Review I

CMPSCI 383
December 6, 2011

General Information about the Final

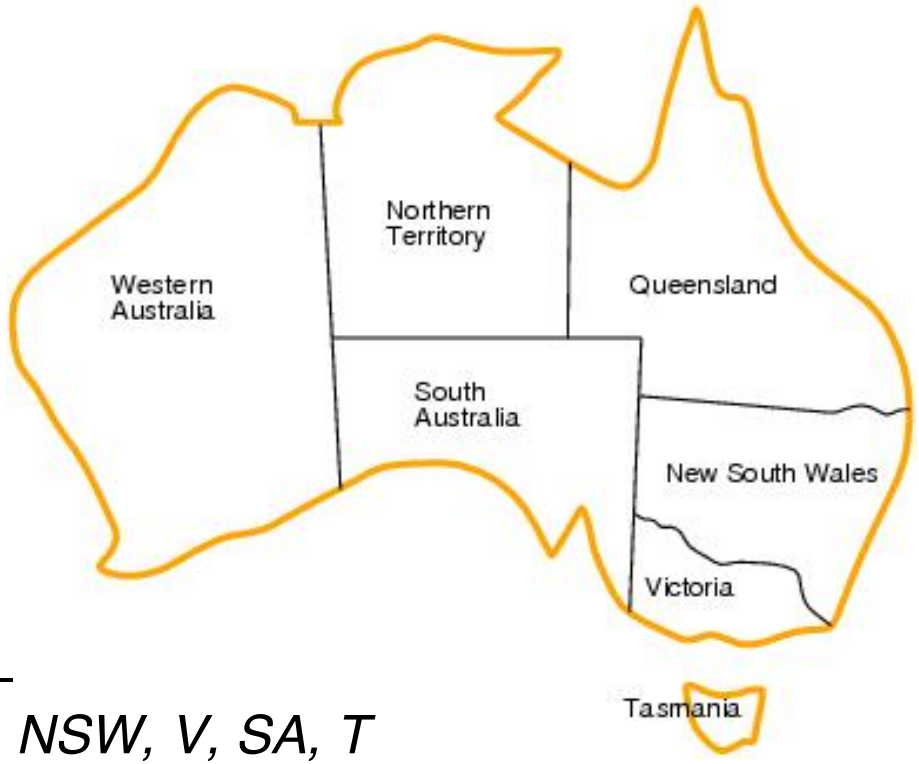
- Closed book closed notes
- Includes midterm material too
- But expect more emphasis on later material

What you should know

Chapter 6: Constraint Satisfaction Problems

- Representations: atomic, factored, structured
- Definition of a constraint satisfaction problem:
  - In CSPs, states are defined by assignments of values to a set of variables $X_1 ... X_n$. Each variable X_i has a domain D_i of possible values.
  - States are evaluated based on their consistency with a set of constraints $C_1 ... C_m$ over the values of the variables.
  - A goal state is a complete assignment to all variables that satisfies all the constraints.

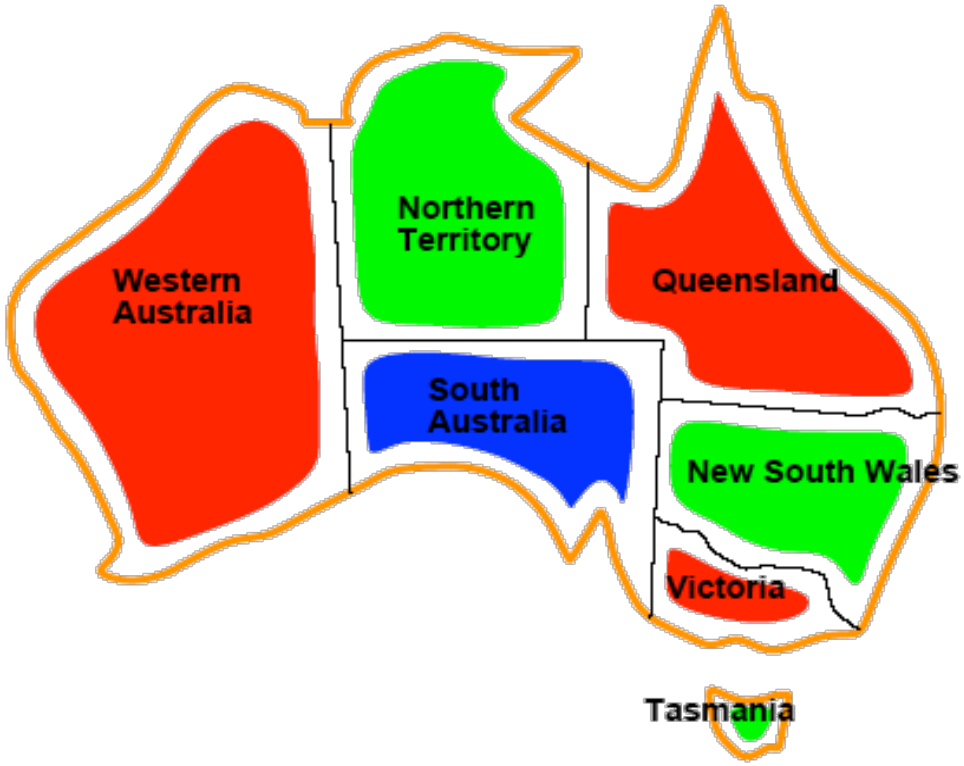
Example: Map coloring



- Variables — *WA, NT, Q, NSW, V, SA, T*
- Domains — $D_i = \{red,green,blue\}$
- Constraints — adjacent regions must have different colors.
  - E.g. $WA \neq NT$ (if the language allows this)
  - E.g. ((*WA,NT*), [(*red,green),(red,blue),(green,red),…*])

Allowable combinations of variables

Example: Map coloring

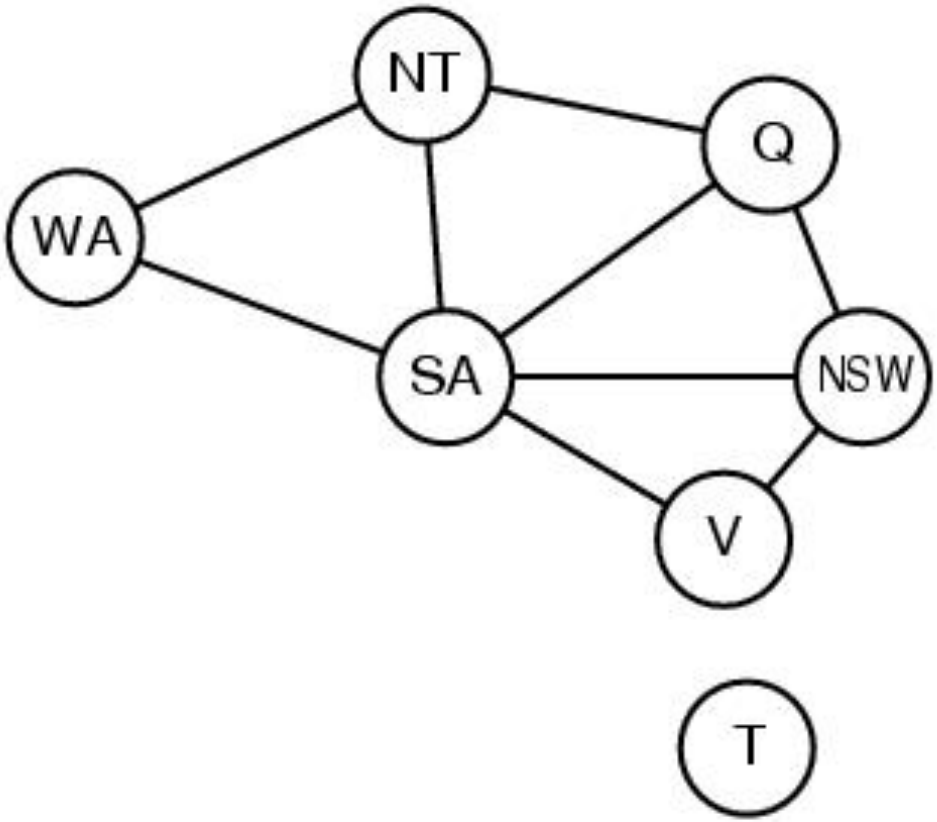


- Solutions are **complete** and **consistent** assignments:
  every variable assigned, all assignments legal, e.g.:
  {WA=red,NT=green,Q=red,NSW=green,V=red,SA=blue,T=green}

Types of Constraints

- Unary constraint: concerns only a single value; e.g., SA ≠ green
- Binary constraint: concerns the relative values of two variables
- Global constraint: concerns an arbitrary number of variables, e.g., *Alldiff*

Constraint graph



Local Consistency

- Node Consistency: Xi is node-consistent if every value in the domain Di satisfies all of Xi's unary constraints.
  - A network is node-consistent if every variable is node-consistent
- Arc Consistency: Xi is arc-consistent with respect to Xj if for every value in the domain Di there is some value in Dj that satisfies the binary constraint on arc (Xi,Xj)
  - A network is arc-consistent if every variable is arc-consistent with every other variable

Arc Consistency (slightly different from the book)

```
function AC-3( csp) returns the CSP, possibly with reduced domains
  inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_n}
  local variables: queue, a queue of arcs, initially all the arcs in csp

  while queue is not empty do
    (X_i, X_j) ← REMOVE-FIRST(queue)
    if REMOVE-INCONSISTENT-VALUES(X_i, X_j) then
      for each X_k in NEIGHBORS[X_i] do
        add (X_k, X_i) to queue
```

```
function REMOVE-INCONSISTENT-VALUES( X_i, X_j) returns true iff succeeds
  removed ← false
  for each x in DOMAIN[X_i] do
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint X_i ↔ X_j
      then delete x from DOMAIN[X_i]; removed ← true
  return removed
```

Naive Search Formulation

Let's start with the straightforward approach, then fix it

States are defined by the values assigned so far

- Initial state: the empty assignment { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment
  → fail if no legal assignments

- Goal test: the current assignment is complete

1. This is the same for all CSPs
2. Every solution appears at depth *n* with *n* variables
   → use depth-first search
3. Path is irrelevant, so can also use complete-state formulation
4. b = (n - ℓ)d at depth ℓ, hence n! · d^n leaves (d is domain size)

Backtracking Search

- Variable assignments are commutative, i.e.,

[WA = red then NT = green] same as [NT = green then WA = red]

- Only need to consider assignments to a single variable at each node

  → b = d and there are d^n leaves

- Depth-first search for CSPs with single-variable assignments is called backtracking search

- Backtracking search is the basic uninformed algorithm for CSPs

Simple backtracking search

- Depth-first search
- Choose values for one variable at a time
- Backtrack when a variable has no legal values left to assign.
- If search is uninformed, then general performance is relatively poor

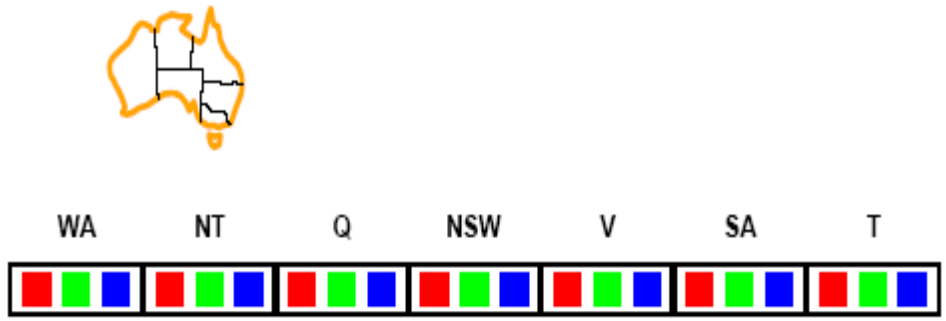
Improving backtracking efficiency

- Approaches
  - Minimum remaining values heuristic (MRV)
    - Select the *most constrained* variable (the variable with the smallest number of remaining values)
  - Degree heuristic
    - Select the variable that is involved in the largest number of constraints with other unassigned variables: The most constraining variable.
  - Least-constraining value heuristic
    - Given a variable, choose the *least constraining value* — the value that leaves the maximum flexibility for subsequent variable assignments.

Combining Search with Inference

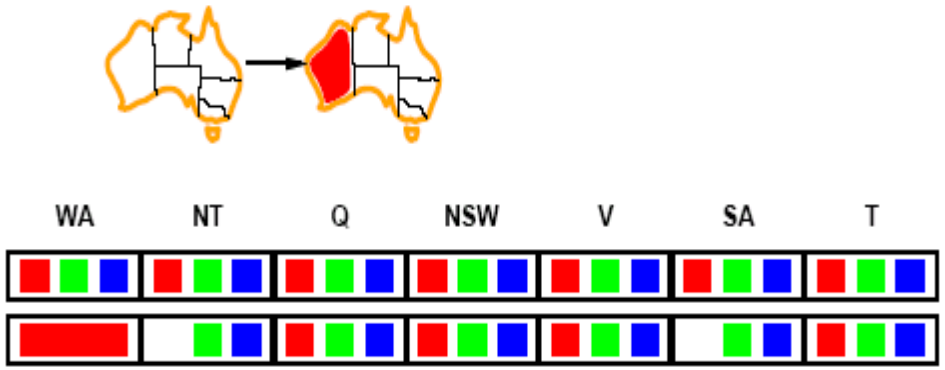
- Forward checking
  - Precomputing information needed by MRV
  - Early stopping
- Constraint propagation
  - Arc consistency (2-consistency)

Forward checking



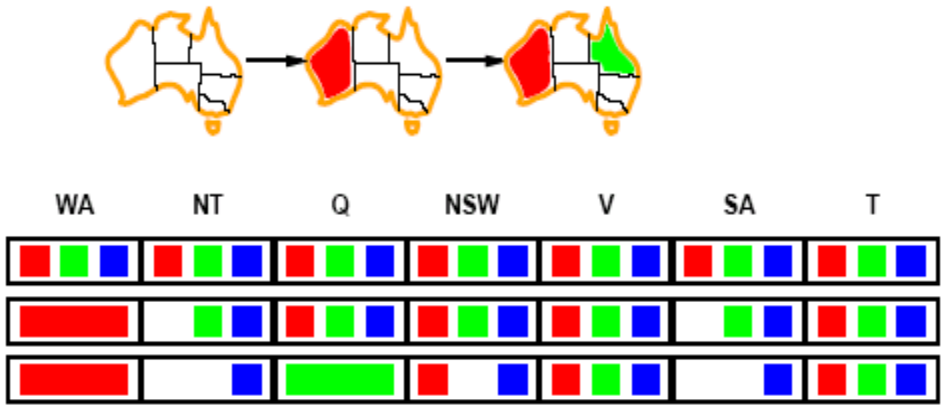
- Can we detect inevitable failure early?
  - *And avoid it later?*
- Yes — track remaining legal values for unassigned variables
- Terminate search when any variable has no legal values.

Forward checking



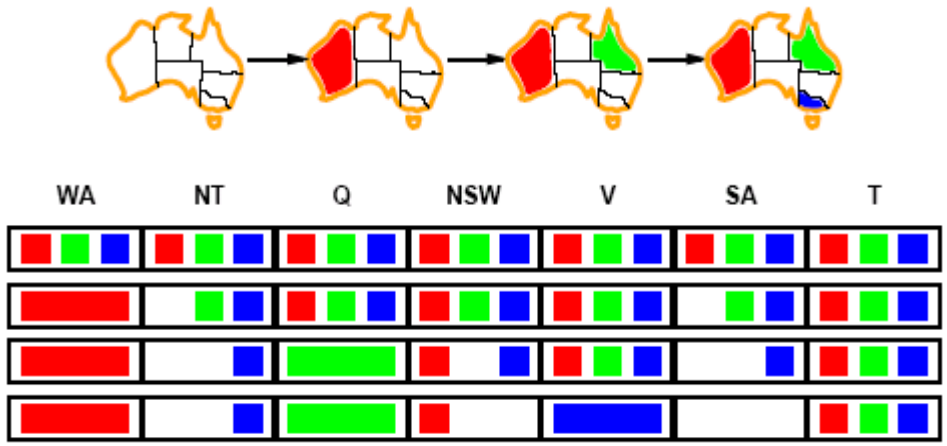
- Assign *{WA=red}*
- Effects on other variables connected by constraints with WA
  - *NT can no longer be red*
  - *SA can no longer be red*

Forward checking



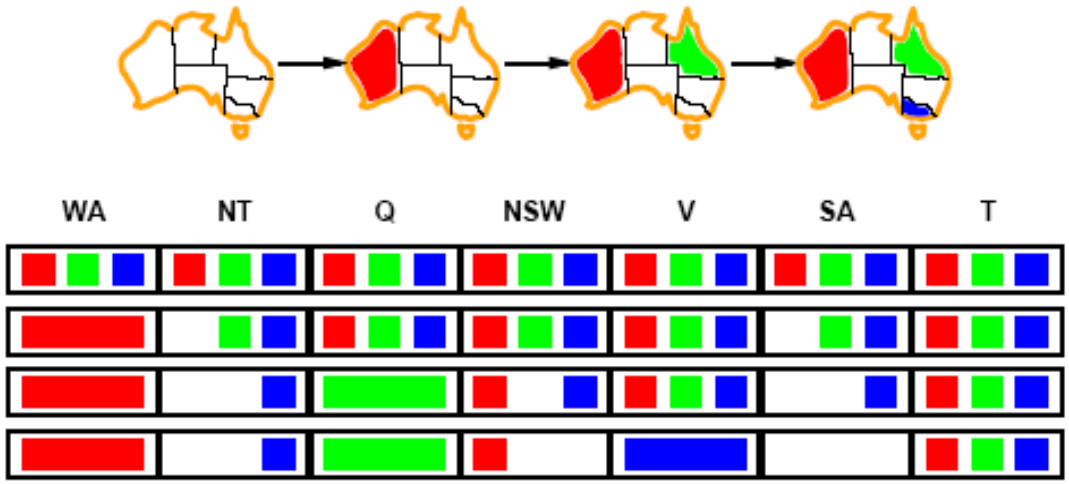
- Assign *{Q=green}*
- Effects on other variables connected by constraints with WA
  - *NT can no longer be green*
  - *NSW can no longer be green*
  - *SA can no longer be green*

Forward checking



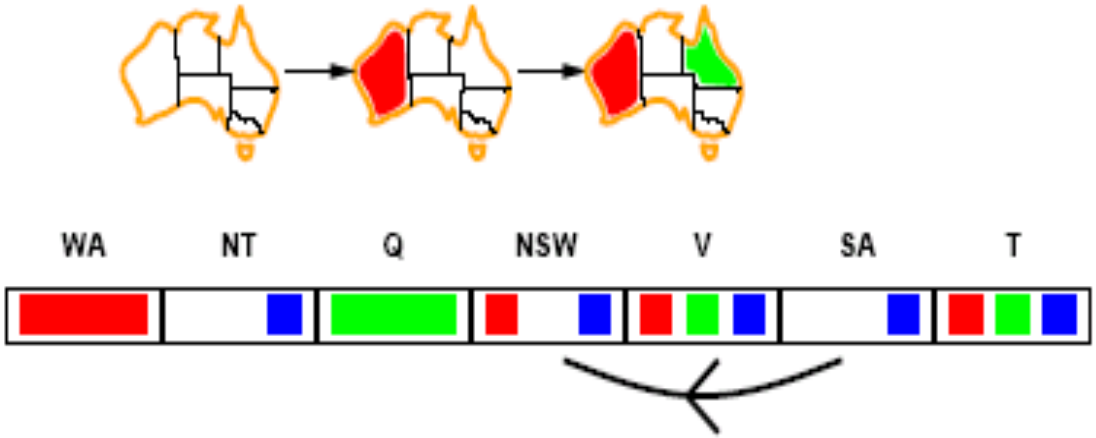
- If *V* is assigned *blue*
- Effects on other variables connected to WA
  - *SA is empty*
  - *NSW can no longer be blue*
- FC has detected a partial assignment that is *inconsistent* with the constraints.

Forward checking



- Solving CSPs with combination of heuristics plus forward checking is more efficient than either approach alone.
- FC checking propagates information from assigned to unassigned variables but does not provide detection for all failures.
  - NT and SA cannot be blue!
- Makes each current variable assignment arc consistent, but does not look far enough ahead to detect all inconsistencies (as AC-3 would)

Arc consistency

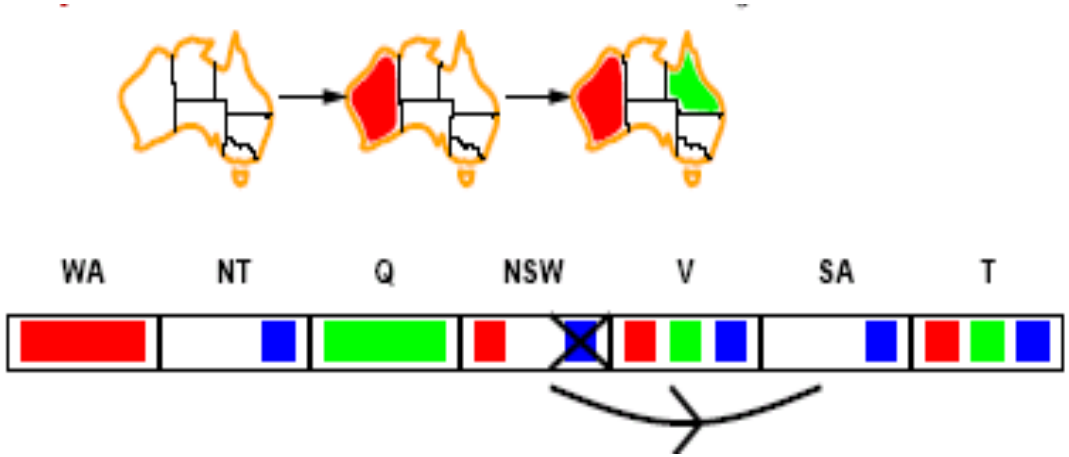


- *X* → *Y* is consistent iff

  for *every* value *x* of *X* there is some allowed *y*
- *SA* → *NSW* is consistent iff

  SA=blue and *NSW=red*

Arc consistency



- *X* → *Y* is consistent iff

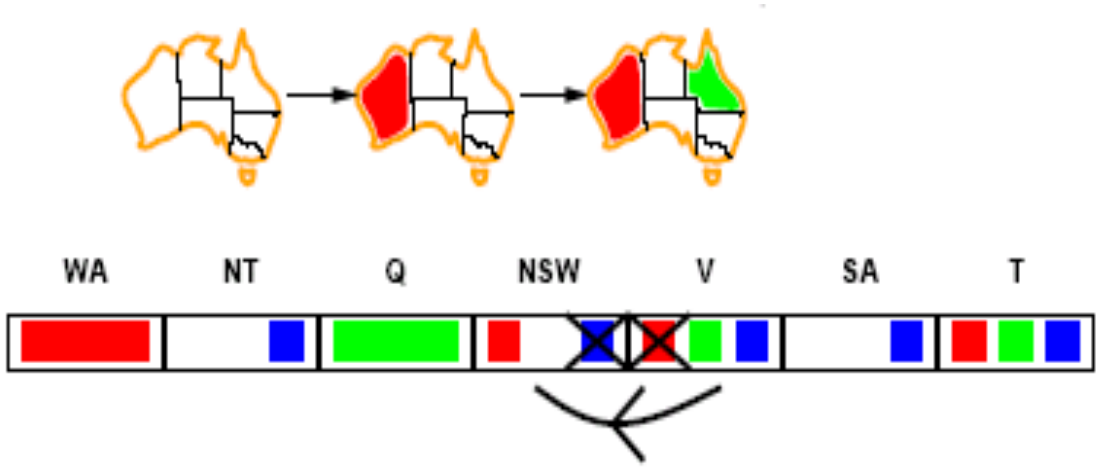
  for *every* value *x* of *X* there is some allowed *y*
- *NSW* → *SA* is consistent iff

  NSW=red and *SA=blue*

  NSW=blue and SA=???

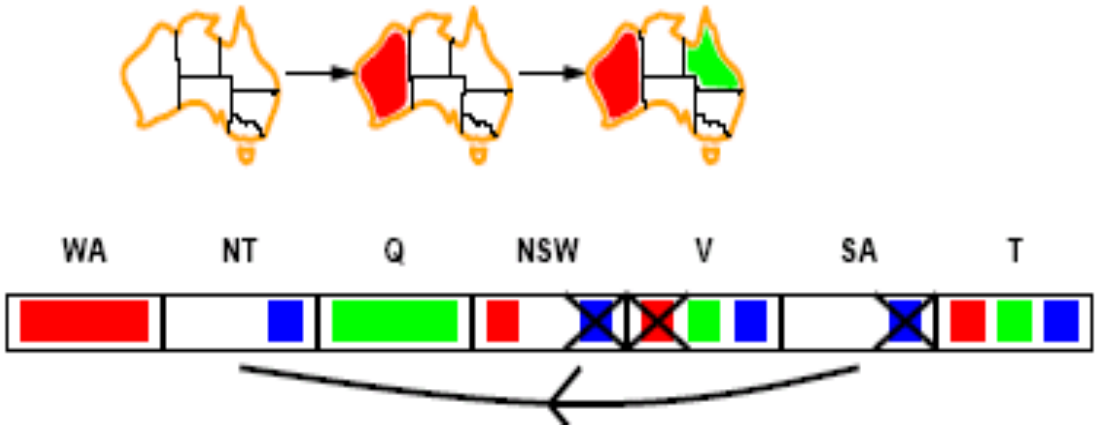
Arc can be made consistent by removing *blue* from *NSW*

Arc consistency



- Arc can be made consistent by removing *blue* from *NSW*
- Recheck *neighbours*
  - Remove red from *V*

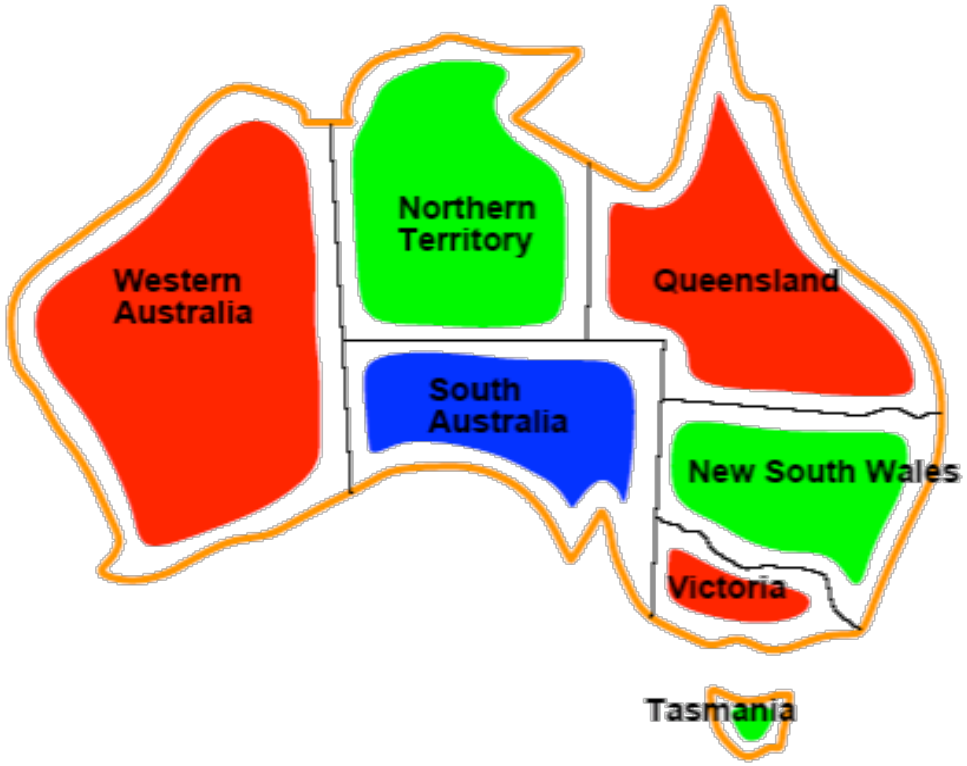
Arc consistency



- Arc can be made consistent by removing *blue* from *NSW*
- Recheck *neighbours*
  - Remove red from *V*
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment.
  - Repeated until no inconsistency remains

Local search for csp

- Do we need the path to the solution or only the solution itself?
- Can we apply local search methods?
  - Hillclimbing
  - Simulated annealing
  - Genetic algorithms
- What's a state?



Min-conflicts heuristic for local search

- To enable local search
  - allow states with unsatisfied constraints
  - operators reassign variable values
- Variable selection: randomly select any conflicted variable
- Value selection by min-conflicts heuristic
  - choose value that violates the fewest constraints
  - i.e., hill-climb with *h(n)* = total number of violated constraints

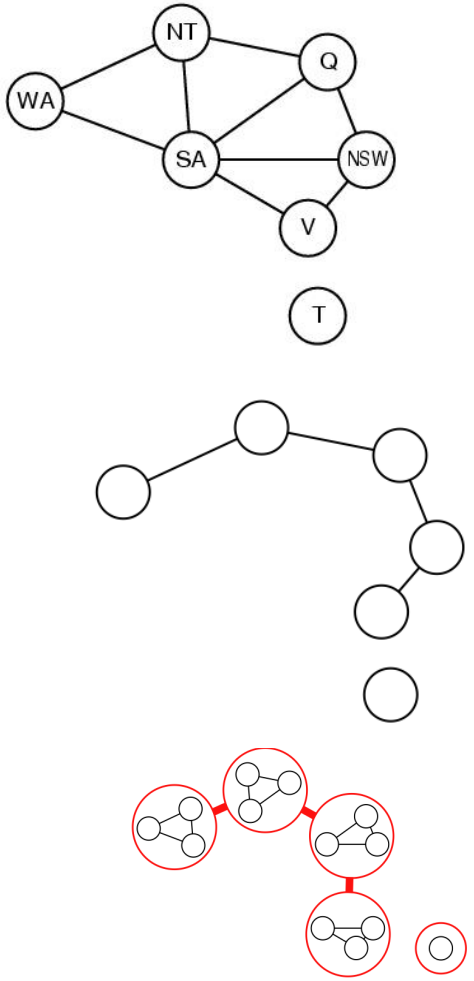
Example: 4-Queens

- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Actions: move queen in column
- Goal test: no attacks; $h(n) = 0$
- Evaluation: $h(n)$ = number of attacks

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000). Average of 50 steps for n = 1M.

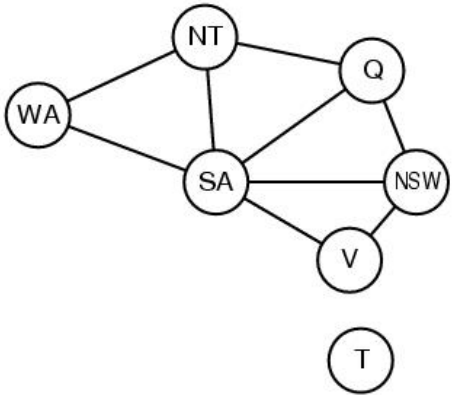
Exploiting the structure of CSPs

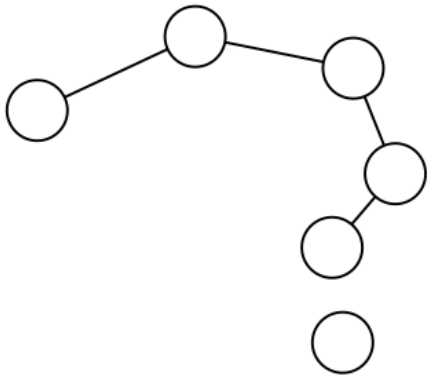
- Decompose into independent problems
- Tree-structured CSPs can be solved in linear time
- Reduce problems to tree-structured CSPs
  - Cycle cutset conditioning — Remove nodes to create trees
  - Tree decomposition — Decompose problem into a tree-structured set of subproblems



Cycle cutset conditioning

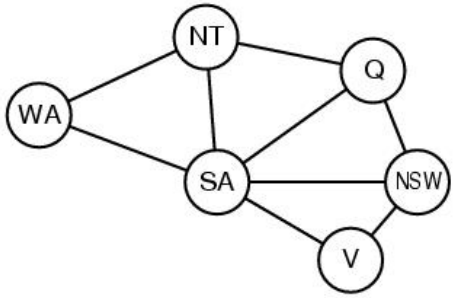
- Want to create a tree
  - What is a tree?
  - Why do we want to create one?
  - *Tree-structured CSPs solvable in linear time*
- Create a tree by deleting nodes
  - How can you delete nodes in CSPs?
  - *Set value and restrict domains*
- Does this always work well?
  - No, what can we do about that?
  - *Step through possible settings*
- What's the payoff?
  - *Big efficiency gains*



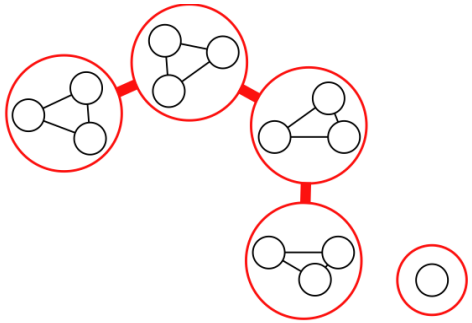


Tree decomposition

- Again, want to create a tree
  - What's another way of creating a tree?
  - *Merging nodes*



- *Don't need to memorize the following:*
  - Rules for doing this:
    - *Every variable in ≥1 subproblems*
    - *All connected variable pairs, and assoc. constraints, in ≥1 subproblems*
    - *If a variable appears in 2 subproblems, it must appear in all subproblems on the path connecting the two subproblems*



- Now, how can we solve this new problem?

Things you should know about…

- Basic form of a CSP
- Be able to formulate a problem as a CSP
- Types of constraints
- Consistent assignment
- Complete assignment
- Constraint graph
- Constraint propagation
- Backtracking search for CSPs
- Heuristics to improve backtracking search
  - MRV
  - Degree heuristic
  - Least-constraining value
- Interleaving search and inference
  - Forward checking
  - Arc consistency
- Local search
  - Complete state formulation
  - Min-conflicts heuristic
- Using problem structure
  - Decomposing into independent subproblems
  - Turn into a tree structured problem
    - Basic knowledge of:
      - Cutset conditioning
      - Tree decomposition

What you don't need to know for the exam

- Continuous domains
- Bounds propagation
- Bounds consistent
- MAC
- Details of Intelligent backtracking
- Constraint weighting
- Directed arc consistency
- Value symmetry
- Symmetry-breaking constraint
- Details of cycle cutset conditioning
- Details of tree decomposition

Chapter 13: Quantifying Uncertainty

- Sources of uncertainty
- MEU principle
- Basics of probability theory
  - Sample point/atomic event/possible world
  - Random variables
  - Joint and conditional distributions
  - Independence: absolute and conditional
  - Bayes Rule

Conditional probability

- Definition of conditional probability:
  $P(a \mid b) = P(a \wedge b) / P(b)$ if $P(b) > 0$

- Product rule gives an alternative formulation:
  $P(a \wedge b) = P(a \mid b)\, P(b) = P(b \mid a)\, P(a)$

- Chain rule is derived by successive application of product rule:

$$\begin{aligned}
\mathbf{P}(X_1, \ldots, X_n) &= \mathbf{P}(X_1,\ldots,X_{n-1})\ \mathbf{P}(X_n \mid X_1,\ldots,X_{n-1}) \\
&= \mathbf{P}(X_1,\ldots,X_{n-2})\ \mathbf{P}(X_{n-1} \mid X_1,\ldots,X_{n-2})\ \mathbf{P}(X_n \mid X_1,\ldots,X_{n-1}) \\
&= \ldots \\
&= \prod_{i=1}^{n} \mathbf{P}(X_i \mid X_1, \ldots, X_{i-1})
\end{aligned}$$

Independence

A and *B* **independent** (absolute, marginal) iff
$\mathbf{P}(A \mid B) = \mathbf{P}(A)$ or $\mathbf{P}(B \mid A) = \mathbf{P}(B)$ or $\mathbf{P}(A, B) = \mathbf{P}(A)\,\mathbf{P}(B)$

A and *B* **conditionally independent given C** iff
$\mathbf{P}(A \mid B,C) = \mathbf{P}(A \mid C)$ or $\mathbf{P}(B \mid A,C) = \mathbf{P}(B \mid C)$
or $\mathbf{P}(A, B \mid C) = \mathbf{P}(A \mid C)\,\mathbf{P}(B \mid C)$

Bayes Rule

- Product rule $P(a \wedge b) = P(a \mid b)\, P(b) = P(b \mid a)\, P(a)$

  $\Rightarrow$ Bayes' rule: $P(a \mid b) = P(b \mid a)\, P(a) / P(b)$

- or in distribution form

  $$\mathbf{P}(Y|X) = \mathbf{P}(X|Y)\, \mathbf{P}(Y) / \mathbf{P}(X) = \alpha \mathbf{P}(X|Y)\, \mathbf{P}(Y)$$

- Useful for assessing diagnostic probability from causal probability:
  - P(Cause|Effect) = P(Effect|Cause) P(Cause) / P(Effect)
  - E.g., let *M* be meningitis, *S* be stiff neck:

    $P(m|s) = P(s|m)\, P(m) / P(s) = 0.8 \times 0.0001 / 0.1 = 0.0008$
  - Note: posterior probability of meningitis still very small!

Specifically (but not limited to) ----

- Be able to compute various probabilities, and probability distributions, from the full joint dist. (cf. problem 13.8 p. 507)
- Applying Bayes rule
  - E.g., diagnosis: p(disease I symptoms) cf. problem 13.15 p. 508
- Equivalent statements for conditional independence: cf. problem 13.17, p. 508

What you don't need to know for the exam

- Wumpus world

Chapter 14: Probabilistic Reasoning

- Bayesian networks
- Exact Inference in Bayesian networks
- Approximate Inference in Bayesian networks

Bayesian networks

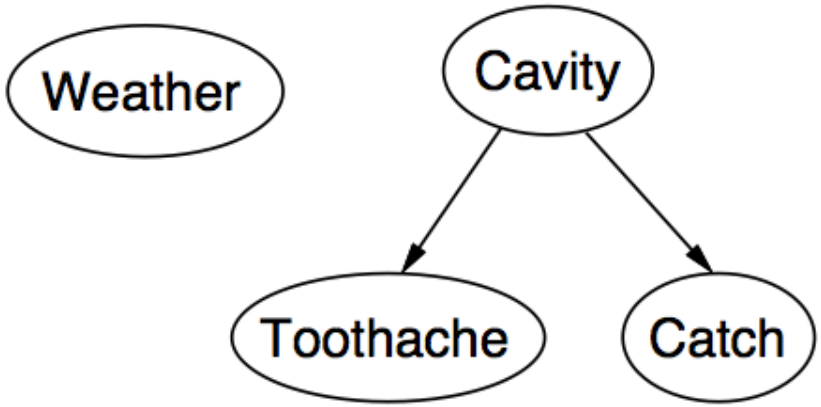
- A simple, graphical notation for conditional independence assertions and hence for compact specification of joint distributions
- Syntax:
  - a set of nodes, one per variable
  - a directed, acyclic graph (link ≈ "directly influences")
  - a conditional distribution for each node given its parents:

$$\mathbf{P}\,(X_i \mid \text{Parents}\,(X_i))$$

- In the simplest case, conditional distribution represented as a conditional probability table (CPT) giving the distribution over X_i for each combination of parent values

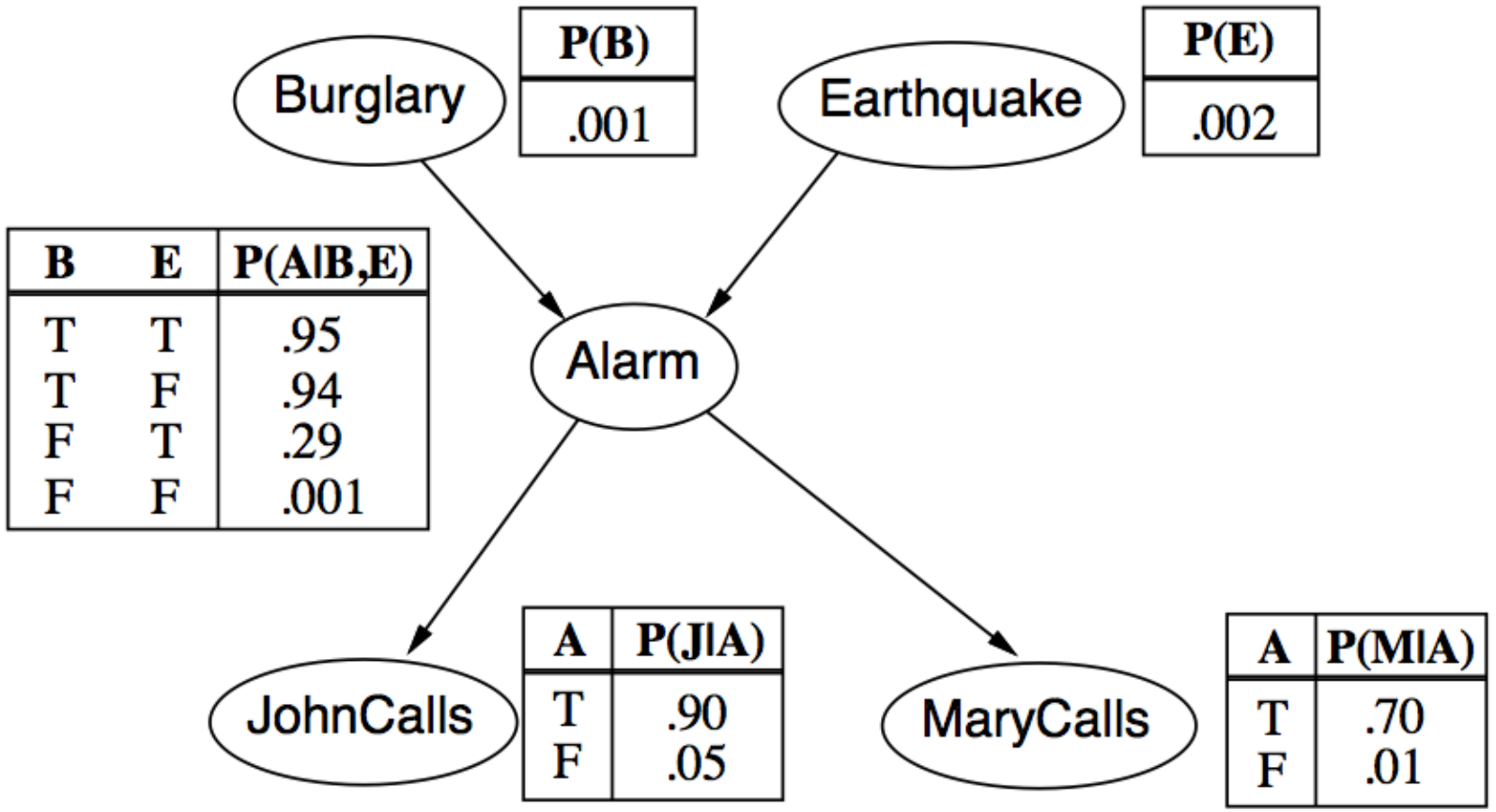
Example

- Topology of network encodes conditional independence assertions:

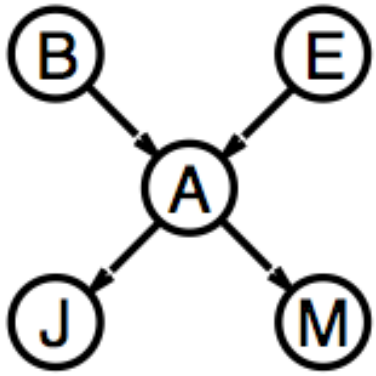


- *Weather* is independent of the other variables
- *Toothache* and *Catch* are conditionally independent given *Cavity*

Example: Home security



Benefits: Compactness



- A CPT for Boolean X_i with k Boolean parents has 2^k rows for the combinations of parent values
- Each row requires one number p for $X_i = true$ (the number for $X_i = false$ is just $1-p$)
- If each variable has no more than k parents, the complete network requires $O(n \cdot 2^k)$ numbers
- i.e., grows linearly with n, vs. $O(2^n)$ for the full joint distribution
- For burglary net, 1 + 1 + 4 + 2 + 2 = 10 numbers (vs. $2^5-1 = 31$)

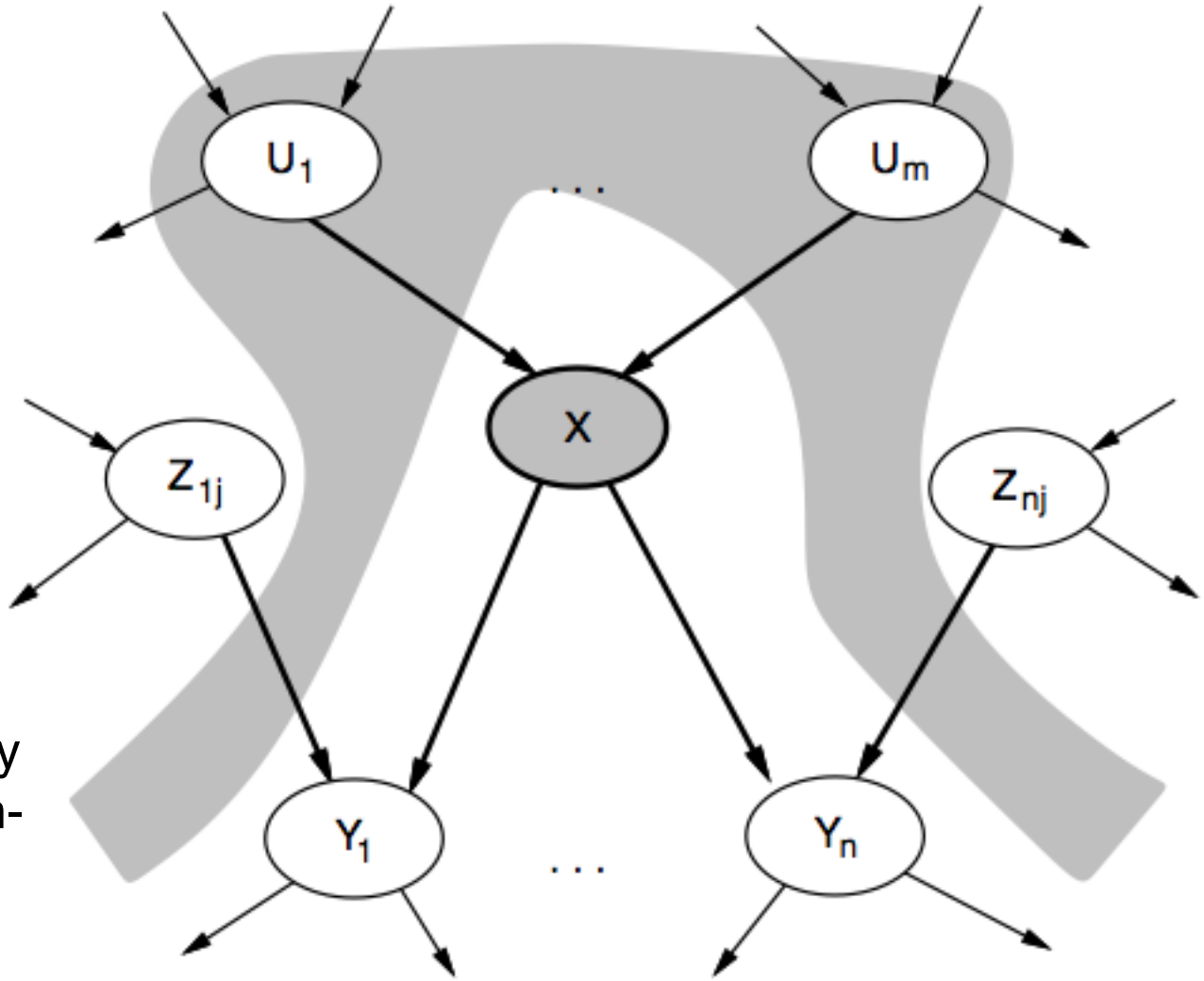
Semantics

- The full joint distribution is defined as the product of the local conditional distributions:

$$\mathbf{P}(X_1, \ldots, X_n) = \pi_{i=1}^{n} \mathbf{P}(X_i \mid Parents(X_i))$$

- Example
  $\mathbf{P}(j \wedge m \wedge a \wedge \neg b \wedge \neg e)$
  $= \mathbf{P}(j|a)\ \mathbf{P}(m|a)\ \mathbf{P}(a|\neg b, \neg e)\ \mathbf{P}(\neg b)\ \mathbf{P}(\neg e)$

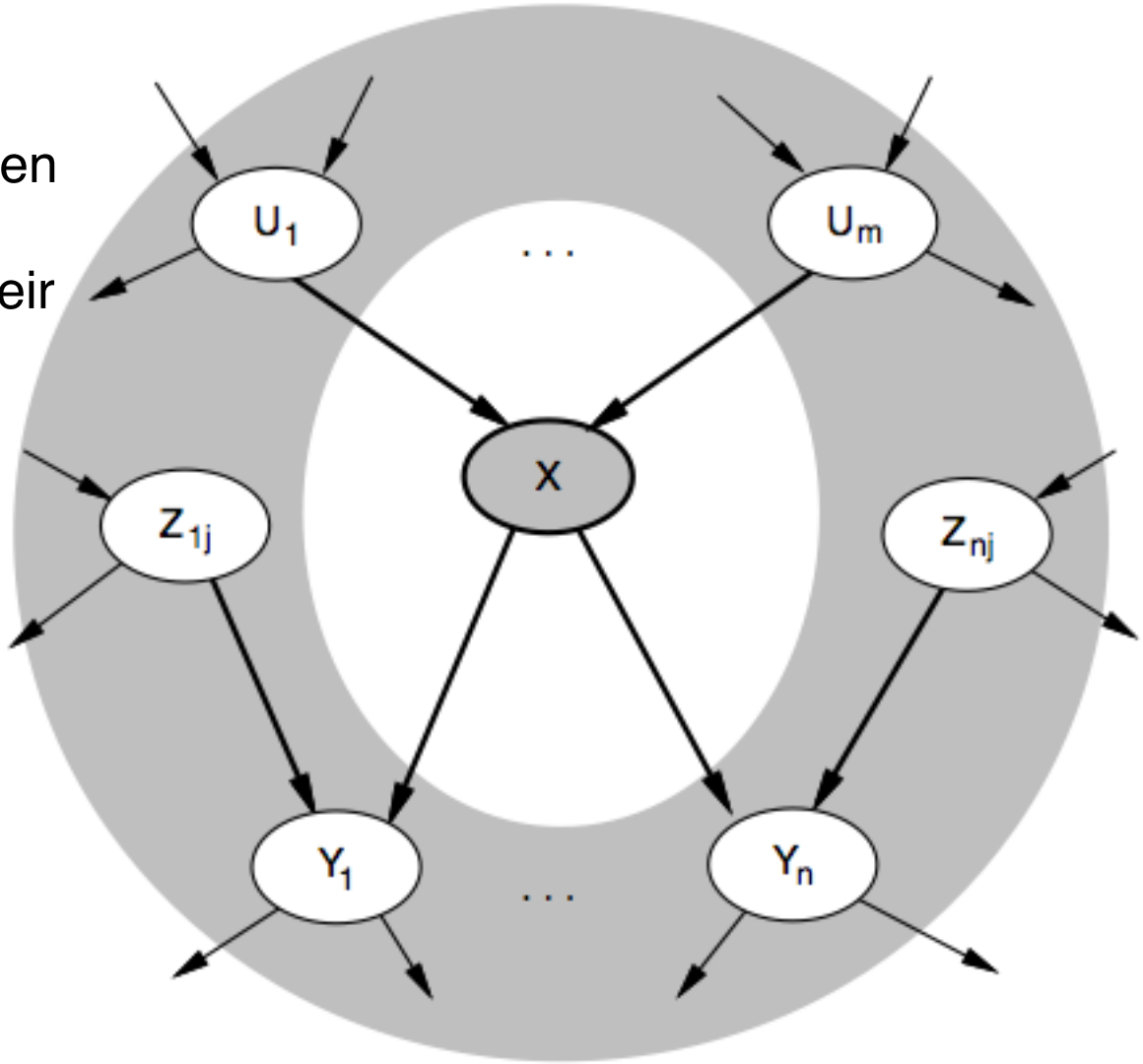
Conditional Independence



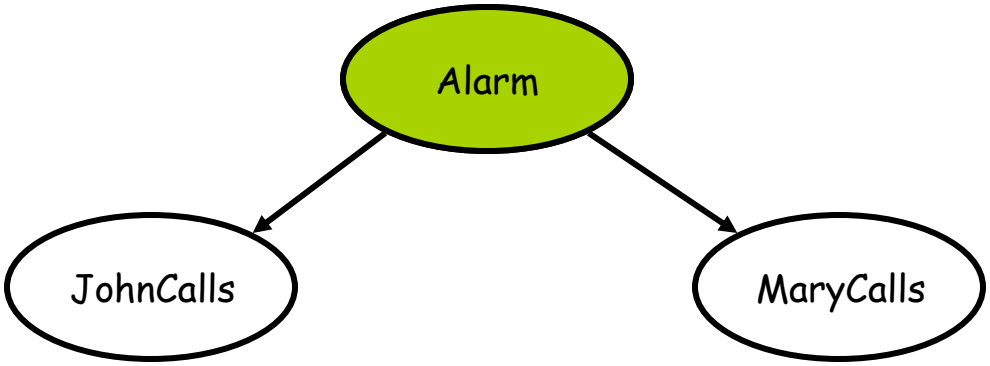
Node X is conditionally independent of its non-descendants given its parents.

Conditional Independence

Node X is conditionally independent of all other nodes in the network given its “Markov blanket” (its parents, children, and their parents).

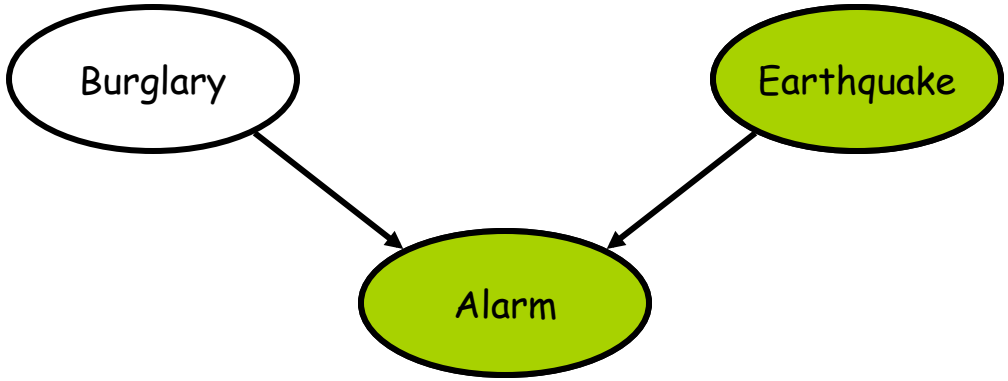


Conditional independence



- Are *JohnCalls* and *MaryCalls* independent?
  - No, they are not completely independent
- If the value of *Alarm* is known, are *JohnCalls* and *MaryCalls* independent?
  - Yes, for each known value of *A, J* and *M* are independent

Conditional independence



- Are *Burglary* and *Earthquake* cond. independent?
  - Yes, nodes are conditionally independent of their non-descendents given their parents
- Are they completely independent?
  - No, one can 'explain away' the other if *Alarm* is known.

Constructing Bayesian networks

- 1. Choose an ordering of variables $X_1, \ldots, X_n$
- 2. For $i = 1$ to n
  - add X_i to the network
  - select parents from $X_1, \ldots, X_{i-1}$ such that

$$\mathbf{P}(X_i \mid \text{Parents}(X_i)) = \mathbf{P}(X_i \mid X_1, \ldots X_{i-1})$$

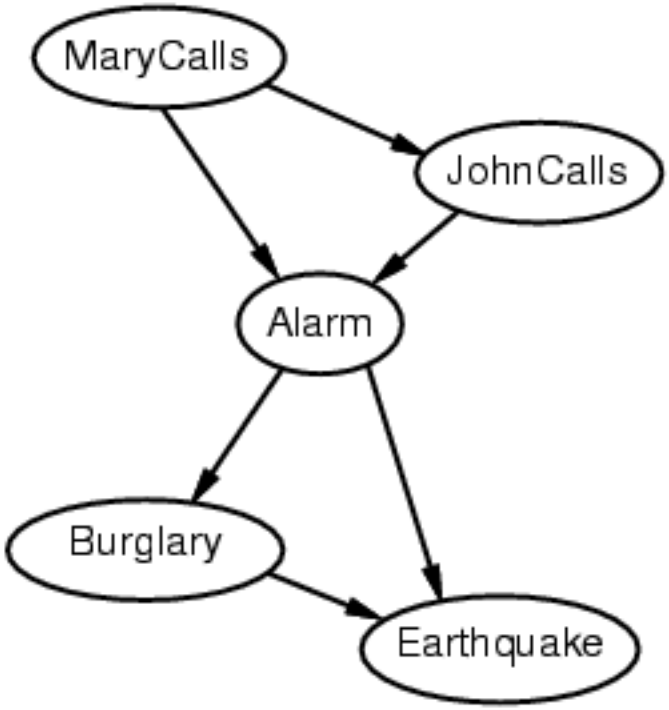
This choice of parents guarantees:

$$\mathbf{P}(X_1, \ldots, X_n) = \pi_{i=1}^{n} \mathbf{P}(X_i \mid X_1, \ldots, X_{i-1}) \quad \text{(chain rule)}$$

$$= \pi_{i=1}^{n} \mathbf{P}(X_i \mid \text{Parents}(X_i)) \quad \text{(by constr.)}$$

Example

- Suppose we choose the ordering M, J, A, B, E



Summary

- Bayesian network:
  - Directed acyclic graph whose nodes correspond to r.v.s; each note has a conditional distribution for its values given its parents.
  - Provides a concise way to represent conditional independence relations
  - Specifies the full joint distribution
  - Often exponentially smaller than explicit representation of the joint distribution

What you don't need to know for the exam

- Noisy-OR, noisy-MAX, leak node
- Bayes nets with continuous variables

Inference in Bayesian networks

- Exact
  - Inference with joint probability distributions
  - Exact inference in Bayesian networks
  - Inference by enumeration
  - Complexity of exact inference
- Approximate
  - Inference by stochastic simulation
  - Simple sampling
  - Rejection sampling
  - Markov chain Monte Carlo (MCMC)

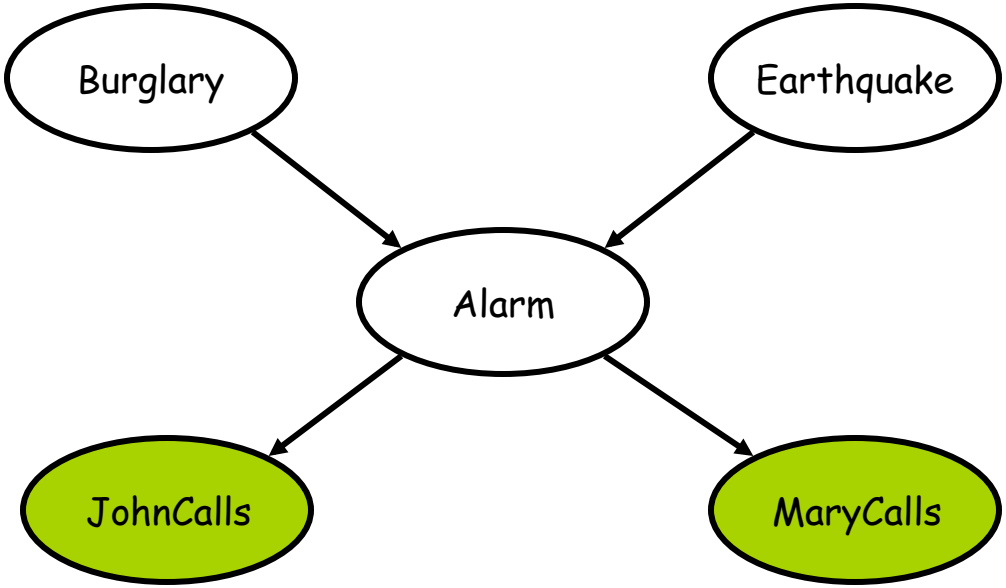
Inference terminology

- **Conditional probability table:** data structure that lists probabilities of a variable given one or more other variables.

- **Joint distribution:**
  distribution that is specified by a Bayesian network

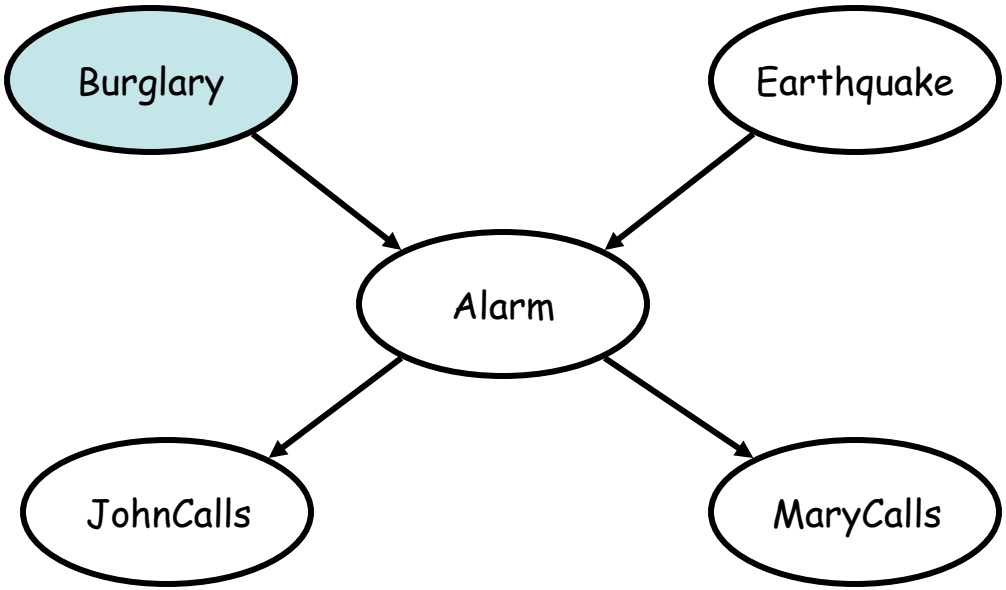
- **Inference**: produces the probability distribution of one or more variables given one or more other variables.

Types of nodes in inference



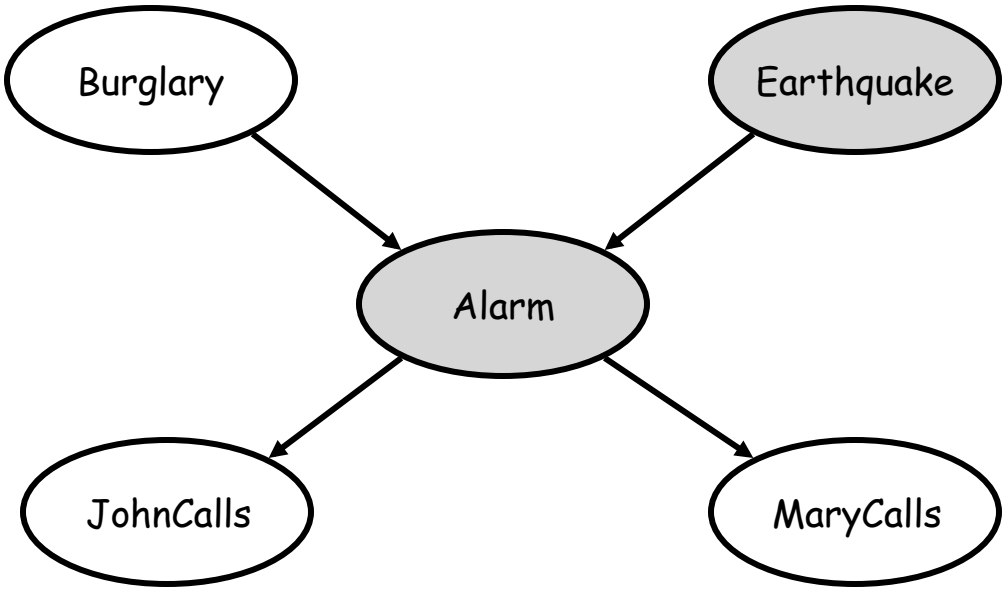
Evidence (or "observed") variables

Types of nodes in inference



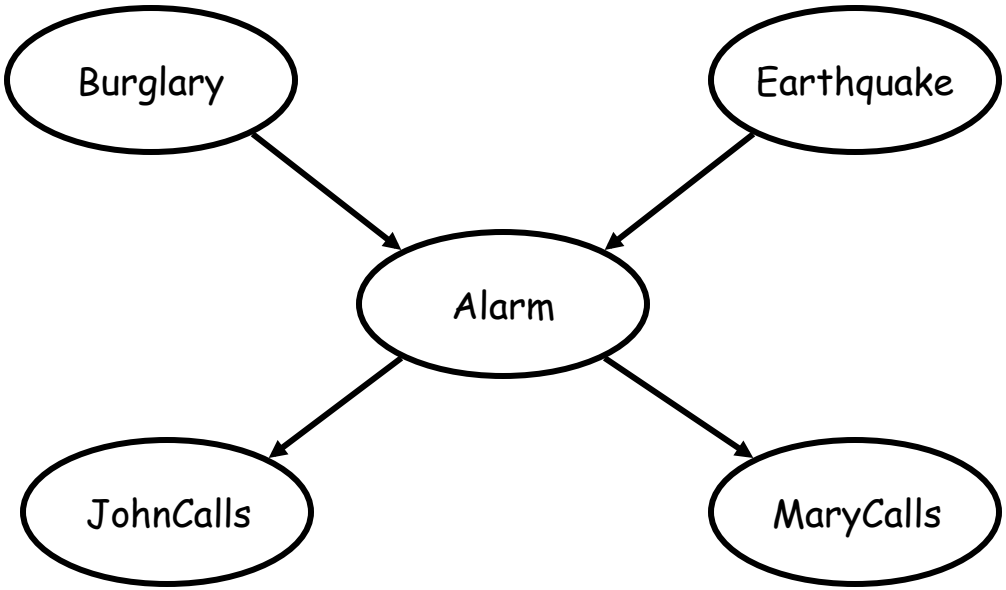
Query variables

Types of nodes in inference

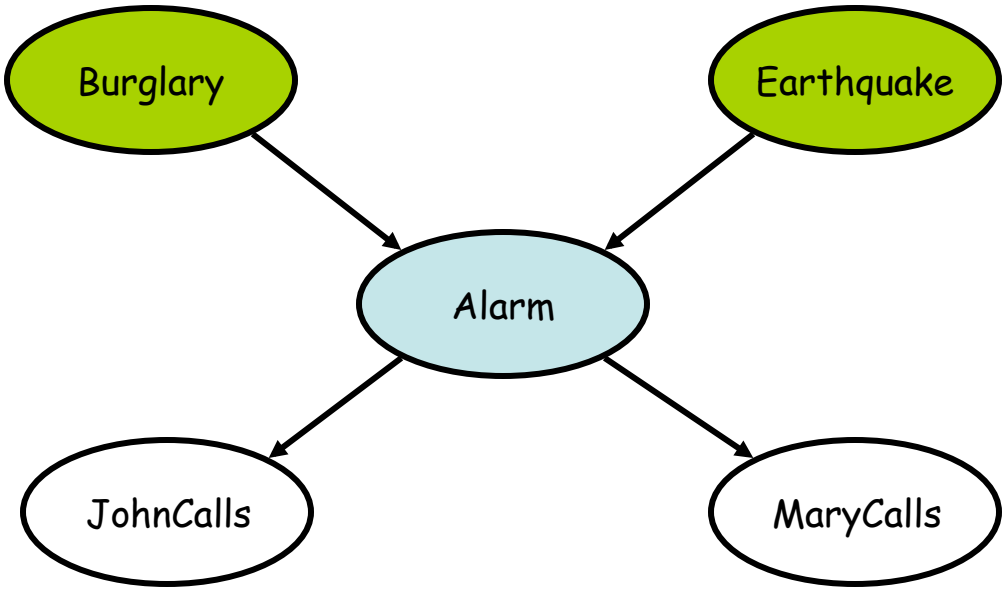


Hidden variables

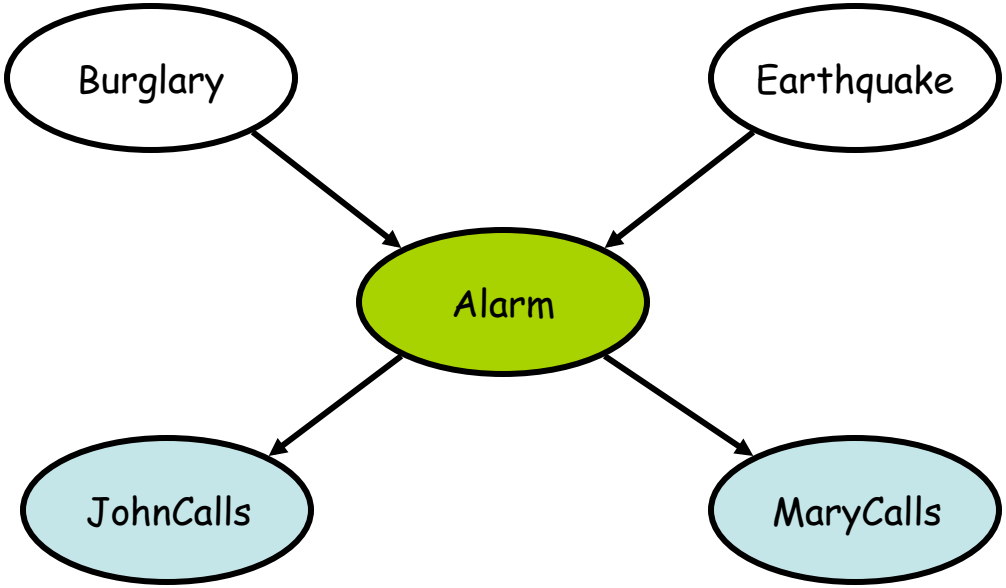
Simple inferences



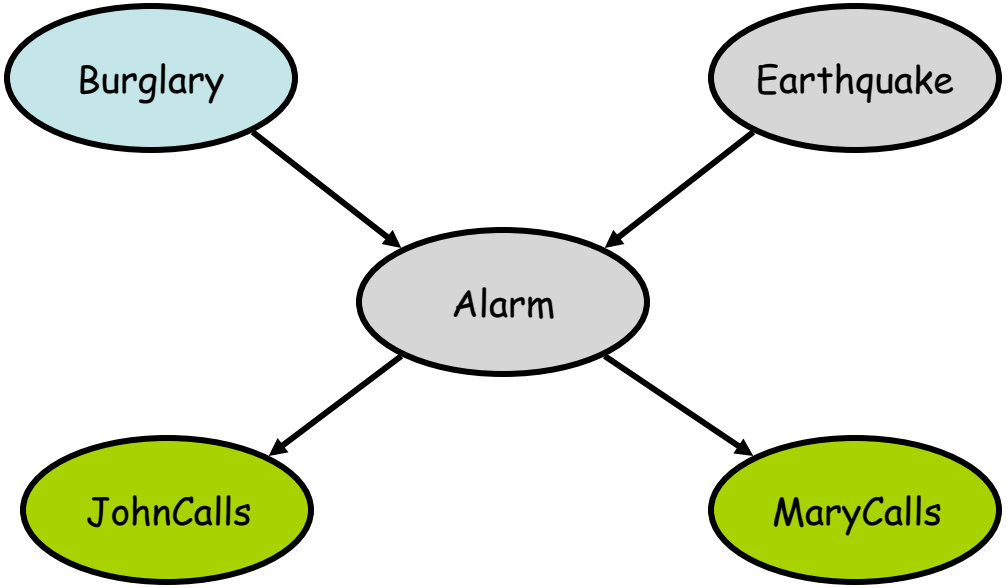
Simple inferences



Simple inferences

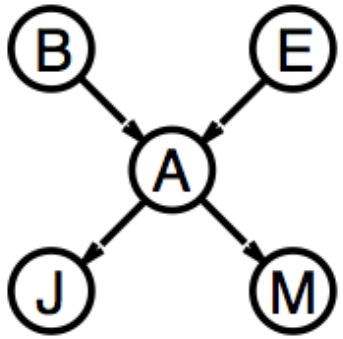


More difficult inferences



Inference by enumeration

$$\mathbf{P}(B|j,m)$$



P(B|j,m) = <0.284, 0.716>

Why approximate inference?

- Inference in singly connected networks is linear!
- ...but many networks are not singly connected
- Inference in multiply connected networks is exponential, even when the number of parents/node is bounded
- May be willing to trade some small error for more tractable inference

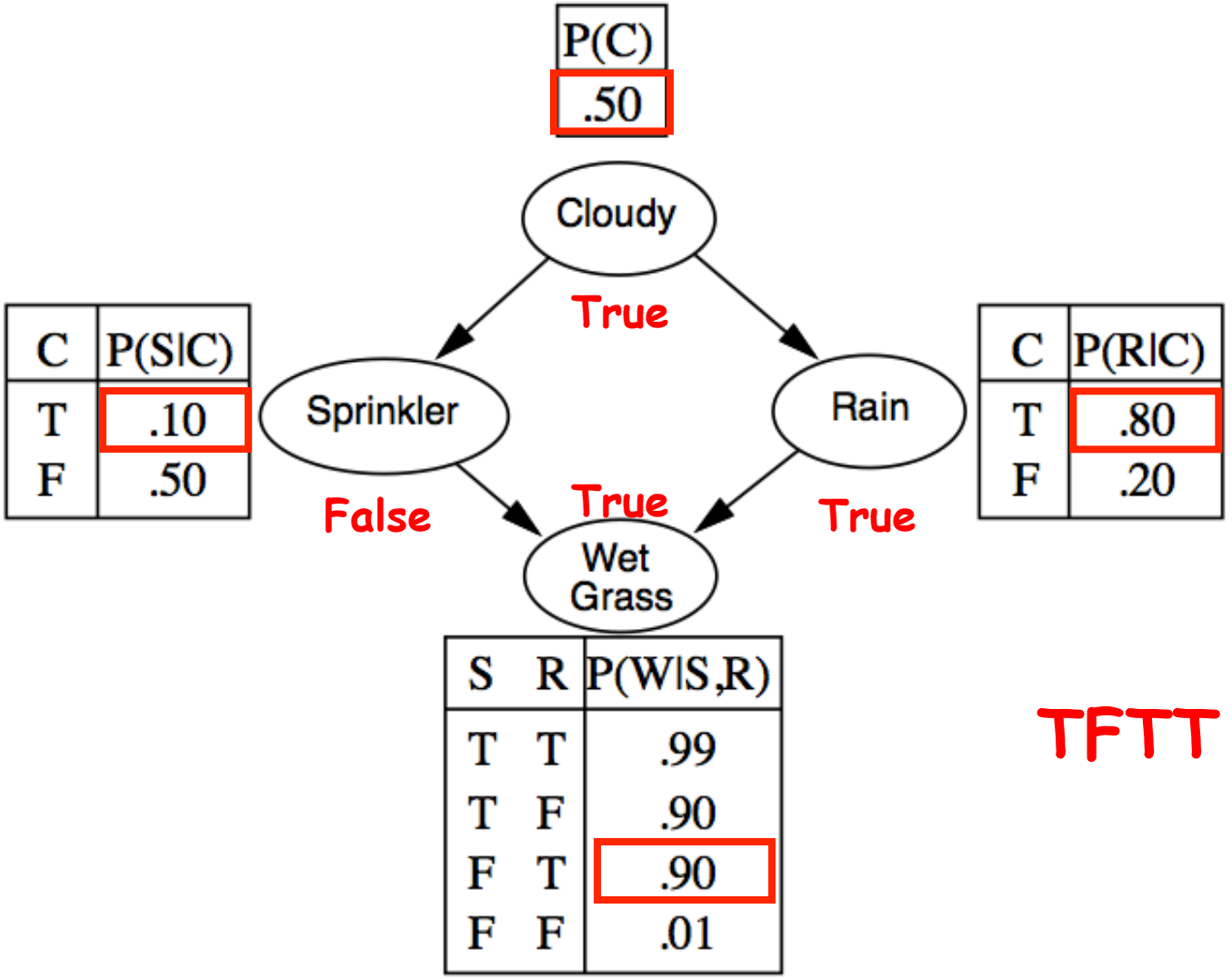
Stochastic simulation

- Core idea
  - Draw samples from a sampling distribution defined by the network
  - Compute an approximate posterior probability in a way that converges to the true probability
- Methods
  - Simple sampling from an empty network
  - Rejection sampling — reject samples that don't agree with the evidence
  - Likelihood weighting — weight samples based on evidence
  - Markov chain Monte Carlo — sample from a stochastic process whose stationary distribution is the true posterior

Simple sampling

- Given an empty network...
- And beginning with nodes without parents...
- We can sample from conditional distributions and instantiate all nodes.
- This will produce one element of the joint distribution.
- Doing this many times will produce an empirical distribution that approximates the full joint distribution.

Example


P(C)
.50
Cloudy
True
C P(S|C)
T .10
F .50
Sprinkler
False
Rain
True
C P(R|C)
T .80
F .20
True
Wet Grass
S R P(W|S,R)
T T .99
T F .90
F T .90
F F .01
TFTT

Benefits and problems of simple sampling

- Works well for an empty network
  - Simple
  - In the limit (many samples), the estimated distribution approaches the true posterior
- But in nearly all cases, we have evidence, rather than an empty network
- What can we do?
- Throw out cases that don't match the evidence

Rejection sampling

- Sample the network as before...
- But discard samples that don't correspond with the evidence.
- Similar to real-world estimation procedures, but the network is the stand-in for the world (much cheaper and easier).
- However, hopelessly expensive for large networks where P(e) is small.

Likelihood weighting

- Do simple sampling as before...
- But weight the likelihood of each sample based on the evidence
- Don’t need to know details…

MCMC: Markov Chain Monte Carlo

- The “state” of the system is the current assignment of all variables
- Algorithm
  - Initialize all variables randomly
  - Generate next state by sampling one variable given its Markov blanket
  - Sample each variable in turn, keeping other evidence fixed.
- Variable selection can be sequential or random

MCMC Problems

- Difficult to tell if it has converged
- Multiple parameters (e.g., burn-in period)
- Can be wasteful if the Markov blanket is large because probabilities don't change much

Specifically (but not limited to)…

- Be able to write down probabilistic statements asserted by a Bayes net.
- Be able to draw a Bayes net from a verbal description: cf. problem 14.1, p. 558
- Be able to compute prob of query given evidence and a Bayes net

What you don't need to know for the exam

- Variable elimination algorithm (14.4.2)
- Clustering algorithms (14.4.4)
- Likelihood weighting
- Why Gibbs sampling works
- Secs. 14.6, 14.7

Problem 14.14

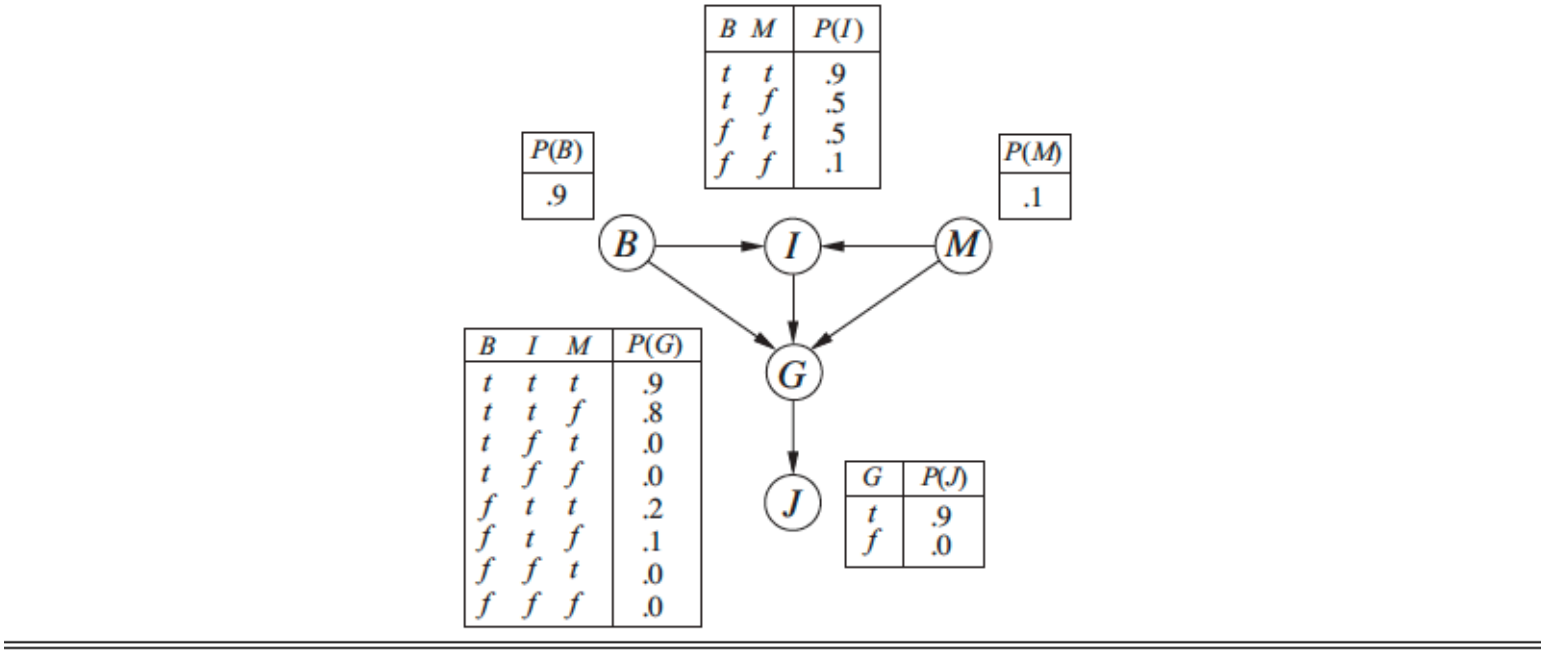


Figure 14.23 **FILES: figures/politics.eps (Tue Nov 3 16:23:20 2009).** A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

Which of the following are asserted by the network?

X P(B,I,M) = P(B)P(I)P(M)

→ P(J|G) = P(J|G,I)

→ P(M|G,B,I) = P(M|G,B,I,J)

Problem 14.14 contd.

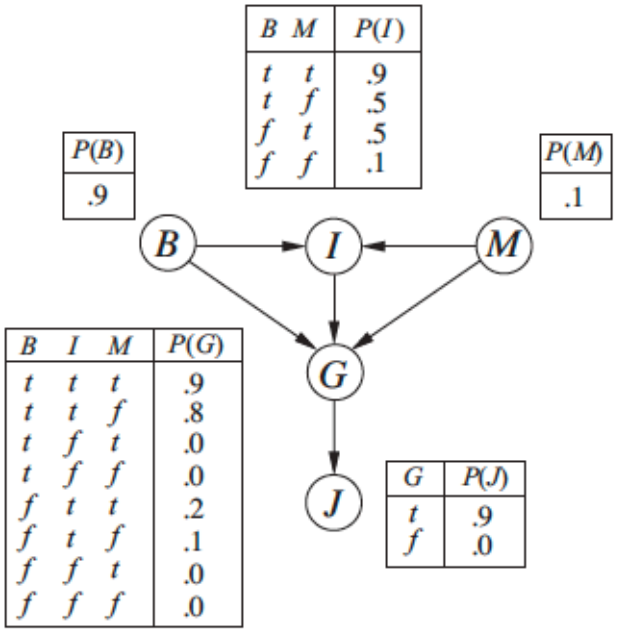


Figure 14.23 **FILES: figures/politics.eps (Tue Nov 3 16:23:20 2009).** A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

Calculate P(b,i,–m,g,j)

$$P(b, i, \neg m, g, j) = P(b)P(\neg m)P(i|b, \neg m)P(g|b, i, \neg m)P(j|g)$$
$$= .9 \times .9 \times .5 \times .8 \times .9 = .2916$$

Problem 14.14 contd.

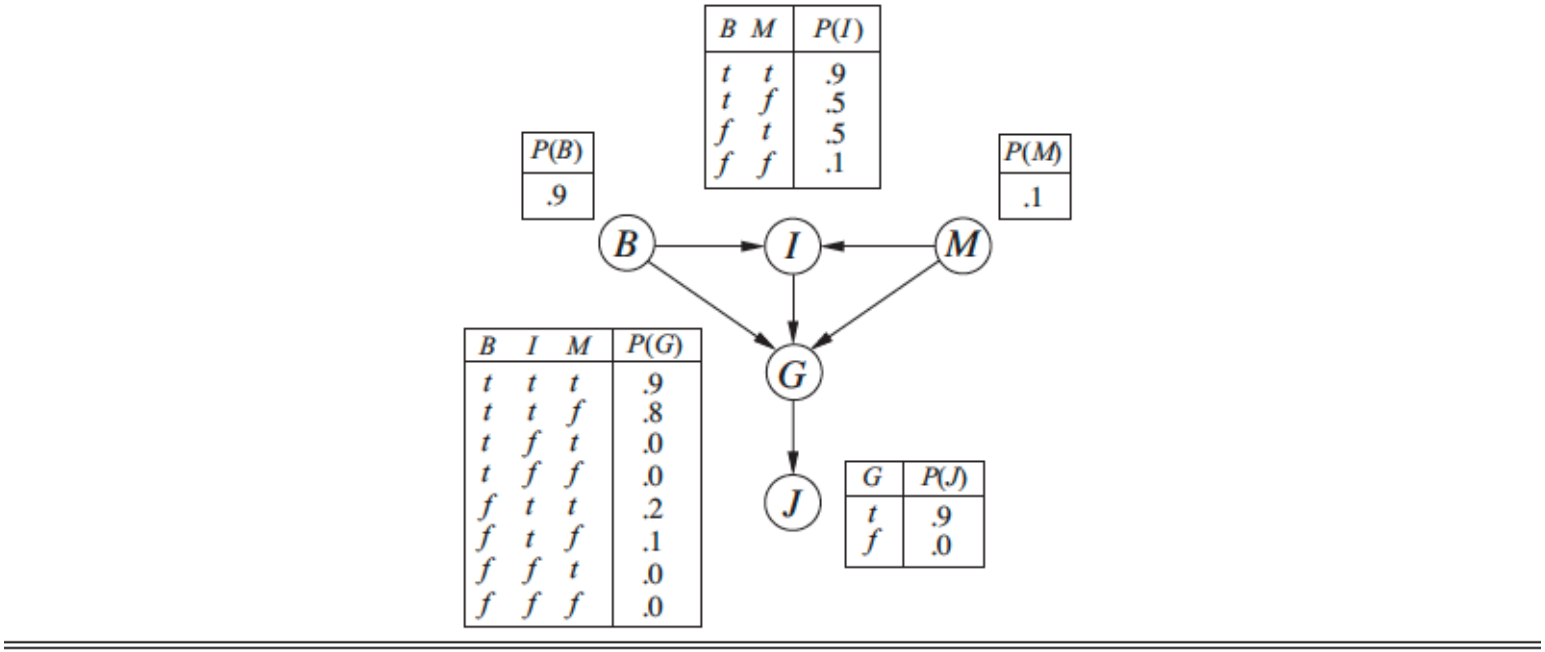
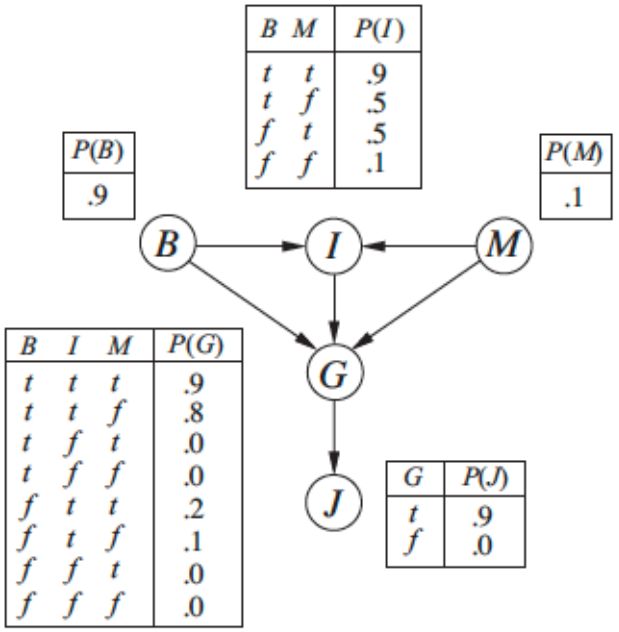


Figure 14.23 FILES: figures/politics.eps (Tue Nov 3 16:23:20 2009). A simple Bayes net with Boolean variables $B = BrokeElectionLaw$, $I = Indicted$, $M = PoliticallyMotivatedProsecutor$, $G = FoundGuilty$, $J = Jailed$.

What is the prob that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor?

Problem 14.4 contd.



Since B, I, M are fixed true in the evidence, we can treat G as having a prior of 0.9 and just look at the submodel with G, J:

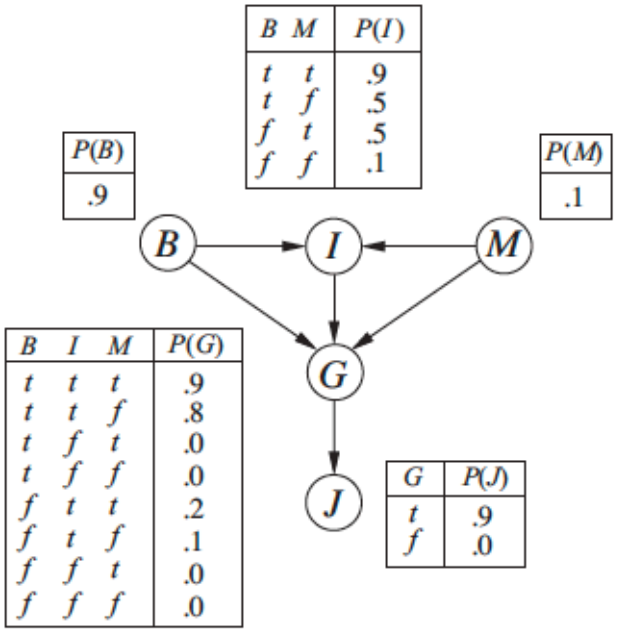
$\mathbf{P}(J|b,i,m) = \alpha \sum_g \mathbf{P}(J,g) = \alpha[\mathbf{P}(J,g) + \mathbf{P}(J,\neg g)]$

$= \alpha[\langle P(j,g), P(\neg j,g)\rangle + \langle P(j,\neg g), P(\neg j,\neg g)\rangle$

$= \alpha[\langle .81, .09\rangle + \langle 0, 0.1\rangle] = \langle .81, .19\rangle$

That is, the probability of going to jail is 0.81.

Problem 14.4 contd.



The long way:

$$P(J=t|B=t,I=t,M=t) = P(j|b,i,m) = P(j,b,i,m)/P(b,i,m) =$$

$$\alpha\sum_g P(j,g,b,i,m) = \alpha\sum_g P(b)P(m)P(i|b,m)P(g|b,i,m)P(j|g) =$$

$$\alpha\sum_g 1\times 1\times 1\times P(g|b,i,m)P(j|g) =$$

$$\alpha\Big(P(g|b,i,m)P(j|g) + P(\neg g|b,i,m)P(j|\neg g)\Big) =$$

$$\alpha(.9\times .9 + .1\times 0) = \alpha\times .81$$