NAME: ________________________

SPIRE ID: ________________________

INFO 150
A Mathematical Foundation for Informatics
First Midterm Exam Fall 2024

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are six problems on pages 2-7, some with multiple parts, for 100 total points. Probable scale is somewhere around A=90, C=60, but will be determined after I grade the exam.
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.

1	/15
2	/20
3	/15
4	/15
5	/15
6	/20
Total	/100

Question 1 (15): Briefly explain the meaning of each of these terms or concepts (3 points each):

- (a) a **closed formula** for a number sequence

- (b) the **negation** of a proposition

- (c) a **universally quantified predicate**

- (d) the **converse** of an implication

- (e) the **Division Theorem** in number theory

Question 2 (20): Translate these four statements as indicated. We have a set of dogs D including (among others) the named dogs Blaze, Gwen, Rhonda, and Wallace who are denoted symbolically by b, g, r and w respectively. We define three predicates on dogs so that $H(x)$ means "dog x likes to dig holes", $S(x, y)$ means "dog x is smaller than dog y", and $L(x, y)$ means "dog x is larger than dog y". (5 points each)

- (a) (to symbols) There is a dog who is both larger than Wallace and smaller than Rhonda, and also does not like to dig holes.

- (b) (to English) $\forall x \in D, \forall y \in D, (S(x, y) \rightarrow L(y, x)) \wedge (L(x, y) \rightarrow S(y, x))$

- (c) (to symbols) If Rhonda likes to dig holes, then she is larger than Wallace.

- (d) (to English) $\neg H(g) \wedge (S(b, r) \vee H(w))$

Question 3 (15): Prove the following using truth tables:

- The two compound propositions $p \wedge (\neg q \rightarrow \neg p)$ and $\neg(q \rightarrow \neg p)$ are logically equivalent.

Question 4 (15): Consider the following statement:

"$\forall n \in \mathbb{Z}$, the number $(n+3)(n+4)$ is even".

1. What would be a **counterexample** disproving this statement?

2. Find three numbers that are *not* counterexamples, showing why they are not counterexamples.

3. Write a letter from the Author to the Reader, that should convince the Reader to stop looking for any counterexamples.

Question 5 (15): Prove the following statement:

"If x is any nonzero rational number, then the number $x + \frac{1}{x}$ is also a rational number."

(**Hint:** Recall the definition of a **rational number**: It is a real number that can be expressed as p/q where both p and q are integers, and $q \neq 0$.

Question 6 (20): Here are ten **true/false** questions, worth two points each. There is no credit for blank answers, so you should answer all the questions.

- (a) Let a be a number sequence with a recursive formula such that $a_1 = 3$ and, for larger n, $a_n = a_{n-1} + 2$. Then a has a closed formula with $a_n = 2n + 1$.

- (b) Let b be a number sequence with the closed formula $b_n = 2^n - 3$. Then b has a recursive formula with $b_1 = 2$ and, for larger n, $b_n = b_{n-1} + 2^{n-1}$.

- (c) The negation of the statement "Both Blaze and Rhonda have curly tails" is "Either Blaze does not have a curly tail, or Rhonda does not have a curly tail, or both".

- (d) If p, q, and r are propositions, and the statement $(p \vee q) \wedge (q \vee r)$ is true, then the three propositions cannot all be true.

- (e) Let D be a set of dogs, and let B be a set of breeds including "pointer (p)" and "spaniel (s)". Define a predicate I such that $I(x, y)$ means "dog x is of breed y". Then the English statement "Some dog is both a pointer and a spaniel" is logically equivalent to the symbolic statement "$(\exists x \in D, I(x, p)) \wedge (\exists y \in D, I(y, s))$".

- (f) In the setting of part (e) of this problem, let Wallace be a particular member of D. That a symbolic statement equivalent to "Every dog except Wallace is a pointer or a spaniel" would have a free variable.

- (g) The statement "Every dog who is a terrier is also cute" is equivalent to the statement "There does not exists a dog who is not cute and is a terrier".

- (h) The negation of the statement "For every breed, there is a dog of that breed" is "There exists a breed such that there is no dog of that breed".

- (i) Any multiple of any odd integer must be odd.

- (j) Let n and b be positive integers. Then there exist positive integers q and r such that $n = qb + r$ and $0 \leq r < b$.