NAME: ____________________________

COMPSCI 250
Introduction to Computation
Final Exam Spring 2022

D. A. M. Barrington and K.Doney 10 May 2022

DIRECTIONS:

- Answer the problems on the exam pages.
- There are five problems on pages 2-8, some with multiple parts, for 120 total points plus 10 extra credit. Probable scale is somewhere around A=105, C=70, but will be determined after we grade the exam.
- Page 9 contains useful definitions and is given to you separately – do not put answers on it!
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.

Last month Blaze and Rhondda were invited to an Easter Egg Hunt at the home of their neighbor Clover. Their neighbor Kiké also joined them. Clover's family placed 23 plastic eggs in the yard, and each dog was given credit for each egg that they first found. For any of the four dogs, let $e(d)$ be the (natural) number of eggs found by dog d. Of the four dogs, Clover and Rhondda are Goldens, and the other two not. Rhondda and Blaze are female, and the other two not. The predicate $G(x)$ means "x is a Golden" and $F(y)$ means "y is female".

Question 1 (15): Translate the five statements as indicated:

- (a, 3) (to symbols) (Statement I) It is not the case that every dog found at least one egg.

- (b, 3) (to English) (Statement II) $\forall d : (e(d) \neq 5) \rightarrow (\neg F(d) \vee G(d))$

- (c, 3) (to symbols) (Statement III) Rhondda found exactly twice as many eggs as were found by some non-female dog.

- (d, 3) (to English) (Statement IV) $\forall x : \forall y : (G(x) \wedge \neg G(y)) \rightarrow (e(x) > e(y))$

- (e, 3) (to symbols) (Statement V) Some female dog found exactly five eggs.

Question 2 (15): Using the definitions and statements from Question 1, determine exactly how the 23 eggs were distributed among the four dogs. Prove your answers, and indicate where you have used any quantifier rules.

Question 3 (20+10): Let $\Sigma = \{a, b, c\}$. Let the language X be the set of all strings over Σ that never have a double letter (that is, with no consecutive appearances of the same letter). Let Y be the language of the regular expression $(abcab + abc)^*$.

- (a, 10) Let $f(n)$, for any natural n, be the number of strings of length n in X. Prove, for all *positive* naturals n, that $f(n) = 3 \times 2^{n-1}$. (**Hint:** Use ordinary induction on all positive naturals. Remember that your argument must refer to strings, not just numbers.)

- (b, 10) Prove that for any natural t such that $t \geq 8$, there exists at least one string of length t in Y. (**Hint:** Use strong induction, using three base cases.)

- (c, 10 XC) Prove that Y is a subset of X. (**Hint:** Use induction on the star language Y. It may be useful to first prove by induction that no string in Y ends with an a.)

Question 4 (40): Here is the usual question about Kleene's Theorem constructions. We begin with the language Y above, which is denoted by the regular expression $(abcab + abc)^*$.

- (a, 10) Using the construction from lecture, construct a λ-NFA N whose language is Y.

- (b, 10) Using the construction from lecture, construct an ordinary NFA N' whose language is $L(N)$.

- (c, 10) Using the Subset Construction on N', construct a DFA whose language is $L(N')$. (**Hint:** You may simplify the diagram by saying where there are edges to a death state, without drawing them.)

- (d, 10) Using State Elimination, starting from D, construct a regular expression whose language is the same as $L(D)$. (Note that we already have a regular expression for Y, but we are asking you to use the construction. Your new regular expression may not be the same as the earlier one, but it should be equivalent, meaning that the two expressions will have the same language.)

Question 5 (30): The following are fifteen true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing.

- (a) Because the divisibility relation over the natural numbers is reflexive, antisymmetric and transitive, it is therefore a total order.

- (b) A bipartite graph of $2n$ nodes can contain as many as $\frac{(2n)^2}{4}$ edges.

- (c) If the inverse of a statement is false, then the contrapositive could still be true.

- (d) The following system of congruences can be simplified to a single congruence modulo $3 \times 8 \times 7 \times 15$, using the Chinese Remainder Theorem: $x \equiv 20 (\text{mod } 3)$, $x \equiv 18 (\text{mod } 8)$, $x \equiv 21 (\text{mod } 7)$, and $x \equiv 5 (\text{mod } 15)$.

- (e) Let $h_1(x)$ and $h_2(x)$ be two heuristics for the same search problem.
  Let $h_3(x) = \min(h_1(x), h_2(x))$. It is not possible for $h_3(x)$ to be admissible if both $h_1(x)$ and $h_2(x)$ are non-admissible heuristics.

- (f) Let L be the language denoted by the regular expression $(bbb + ccc)^* + (b + c)a^*$. Then the strings $bbbbb$ and $ccccbc$ are $L-$equivalent in the sense of the Myhill-Nerode Theorem.

- (g) There exists a two-way DFA which decides the unary language $\{1^{a^2} | a \geq 0\}$.

- (h) There exists an NFA which decides the language $\{w \mid w$ contains an equal number of appearances of the substrings 01 and 10$\}$.

- (i) Let us define a Turing machine with a two-way infinite tape (2-way TM) to be identical to our standard TM, except the tape extends infinitely to the left as well as the right. Then these two-way TMs recognize the same class of languages as do standard TMs.

- (j) The following language EMPTY-DFA is not Turing decidable: It is the set of all DFA's D such that $L(D) = \emptyset$.

- (k) Suppose that we have a function f from pairs of strings to pairs of strings that is a reduction from the Halting Problem $H(M, w)$ to some arbitrary language $L(M, w)$, such that for any Turing machine M and any string w, we have that $(M, w) \in H \iff f(M, w) \in L$. From this we know that language L is undecidable.

- (l) The class of regular languages is closed under reversal ($L^R = \{w^R | w \in L\}$).

- (m) For any composite integer n, the sum of the prime factors of n is strictly less than n.

- (n) In any version of generic search, the search may terminate as soon as the goal vertex has been added to the open list.

- (o) If D is a minimal DFA with k states that decides language L, and the strings λ and a^{k+1} are L-equivalent, then the equivalence class of λ must contain at least three different strings.

Supplemental Page for CS250 Final Exam, 10 May 2022, Barrington/Doney

The following situations, definitions, and statements are used in Questions 1 and 2.

Last month Blaze and Rhondda were invited to an Easter Egg Hunt at the home of their neighbor Clover. Their neighbor Kiké also joined them. Clover's family placed 23 plastic eggs in the yard, and each dog was given credit for each egg that they first found. For any of the four dogs, let $e(d)$ be the (natural) number of eggs found by dog d. Of the four dogs, Clover and Rhondda are Goldens, and the other two are not. Rhondda and Blaze are female, and the other two are not. The predicate $G(x)$ means "x is a Golden" and $F(y)$ means "y is female".

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- (d, 3) (to English) (Statement IV) $\forall x : \forall y : (G(x) \wedge \neg G(y)) \rightarrow (e(x) > e(y))$
- (e, 3) (to symbols) (Statement V) Some female dog found exactly five eggs.

Question 3 refers to the following two languages:

Let $\Sigma = \{a, b, c\}$. Let the language X be the set of all strings over Σ that never have a double letter (that is, with no consecutive appearances of the same letter). Let Y be the language of the regular expression $(abcab + abc)^*$. Let $f(n)$, for any natural n, be the number of strings of length n in X.