NAME: ______________________________

SPIRE ID: ______________________________

INFO 150
A Mathematical Foundation for Informatics
First Midterm Exam Fall 2025

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are six problems on pages 2-7, some with multiple parts, for 100 total points. Probable scale is somewhere around A=90, C=60, but will be determined after I grade the exam.
- If you need extra space use the back of a page.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.

1	/15
2	/20
3	/15
4	/15
5	/15
6	/20
Total	/100

Question 1 (15): Briefly explain the meaning of each of these terms or concepts (3 points each):

- (a) when a compound proposition is a **contradiction**
- (b) a **negative integer**
- (c) a **rational number**
- (d) the **DeMorgan Law** involving the statement $\neg(p \vee q)$
- (e) the **base case** of a proof by induction

Question 2 (20): Translate these four statements as indicated (5 points each). We have a set of dogs D including (among others) the named dogs Blaze, Clover, Rhonda, and Teddy who are denoted symbolically by b, c, r and t respectively. We define three predicates on dogs so that $F(x)$ means "dog x is female, $G(x)$ means "dog x is a Golden retriever", and "$PN(x, y)$" means "dog x plays nicely with dog y".

- (a) (to symbols) Blaze plays nicely with all Golden retrievers.

- (b) (to English) $\exists x \in D, \forall y \in D, F(x) \wedge (PN(x, y) \rightarrow G(y))$

- (c) (to symbols) Teddy does not play nicely with either Blaze or Rhonda.

- (d) (to English) $G(c) \wedge (\neg F(t) \rightarrow PN(t, c))$

Question 3 (15): Prove the following using truth tables:

- The compound proposition $(p \wedge (q \oplus r)) \rightarrow (r \vee q)$ is a tautology. (Note that $\oplus$ is the **exclusive or** symbol.)

Question 4 (15): Consider the following statement:

"$\forall n \in \mathbb{Z}$, $n^2 + n - 1$ is not divisible by 3.

1. What would be a **counterexample** disproving this statement? (No actual counterexample exists, because the statement is true, but we are asking which properties would an integer need in order to be a counterexample.)

2. Find three numbers that are *not* counterexamples, showing why they are not counterexamples.

3. Write a letter from the Author to the Reader, that should convince the Reader to stop looking for any counterexamples.

Question 5 (15): For any positive integer n, let $S(n)$ be the sum $\sum_{i=1}^{n}(i^2 - 4i)$. That is, $S(n)$ is the sum of the numbers $(1^2 - 4) + (2^2 - 8) + \ldots + (n^2 - 4n)$. For example, $S(1) = 1 - 4 = -3$ and $S(3) = (1 - 4) + (4 - 8) + (9 - 12) = -10$. We would like to prove by induction that for all positive naturals n, $S(n)$ is given by the closed formula $S(n) = n(n + 1)(2n - 11)/6$.

- (a, 3) Write the precise boolean statement $P(n)$ that we would like to prove to be true for all positive integers n.

- (b, 3) State and prove the **base case** (or base cases) for your induction.

- (c, 3) State the **inductive hypothesis** and **inductive goal** for your inductive step.

- (d, 6) Prove your inductive step, completing the proof.

Question 6 (20): Here are ten **true/false** questions, worth two points each. There is no credit for blank answers, so you should answer all the questions. No explanation is needed or wanted.

- (a) Let a be a number sequence with a recursive formula such that $a_1 = 3$ and, for larger n, $a_n = 2a_{n-1}$. Then a has a closed formula with $a_n = 3(2^{n-1})$.

- (b) If an islander (as on Smullyan's Island) says "I am telling the truth", we may conclude that they are telling the truth.

- (c) If p, q, and r are propositions, then it is possible that the statement $(p \vee q) \wedge \neg(q \vee \neg r)$ is true.

- (d) Let D be a set of dogs, and T be a unary predicate on D such that "$T(x)$" means "dog x is not a terrier". Then the statement "Every dog in D is not a terrier" may be translated as "$\forall x \in D, \neg T(D)$".

- (e) The negation of the statement "Some dog is smaller than all black dogs" is "Every dog is larger than all black dogs".

- (f) The contrapositive of the statement "If Blaze is smaller than this dog, then it is a Rottweiler" is the statement "If Blaze is not smaller than this dog, then it is not a Rottweiler".

- (g) Let x be an even integer and y be an odd integer. Then there must exist some integer k such that $x = 2k$ and $y = 2k + 1$.

- (h) If x and y are both irrational numbers, then $x + y$ must be irrational.

- (i) The sum $S(5) = \sum_{i=1}^{5}(i - 3)$ is equal to 0.

- (j) If we prove $P(m)$, for any $m > 1$, using the assumptions $P(1)$, $P(2)$,..., $P(m-1)$, then we have proved $P(n)$ to be true for all positive integers n.