NAME: _____

COMPSCI 250
Introduction to Computation
First Midterm Fall 2021

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are four problems on pages 2-7, some with multiple parts, for 100 total points plus 5 extra credit. Final scale was A = 92, C = 60.
- Page 8 contains useful definitions and is given to you separately – do not put answers on it!
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.

1	/20
2	/30
3	/30
4	/20+5
Total	/100+5

Question 1 (20): Blaze has a set $T = \{a, d, m, o\}$ of chew toys, consisting of an Antler, a Dinosaur, a Marrowbone, and an Octopus.

Every morning Blaze chooses *one or more* of her toys to play with while waiting for her walk. Let $W = \{\text{Mon}, \text{Tue}, \text{Wed}, \text{Thu}\}$ to a particular set of weekdays, and define the predicate $C : T \times W$ so that $C(x, y)$ means "Blaze chooses toy x on weekday y".

Translate each statement as according to the directions:

- (a, 2) (to English) (Statement I) $C(a, \text{Wed}) \oplus C(o, \text{Wed})$

- (b, 3) (to symbols) (Statement II) If Blaze chose the Antler on either Tuesday or Wednesday, then she also chose the Octopus on Wednesday.

- (c, 3) (to English) (Statement III) $(C(a, \text{Wed}) \vee C(o, \text{Tue})) \rightarrow (\neg C(a, \text{Tue}) \wedge \neg C(o, \text{Tue}))$

- (d, 2) (to symbols) (Statement IV) Blaze chose the Dinosaur on each day of the week.

- (e, 3) (to English) (Statement V) $\forall x : \exists y : C(x, y)$

- (f, 3) (to symbols) (Statement VI) The Marrowbone was chosen on Thursday and on no other days of the week.

- (g, 4) (to English) (Statement VII) $\exists u : \exists v : \forall w : C(w, \text{Thu}) \rightarrow [(u = w) \vee (v = w)]$

Question 2 (30): These questions use the definitions, predicates, and premises on the supplementary sheet.

- (a, 10) Given *only* that Statements I, II and III are true, determine the truth values of the four propositions $p = C(a, \text{Tue})$, $q = C(a, \text{Wed})$, $r = C(o, \text{Tue})$, and $s = C(o, \text{Wed})$. You may use a truth table or a deductive sequence proof. (**Hint:** Exactly one of the four propositions is true. **I intended for there to be only one solution, but there are two. You need to find both, one of which is consistent with the hint.**)

- (b, 20) Assuming that Statements I-VII are all true, prove that Blaze chose the Antler on Monday. (You may want to quote consequences of I-III from part (a) of this question.) You may use either English or symbols, but make it clear each time you use a quantifier proof rule. (**Use the hint from 2a for this problem, so that you use the solution to 2a that is consistent with that.**)

Question 3 (20): The following are ten true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing. Some of them use the sets, relations, and functions defined on the supplemental sheet, but you should assume the truth of Statements I-VII only if explicitly told to.

- (a) Assuming that Statements I-VII in Question 1 are all true, we cannot determine whether Blaze chose the Octopus on Monday.

- (b) Let R be a relation from a set A to a set B. Then R is a one-to-one function if and only if for any element a of A, there exists at most one element b of B such that $R(a, b)$ is true.

- (c) $|\emptyset| = 1$.

- (d) If p is any prime number, and "!" is the factorial function, then $p! + 1$ must be a prime number.

- (e) If R is an equivalence relation on a set A, and x and y are any elements of A, then if $[x] \neq [y]$, then $[x] \cap [y] = \emptyset$.

- (f) $\{\emptyset\} \subseteq \{a\}$, where a is a string.

- (g) It is not the case that every surjection (onto function) has an inverse function.

- (h) If A and B are sets, and $A \subseteq B$, then $A \cap B = \emptyset$ and $A \cup B = B$ must both be true.

- (i) The statement "If I am arguing, then you paid" is the contrapositive of the statement "If I am not arguing, then you did not pay".

- (j) Let Σ be the set $\{a, b\}$ and let Σ^* be the set of all finite strings over Σ. Define the relation $R \subseteq \Sigma^* \times \Sigma^*$ such that $R(u, v)$ means "u is either a prefix of v or a suffix of v, or both". Then R is not a partial order.

Question 4 (30+5): Here are some straightforward number theory questions.

- (a, 5) Show the steps of the Euclidean Algorithm to determine the natural g that is the greatest common divisor of 726 and 1463.

- (b, 5) Give complete factorizations of 726 and 1463. (The solution of part (a) should save you some calculation.)

- (c, 10) Let y and z be any two positive naturals. Let a be the greatest common divisor of y and z. Prove that y/a has a multiplicative inverse modulo z/a and that z/a has an inverse modulo y/a. Describe how you would compute these two inverses.

- (d, 10) Let g be the greatest common divisor found in part (a). Find the inverse of $726/g$ modulo $1463/g$ and the inverse of $1463/g$ modulo $726/g$, making clear which is which.

- (e, 5XC) Let D be the division relation, so that for any two naturals $D(a, b)$ means "a divides b, or equivalently $b\%a = 0$". Let S be the set $\{x : D(x, 726) \vee D(x, 1463)\}$. Draw a Hasse diagram for the set S using the relation D.

COMPSCI 250 First Midterm Supplementary Handout: 12 October 2021

Here are definitions of sets, predicates, and statements used on the exam.

Remember that the scope of any quantifier is always to the end of the statement it is in.

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We have abbreviated the four propositions $p = C(a, \text{Tue})$, $q = C(a, \text{Wed})$, $r = C(o, \text{Tue})$, and $s = C(o, \text{Wed})$.

The seven statements of Question 1 are:

- (a, 2) (to English) (Statement I) $C(a, \text{Wed}) \oplus C(o, \text{Wed})$
- (b, 3) (to symbols) (Statement II) If Blaze chose the Antler on either Tuesday or Wednesday, then she also chose the Octopus on Wednesday.
- (c, 3) (to English) (Statement III) $(C(a, \text{Wed}) \vee C(o, \text{Tue})) \rightarrow (\neg C(a, \text{Tue}) \wedge \neg C(o, \text{Tue}))$
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