NAME: ____________________________

SPIRE ID: ____________________________

COMPSCI 250
Introduction to Computation
First Midterm Spring 2025

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are 6 problems on pages 2-10, some with multiple parts, for 100 total points plus 5 extra credit. Final scale will be determined after the exam.
- Page 11 contains useful definitions and is given to you separately – do not put answers on it!
- If you need extra space use the back of a page – both sides are scanned.
- But, if you do write on the back, you must explicitly add a note on the front side stating that you are continuing on the back page. Otherwise , we might not see your solution on Gradescope.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.
- Your answers must be LEGIBLE, and not cramped. Write only short paragraphs with space between paragraphs

1	/10
2	/10
3	/20
4	/20
5	/20+5
6	/20
Total	/100+5

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Definitions for Questions 1-3: While each of the dogs in Dave's neighborhood is perfect in their own way, some of them are given to certain undesirable activities.

Let D be the set of dogs $\{b, c, i, k, r\}$, containing the five dogs Blaze, Clover, Indie, Kiké, and Rhonda.

Let U be a set of behaviors, containing exactly $\{CR, GP, PB, WHO\}$, referring to "chewing rugs", "growling at pedestrians", "peremptory[1] barking", and 'waking humans overnight".

Let E be a binary predicate from D to U, so that $E(d, u)$ means "dog d exhibits behavior u". Your eventual goal is to determine all the 20 truth values of this relation, based on the following five statements in Question 1.

Question 1 (10): (Translations) (2 points each)

- (a, to symbols) **Statement I:** If Kiké chews rugs, then Clover peremptorily barks but Blaze does not wake humans overnight.

- (b, to English) **Statement II:** $(E(c, PB) \leftrightarrow E(b, WHO)) \wedge \neg(E(k, CR) \oplus E(c, PB))$

- (c, to symbols) **Statement III:** Blaze growls at pedestrians, and no other dog does so.

- (d, to English) **Statement IV:** $(\exists u : E(i, u)) \rightarrow (\forall v : E(b, v) \leftrightarrow E(r, v))$

- (e, to symbols) **Statement V:** Each of the behaviors, except for growling at pedestrians, is exhibited by exactly three of the dogs. (**Note:** This statement does not say whether these three dogs are the same dogs or different ones.)

[1] **peremptory**: in a manner insisting on immediate obedience or attention

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Question 2 (10): (Boolean Proof)

Using Statements I and II *only*, determine the truth values of the three propositions $p = E(k, CR)$, $q = E(c, PB)$, and $r = E(b, WHO)$, using either a truth table or a deductive sequence proof. Note that (particularly with the deductive sequence approach) you are responsible for both showing that your solution agrees with the statements, and that no other solution does so.

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Question 3 (20) (Predicate Proof) :

Using Statements I-V, fill in the table below to show all 20 truth values of the propositions in the relation E. Justify your conclusion, making clear where you are using quantifier rules. (One way to do this is to number the lines of your argument, and indicate in your table which entry is determined by which line.)

Hint: It follows from Statements I and II that *all three* of the propositions p, q, and r are *false*. (We've thus filled in those three entries of the table for you already.) You still need to prove this fact on Question 2, but *here* you may assume that without further proof.

In this table a **0** denotes False, and a **1** should be used to denote True.

$E(b, CR) =$		$E(b, GP) =$		$E(b, PB) =$		$E(b, WHO) =$ **0**
$E(c, CR) =$		$E(c, GP) =$		$E(c, PB) =$ **0**		$E(c, WHO) =$
$E(i, CR) =$		$E(i, GP) =$		$E(i, PB) =$		$E(i, WHO) =$
$E(k, CR) =$ **0**		$E(k, GP) =$		$E(k, PB) =$		$E(k, WHO) =$
$E(r, CR) =$		$E(r, GP) =$		$E(r, PB) =$		$E(r, WHO) =$

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Question 4 (20): (Binary Relations on a Set) Parts (a) and (b) deal with two binary relations P and Q, each from a set to itself. Part (a) is on this page, part (b) on the next.

(a, 10) Let $A = \{a, b, c, d\}$. The relation $P \subseteq A \times A$ is a **partial order**.
It is known that $(b, c) \in P$, $(c, a) \in P$, $(d, a) \in P$ and $(d, c) \notin P$.

(i) In the table given below, label all pairs that must be in P with a "✓" and all pairs that cannot be in P with a "×." For pairs that are neither, leave their entry blank. We start you off by labeling the four known pairs with the appropriate "✓" or ×.

Justify your answers. That is, you must provide a brief explanation as to why each of the pairs you marked "✓" are in P and those you marked "×" are not in P.

Your explanations must reference the properties of P that are being used. They must also be consistent. That is, if you somewhere use the fact that $(x, y) \notin P$ where $(x, y) \neq (d, c)$, then you must have already previously proven that $(x, y) \notin P$.

(ii) How many partial orders P' exist that satisfy $(b, c) \in P'$, $(c, a) \in P'$, $(d, a) \in P'$ and $(d, c) \notin P'$. Briefly explain how you know this. Draw a Hasse diagram illustrating each such P' that could exist.

(a,a)	(a,b)	(a,c)	(a,d)	(b,a)	(b,b)	(b,c)	(b,d)	(c,a)	(c,b)	(c,c)	(c,d)	(d,a)	(d,b)	(d,c)	(d,d)
						✓		✓				✓		×	

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(b, 10) Let $B = \{a, b, c, d, e\}$. The relation $Q \subseteq B \times B$ is an **equivalence relation**. It is known that $(a, b) \in Q$, $(b, c) \in Q$, $(d, a) \in Q$ and $(e, d) \notin Q$.

(i) In the table given below, label all pairs that must be in Q with a "✓" and all pairs that cannot be in Q with a "×." For pairs that are neither, leave their entry blank. We start you off by labelling the three known pairs with the appropriate "✓" or ×.

Justify your answers following the same rules that were given in part (a).

(ii) How many Equivalence Relations Q' exist that satisfy $(a, b) \in Q'$, $(b, c) \in Q'$, $(d, a) \in Q'$ and $(e, d) \notin Q'$. Briefly explain how you know this. Write down the possible equivalence relation(s) in partition form, that is, as a set of sets of items in B.

(a,a)	(a,b)	(a,c)	(a,d)	(a,e)		(b,a)	(b,b)	(b,c)	(b,d)	(b,e)
	✓							✓		
(c,a)	(c,b)	(c,c)	(c,d)	(c,e)		(d,a)	(d,b)	(d,c)	(d,d)	(d,e)
						✓				
(e,a)	(e,b)	(e,c)	(e,d)	(e,e)						
			×							

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Question 5 (20+5): (Number Theory)

- (a, 4) For each of (i) and (ii) below say whether m has a multiplicative inverse modulo n. In each one, if the inverse exists, write down what it is. You do not need to show your work. The number you write down should be between 0 and $n-1$. If the inverse does not exist, prove that it does not exist.

  (i) $m = 3$, $n = 14$.

  (ii) $m = 422$, $n = 446$

- (b, 8) The naturals 87 and 38 are relatively prime.
  Use the Extended GCD algorithm as taught in class to find integers a and b satisfying the equation $a \cdot 87 + b \cdot 38 = 1$.
  Show all of your work.

  After solving the problem, write your final solution here:

  $a =$ __________. $b =$ __________.

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- (c, 4) Using the results from the previous part, determine both an inverse of 38 modulo 87 and an inverse of 87 modulo 38.

  For full credit, your answers should each be the smallest natural numbers that are inverses.

  After solving the problem, write your final solutions here:

  ____ is an inverse of 87 modulo 38.
  ____ is an inverse of 38 modulo 87.

- (d, 4) You are now given that $9 \cdot 45 - 4 \cdot 101 = 1$.

  Using the technique taught in class, determine the smallest natural x that solves both the congruences

  $$x \equiv 3 \pmod{45} \quad \text{and} \quad x \equiv 4 \pmod{101}.$$

  Show all of your work.
  After solving the problem, write your final solution here:
  The smallest natural satisfying both congruences is $x =$ ______________.

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- (ec. 5 extra credit) You are now given that $7 \cdot 23 - 8 \cdot 20 = 1$.
  Find the smallest natural x that satisfies

$$x \equiv 2 \pmod{20} \quad \text{and} \quad x \equiv 1 \pmod{23} \quad \text{and} \quad x \equiv 0 \pmod{14}.$$

  It is not necessary to show your work but, if you do not, and get the answer wrong, we cannot give partial credit for a correct technique.
  After solving the problem, write your final solution here:
  The smallest natural satisfying all three congruences is $x =$ ____________.

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Question 6 (20): The following are ten true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

After reading the questions, write the correct answer, either T (for true) or F (for false), in the corresponding column.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)

- (a) There exists a non-empty subset S of the naturals such that if x is any element of S, x's predecessor is also in S.

- (b) Let R be a partial order on a set A. Then if x and y are different elements of A, we know that $((x, y) \in R) \oplus ((y, x) \in R)$.

- (c) Let $a > 1$ be a natural, and let $z = a! - 1$. Then for any natural b such that $1 < b \leq a$, z is not divisible by b.

- (d) Consider a truth table using two boolean variables a and b. We know that $a \to (a \vee b)$. Then in the truth table the column for the term a has two 1's, and the column for the term $a \vee b$ has three.

- (e) Let a be any natural such that $a \equiv 2 \pmod 5$. Then there exists some natural n such that $a^n \equiv 0 \pmod 5$.

- (f) Hasse Diagrams may be used to represent equivalence relations as well as to represent partial orders.

- (g) Let X be the set of all even naturals, and let Y be the set of all odd naturals. Then there does not exist a set Z such that $Z \subseteq X$ and $Z \subseteq Y$.

- (h) A function $f : X \to Y$ is one-to-one if and only if for every element x of X, there is exactly one element y of Y such that $f(x) = y$.

- (i) Let x, y, and z be three naturals such that $xyz = 4200$. Then one of these three numbers is divisible by 7, and the other two are not.

- (j) The statement $X \Delta Y = (X \cup Y) \setminus (X \cap Y)$ is a set identity.

COMPSCI 250 First Midterm Supplementary Handout: 11 March 2025

Here are the definitions of sets, predicates, and statements used on the exam.

Remember that the scope of any quantifier is always to the end of the statement it is in.

The scenario of Question 1-3 is as follows.

While each of the dogs in Dave's neighborhood is perfect in their own way, some of them are given to certain undesirable activities.

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- **Statement III:** Blaze growls at pedestrians, and no other dog does so.
- **Statement IV:** $(\exists u : E(i, u)) \rightarrow (\forall v : E(b, v) \leftrightarrow E(r, v))$
- **Statement V:** Each of the behaviors, except for growling at pedestrians, is exhibited by exactly three of the dogs. (**Note:** This statement does not say whether these three dogs are the same dogs or different ones.)