Coming up

- Quiz 2 this Thursday, in class
- User reports due April 30
- Today: quiz review and Reasoning About Programs

1

What'll be on quiz 2?

- Types of questions
  - Same as quiz 1
  - True False and Multiple Choice
- Topics
  - User Interfaces
  - Design Patterns
  - Testing
  - Debugging
  - Automated Program Repair and Verification
  - Ethics (video on class webpage)
  - Reasoning about Software

2

User Interfaces

- Which UI elements are appropriate when
- User-centered testing
- Paper prototypes

3

Design Paterns

- Creational design patterns
  - Singleton
  - Interning
  - ~~Flyweight~~
  - Factories

4

Testing

- Know about different kinds of tests
  - unit, integration, regression, etc.
- Know about different kinds of coverage
  - statement, path, etc.
- Know what's hard about testing
  - GUI, usability, covering all behavior, etc.

5

Debugging

- Know four kinds of defense against bugs
  - make impossible
  - don't introduce
  - make errors visible
  - last resort: debugging
- Representation (rep) invariants
- Assertions

6

Automated Program Repair & Verification

- Test-driven repair
- Overfitting
- ML-driven proof synthesis

7

Ethics in Software Engineering

- Focus on the lecture (video on class website)
- Examples of ethics in the workplace
- Focus on high-level ideas only

8

Reasoning about programs (today)

- Ways to verify your code
  - testing, reasoning, proving
- Forward reasoning
- Backward reasoning
- Loop invariants
- Induction
- Practice some examples!

9

Loop example

Find the weakest precondition

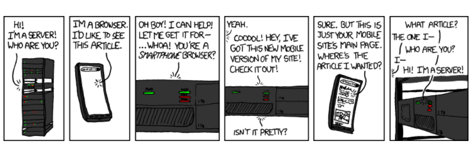
```
for (int x = 1; x <> y;) {
  if (y > x) {
    y = y / 2;
    x=2*x;
  }
}
```

// postcondition: `x=8`, `y=8`, and `x` and `y` are `int`s

you can also find the loop invariant and decrement function

10

Reasoning about programs



11

Ways to verify your code

- The hard way:
  - Make up some inputs
  - If it doesn't crash, ship it
  - When it fails in the field, attempt to debug
- The easier way:
  - Reason about possible behavior and desired outcomes
  - Construct simple tests that exercise that behavior
- Another way that can be easy
  - Prove that the system does what you want
    - Rep invariants are preserved
    - Implementation satisfies specification
  - Proof can be formal or informal (we will be informal)
  - Complementary to testing

12

Reasoning about code

- Determine what facts are true during execution
  - x > 0
  - for all nodes n: n.next.previous == n
  - array a is sorted
  - x + y == z
  - if x != null, then x.a > x.b
- Applications:
  - Ensure code is correct (via reasoning or testing)
  - Understand why code is incorrect

13

Forward reasoning

- You know what is true before running the code
  What is true after running the code?
- Given a precondition, what is the postcondition?
- Applications:
  Representation invariant holds before running code
  Does it still hold after running code?
- Example:

```
// precondition: x is even
x = x + 3;
y = 2x;
x = 5;
// postcondition:  ??
```

14

Backward reasoning

- You know what you want to be true after running the code
  What must be true beforehand in order to ensure that?
- Given a postcondition, what is the corresponding precondition?
- Applications:
  (Re-)establish rep invariant at method exit: what's required?
  Reproduce a bug: what must the input have been?
- Example:

```
// precondition:  ??
x = x + 3;
y = 2x;
x = 5;
// postcondition:  y > x
```

- How did you (informally) compute this?

15

Forward vs. backward reasoning

- Forward reasoning is more intuitive for most people
  - Helps understand what will happen (simulates the code)
  - Introduces facts that may be irrelevant to goal
    Set of current facts may get large
  - Takes longer to realize that the task is hopeless
- Backward reasoning is usually more helpful
  - Helps you understand what should happen
  - Given a specific goal, indicates how to achieve it
  - Given an error, gives a test case that exposes it

16

Forward reasoning example

```
assert x >= 0;
i = x;
    // x ≥ 0  &  i = x
z = 0;
    // x ≥ 0  &  i = x  &  z = 0
while (i != 0) {         ⇐ What property holds here?
  z = z + 1;
  i = i –1;              ⇐ What property holds here?
}
    // x ≥ 0  &  i = 0  &  z = x
assert x == z;
```

17

Backward reasoning

Technique for backward reasoning:

- Compute the weakest precondition (wp)
- There is a wp rule for each statement in the programming language
- Weakest precondition yields strongest specification for the computation (analogous to function specifications)

18

Assignment

// precondition: ??
x = e;
// postcondition: Q

Precondition: Q with all (free) occurrences of x replaced by e

- Example:
  // assert: ??
  x = x + 1;
  // assert x > 0

  Precondition = (x+1) > 0

19

Method calls

// precondition: ??
x = foo();
// postcondition: Q

- If the method has no side effects: just like ordinary assignment
- If it has side effects: an assignment to every variable it modifies

Use the method specification to determine the new value

20

If statements

// precondition: ??
if (b) S1 else S2
// postcondition: Q

Essentially case analysis:

wp("if (b) S1 else S2", Q) =
(b ⇒ wp("S1", Q)
∧ ¬ b ⇒ wp("S2", Q))

21

If: an example

```
// precondition: ??
if (x == 0) {
  x = x + 1;
} else {
  x = (x/x);
}
// postcondition: x ≥ 0
```

Precondition:
wp("if (x==0) {x = x+1} else {x = x/x}", x ≥ 0) =
= (x = 0 ⇒ wp("x = x+1", x ≥ 0)
& x ≠ 0 ⇒ wp("x = x/x", x ≥ 0))
= (x = 0 ⇒ x + 1 ≥ 0) & (x ≠ 0 ⇒ x/x ≥ 0)
= 1 ≥ 0 & 1 ≥ 0
= true

22

Reasoning About Loops

- A loop represents an unknown number of paths
  - Case analysis is problematic
  - Recursion presents the same issue
- Cannot enumerate all paths
  - That is what makes testing and reasoning hard

23

Loops: values and termination

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

1) Pre-assertion guarantees that x ≥ y
2) Every time through loop
   x ≥ y holds and, if body is entered, x > y
   y is incremented by 1
   x is unchanged
   Therefore, y is closer to x (but x ≥ y still holds)
3) Since there are only a finite number of integers between x and y, y will eventually equal x
4) Execution exits the loop as soon as x = y

24

Understanding loops by induction

- We just made an inductive argument
  - Inducting over the number of iterations
- Computation induction
  - Show that conjecture holds if zero iterations
  - Assume it holds after n iterations and show it holds after n+1
- There are two things to prove:
  - Some property is preserved (known as "partial correctness")
    - loop invariant is preserved by each iteration
  - The loop completes (known as "termination")
    - The "decrementing function" is reduced by each iteration

25

Loop invariant for the example

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

- So, what is a suitable invariant?
- What makes the loop work?

LI = $x \geq y$

1) $x \geq 0$ & $y = 0 \Rightarrow$ LI
2) LI & $x \neq y$ {`y = y+1;`} LI
3) (LI & $\neg(x \neq y)$) $\Rightarrow$ $x = y$

26

Is anything missing?

```
// assert x ≥ 0 & y = 0
while (x != y) {
    y = y + 1;
}
// assert x = y
```

Does the loop terminate?

27

Decrementing Function

- Decrementing function D(X)
  - Maps state (program variables) to some well-ordered set
  - This greatly simplifies reasoning about termination
- Consider: `while (b) S;`
- We seek D(X), where X is the state, such that
  1. An execution of the loop reduces the function's value: LI & b {S} $D(X_{post}) < D(X_{pre})$
  2. If the function's value is minimal, the loop terminates: (LI & D(X) = minVal) $\Rightarrow \neg$b

29

Proving Termination

```
// assert x ≥ 0 & y = 0
// Loop invariant: x ≥ y
// Loop decrements: (x-y)
while (x != y) {
    y = y + 1;
}
// assert x = y
```

- Is "x-y" a good decrementing function?

1. Does the loop reduce the decrementing function's value?
   // assert (y □ x); let $d_{pre} = (x - y)$
   y = y + 1;
   // assert $(x_{post} - y_{post}) < d_{pre}$
2. If the function has minimum value, does the loop exit?
   (x □ y & x – y = 0) □ (x = y)

30

Choosing Loop Invariant

- For straight-line code, the wp (weakest precondition) function gives us the appropriate property
- For loops, you have to guess:
  - The loop invariant
  - The decrementing function
- Then, use reasoning techniques to prove the goal property
- If the proof doesn't work:
  - Maybe you chose a bad invariant or decrementing function
    - Choose another and try again
  - Maybe the loop is incorrect
    - Fix the code
- Automatically choosing loop invariants is a research topic

31

In practice

I don't routinely write loop invariants

I do write them when I am unsure about a loop and when I have evidence that a loop is not working
- Add invariant and decrementing function if missing
- Write code to check them
- Understand why the code doesn't work
- Reason to ensure that no similar bugs remain

32

More on Induction

- Induction is a very powerful tool

$$2^n = 1 + \sum_{k=1}^{n} 2^{k-1}$$

Proof by induction: Base Case

For n=1, $1 + \sum_{k=1}^{1} 2^{k-1} = 1 + 2^0 = 1 + 1 = 2 = 2^1$

33

Inductive Step

Assume $2^m = 1 + \sum_{k=1}^{m} 2^{k-1}$ and show that $2^{m+1} = 1 + \sum_{k=1}^{m+1} 2^{k-1}$

$$2^{m+1} = 1 + \sum_{k=1}^{m+1} 2^{k-1} = 1 + \sum_{k=1}^{m} 2^{k-1} + 2^m = 2^m + 2^m = 2 \times 2^m = 2^{m+1}$$

34

Is Induction Too Powerful?

35