as hyperparameters, the order in which the training data is seen, and the encoded richness of the training data — leads to diversity in the sets of theorems the learned models can prove. Using the theorem prover’s unique ability to serve as an oracle for correctness, Diva uses this diversity to significantly increase its proving power [12].

Armed with a predictive model, these search-based tools search through the space of possible proof scripts. They use the model to predict the likely next proof steps, and the theorem prover to compute the new proof states or errors resulting from these steps. They prune search paths unlikely to be successful or that repeat an already explored state; Proverbot9001, in particular, also prunes states that would explore a subgoal for which a solution was already found. This search through the space of proof scripts represents a set of potential partial proof scripts that aim to make progress toward the goal of proving the theorem. We call the set of explored search paths, together, the search tree.

B. The Proofster implementation

Proofster is implemented as a Flask app and uses BeautifulSoup to create the results page with the synthesized proof and the search graph. Proofster allows the developer to enter a theorem into a text box (or select one from several examples, as a demonstration). Proofster then passes the developer-specified theorem to its proof-synthesis backend and retrieves the search tree, and, if the backend is successful, the synthesized proof. Proofster then uses Alectryon to render the proof as an interactive, literate Coq object. Hovering over a tactic displays the context and goals at that stage of the proof.

Proofster uses the the D3.js library display the search tree and allow the developer to interact with it. Subtrees can be collapsed and expanded to see the tactics tried by the proof synthesis model. This information can also be helpful to developers to provide hints to Proofster in the case where Proofster fails to prove the theorem initially.

Proofster is deployed on AWS and is publicly available at https://proofster.cs.umass.edu/. Proofster is open-source, and is publicly available at https://github.com/UCSD-PL/proverbot9001/tree/demowebtool.

Next, we illustrate Proofster’s two use cases using examples.

C. Using Proofster

Supposed a developer has written a function, max_elem_list, that takes a list of natural numbers and returns its largest
Fig. 1. A Proofster screenshot of the developer asking to prove the theorem `every_elem_le_max` about the function `max_elem_list`.

Fig. 2. When Proofster executes the query from Figure 1, it produces a complete proof for the theorem `every_elem_le_max` on this page. Hovering over a tactic in the proof shows the proof state at that point in the proof, which allows the developer to explore and understand how the proof verifies the property.

The developer decides to use Proofster to prove the above property, in Coq. She heads over to https://proofster.cs.umass.edu/ and enters some basic imports, the definition of the `max_elem_list` function, and the theorem `every_elem_le_max`. She does not enter the proof of the theorem, but only starts it with `Proof.` and `Admitted.` to tell Proofster to generate a proof for that theorem. (Proofster will replace `Admitted.` with the proof.)

Fig. 3. A Proofster screenshot of the developer asking to prove the theorem `list_forall2`.

Fig. 4. When Proofster executes the query from Figure 3, it is not able to automatically synthesize a proof for this theorem. Instead of a proof, Proofster displays the search tree for the developer to investigate (Figure 4). She sees that Proofster tried a few forms of induction on the input lists and gets an idea: perhaps inducting over terms of the relation between lists `list_forall2 a₁ b₁`, rather than over the lists directly, will result in a more informative inductive hypothesis. The developer returns to the query page and suggests a hint for Proofster: `induction 1`, which inducts over the first unnamed hypothesis (here,
the term of type $\text{lisl}_{\text{forall2}} a1 b1$), something $\text{Proofster}$ had failed to try. She then admits the rest and queries $\text{Proofster}$. Armed with this hint, $\text{Proofster}$ synthesizes the correct proof (Figure 5).

D. Evaluation Plan

We plan to evaluate $\text{Proofster}$ by soliciting feedback from developers, and by using it in a proof engineering graduate class. $\text{Proofster}$’s backends have been thoroughly evaluated on a benchmark of 68K Coq theorems from 122 open-source projects. ASTactic can fully automatically prove 12.3% of the theorems [54], Passport 12.7% [46], TacTok 12.9 [13], Proverbot9001 [45] 19.2%, and Diva 21.7% [12]. Together with CoqHammer, these tools can prove more than 33% of the theorems.

III. RELATED WORK

The $\text{Proofster}$ web interface provides an environment to interactively explore both the synthesized proof, and the synthesis search process. It uses the Alectryon [39] library to render literate Coq code, which is interactive and easy to read, even when one does not have immediate access to a proof assistant to step through the synthesized proof. jsCoq [15] and PeaCoq [44] also allow you to interact with formal proofs via web interfaces, but neither synthesize proofs. Tactician tactic-learning Coq plugin can be accessed through a web demonstration of two examples using jsCoq [5]. Section 7.1 of “QED at Large” [42] provides a thorough survey of user interfaces for formal proofs.

Automatically synthesizing proofs from scratch is a promising direction in easing formal verification [5], [10], [13], [12], [23], [26], [45], [46], [54]. For the Coq proof assistant, these methods have been able to prove as many as $\frac{1}{3}$ of the theorems [12] in a large benchmark of correctness properties of software systems [54]. However, these efforts have not yet directly addressed usability and adoption, which is $\text{Proofster}$’s goal. Such tools could potentially prove mathematical theorems [27] or nonfunctional software properties, such as privacy [9]. For software properties such as fairness [3], [7], [8], [14], [22] and safety [52], complementary approaches provide high-confidence, probabilistic guarantees based on statistical tests and confidence bounds [2], [16], [21], [31], [52].

Proof repair is an important open problem in formal verification [41], [43], which $\text{Proofster}$ may aid by allowing the user to provide hints based on information gained from failed proof-synthesis attempts. This problem is related to automated program repair, which aims to patch defects in systems [18], e.g., using tests and bug reports [33], [17] or inferred constraints on program behavior [1]. In automated program repair, a major challenge is that the tests used to validate the generated patches only partially describe the expected system behavior, and thus the patches can overfit to those tests, failing to correctly repair the program while appearing to do so [34], [48], [40]. Among other methods, extracting test oracles from natural language specifications [32] or using bug reports to help localize defects [33] can help.

IV. CONTRIBUTIONS AND FUTURE WORK

We have presented $\text{Proofster}$, a web-based tool aimed at assisting developers with the formal verification process via proof synthesis. $\text{Proofster}$ uses a proof synthesis backend to attempt to automatically generate Coq proofs for user-supplied theorems. The user can use $\text{Proofster}$ to explore the proof state at various stages of the synthesized proof, as well as the search tree generated during synthesis. When $\text{Proofster}$ fails to produce a proof, the user can provide hints as partial proofs, helping $\text{Proofster}$ try again. While our implementation currently works with a specific backend [45], its design is general and aims to work with any Coq proof-synthesis tool, e.g., Diva [12], among others. But reifying that ability is left to future work. Similarly, $\text{Proofster}$ works specifically with proofs for the Coq proof assistant, but, in theory, can be made to work with proof-synthesis tools for other proof assistants, e.g., Thor [26] for the Isabelle/HOL proof assistant [36], among others. Finally, while $\text{Proofster}$’s web-based interface makes it accessible to a broad set of users, we are currently building a version as a Coq plugin, that would integrate it into the IDEs more commonly used by proof engineers.

ACKNOWLEDGMENTS

This work is supported by the Defense Advanced Research Projects Agency under grant no. DARPA HR0011-22-9-0063, by the National Science Foundation under grant no. CCF-2210243, and by Amazon.

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