# Context-free Grammar 

## CS 690N, Spring 2017

Advanced Natural Language Processing http://people.cs.umass.edu/~brenocon/anlp2017/

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- Syntax: how do words structurally combine to form sentences and meaning?
- Representations
- Constituents
- [the big dogs] chase cats
- [colorless green clouds] chase cats
- Dependencies
- The dog chased the cat.
- My dog, a big old one, chased the cat.
- Idea of a grammar (G): global template for how sentences / utterances / phrases $w$ are formed, via latent syntactic structure $y$
- Linguistics: what do $G$ and $P(w, y \mid G)$ look like?
- Generation: score with, or sample from, $\mathrm{P}(\mathrm{w}, \mathrm{y} \mid \mathrm{G})$
- Parsing: predict $\mathrm{P}(\mathrm{y} \mid \mathrm{w}, \mathrm{G})$


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- Competence vs. Performance?


## Hierarchical view of syntax

- "a Sentence made of Noun Phrase followed by a Verb Phrase"



## Is language context-free?

- Practical examples where nesting seems like a useful explanation
- The processor has 10 million times fewer transistors on it than todays typical micro- processors, runs much more slowly, and operates at five times the voltage...
- $\quad S \rightarrow N N V P$

$$
\begin{aligned}
V P & \rightarrow V P 3 S|V P N 3 S| \ldots \\
V P 3 S & \rightarrow V P 3 S, V P 3 S \text {, and } V P 3 S|V B Z| V B Z ~ N P \mid \ldots
\end{aligned}
$$

- Regular language $<=>$ RegEx $<=>$ paths in finite state machine
- Context-free language <=> CFG <=> derivations in pushdown automaton
- A context-free grammar is a 4-tuple:

```
N a set of non-terminals
\Sigma a set of terminals (distinct from N)
R a set of productions, each of the form }A->\beta\mathrm{ ,
    where }A\inN\mathrm{ and }\beta\in(\Sigma\cupN)
S a designated start symbol
```

- Derivation: sequence of rewrite steps from $S$ to a string (sequence of terminals, i.e. words)
- Yield: the final string
- A CFG is a "boolean language model"
- A probabilistic CFG is a probabilistic language model:
- Every production rule has a probability; defines prob dist. over strings.


## Example



( $\mathrm{NP}^{\left(\mathrm{NN}^{2}\right.}$ sushi $)$ )
$(\operatorname{pP}(\operatorname{In}$ with $)(\mathrm{NP}($ Nns chopsticks $)))))$

- All useful grammars are ambiguous: multiple derivations with same yield - [Parse tree representations: Nested parens or non-terminal spans]


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( $\mathrm{s}\left(\mathrm{NP}(\mathrm{Prp} S h e)\left(\mathrm{vp}\left(\mathrm{Vbz}_{\mathrm{bz}}\right.\right.\right.$ eats $)$
$\left(\operatorname{pr}(\operatorname{In}\right.$ with $)\left({ }_{\mathrm{NP}}(\mathrm{Nns}\right.$ chopsticks $\left.\left.\left.\left.)\right)\right)\right)\right)$
$\left(\mathrm{NP}\left(\mathrm{NP}^{(\mathrm{Nn}}\right.\right.$ sushi $\left.)\right)(\mathrm{PP}(\mathrm{In}$ with $)(\mathrm{NP}(\mathrm{Nns}$ chopsticks $\left.\left.\left.)))\right)\right)\right)$

- All useful grammars are ambiguous: multiple derivations with same yield - [Parse tree representations: Nested parens or non-terminal spans]
[Examples from Eisenstein (2017)]


## Constituents

- Constituent tree/parse is one representation of sentence's syntax. What should be considered a constituent, or constituents of the same category?
- Substitution tests
- Pronoun substitution
- Coordination tests
- Simple grammar of English
- Must balance overgeneration versus undergeneration
- Noun phrases
- NP modification: adjectives, PPs
- Verb phrases
- Coordination...


## Parsing with a CFG

- Task: given text and a CFG, answer:
- Does there exist at least one parse?
- Enumerate parses (backpointers)
- Cocke-Kasami-Younger algorithm
- Bottom-up dynamic programming:

Find possible nonterminals for short spans of sentence, then possible combinations for higher spans

- Requires converting CFG to Chomsky Normal Form (a.k.a. binarization)


## CKY



For cell [i,j] (loop through them bottom-up) For possible splitpoint $\mathrm{k}=(\mathrm{i}+1) . .(\mathrm{j}-\mathrm{l})$ : For every B in $[\mathrm{i}, \mathrm{k}]$ and C in $[\mathrm{k}, \mathrm{j}]$, If exists rule A -> B C, add $A$ to cell $[i, j] \quad$ Recognizer) ... or ... add (A,B,C, k) to cell [i,j] (Parser)

Recognizer: per span, record list of possible nonterminals

Parser: per span, record possible ways the nonterminal was constructed.

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| Adj -> yummy |
| NP -> foods |
| NP -> store |
| NP -> NP NP |
| NP -> Adj NP |



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