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COMPSCI 250
Introduction to Computation
Final Exam Spring 2023

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DIRECTIONS:

- Answer the problems on the exam pages.
- There are six problems on pages 2-10, some with multiple parts, for 120 total points plus 10 extra credit. The probable scale is around A=110, C=70, but will be determined after we grade the exam.
- If you need extra space, use the back of a page.
- No books, notes, calculators, or collaboration.
- In case of a numerical answer, an arithmetic expression like "$2^{17} - 4$" need not be reduced to a single integer.
- If you don't know the answer to any question, you can just write "Pass" or "I don't know", and you will receive 20% of the points of that question. You can use it as many times as you want. Note that it doesn't apply to wrong answers, and you may get less than 20% if you attempt a question and your answer is not correct. (It would be silly to "Pass" on a true/false question, since it's far better for you guess, and to save us grading effort we will not allow you to pass on those.)

Question 1 (35): Dave's dogs Blaze and Rhonda often see animals during their daily walks. They walk through five locations, the Neighborhood, the Farm, the Woods, the Park, and the Village.

The relation X is a subset of $D \times A \times L$, where $D = \{b, r\}$ is the set of dogs Blaze and Rhonda, $A = \{O, R, S\}$ is the set of species they may have seen, Opossum, Rabbit, or Squirrel, and $L = \{f, n, p, v, w\}$ is the set of five locations. The predicate $X(d, a, \ell)$ means "dog g observed an animal of species a in location ℓ". (Sorry that "Rhonda" and "Rabbit" have the same letter — we will use r for the dog and R for the species.)

- (a, 10) **Translations:** Translate these four statements as indicated (point values as indicated):

  Statement I: (to symbols, 3) An Opossum was seen in exactly one location, both dogs saw it, and it was the only location in which Blaze saw both a Rabbit and a Squirrel.

  Statement II: (to English, 2) $\forall y : X(r, s, y)$

  Statement III: (to symbols, 3) Wherever Rhonda saw a Rabbit, Blaze also saw a Rabbit in the same location.

  Statement IV: (to English, 2) $\neg(X(r, S, f) \leftrightarrow X(b, S, f))$

- (b, 10) **Boolean Proof:**
  Let $q_1 = X(b, R, n)$, $q_2 = X(b, R, p)$, $q_3 = X(b, R, v)$, and $q_4 = X(b, R, w)$. Determine the truth values of q_1, q_2, q_3, and q_4, given the premises
  $(q_2 \vee q_3) \rightarrow \neg q_1$,
  $\neg q_4 \rightarrow (q_1 \wedge \neg q_3)$, and
  $q_4 \rightarrow (q_1 \wedge q_2)$.
  You may use either a truth table or deductive proof rules.

- (c, 15) **Dog Proof:** Using the four premises from part (a) *and* the three premises from part (b), determine where the Opossum was seen, and prove your answer.

Question 2 (15+10): Recall that the Fibonacci function $F(n)$ is defined by the rules $F(0) = 0$, $F(1) = 1$, and for all n with $n > 1$, $F(n) = F(n-1) + F(n-2)$.

- (a, 15) Show that for *all* naturals n, $F(1) + F(3) + \ldots + F(2n+1) = F(2n+2)$, that is:

$$\sum_{i=0}^{n} F(2i+1) = F(2n+2)$$

- (b, 10XC) In Midterm #2, we learned that in the village of Gigili there is a single child for each age in the set $\{1, 2, 4, 8, \ldots\}$, that is, for all naturals of the form 2^k. We were asked to prove that any positive natural can be expressed as a sum of *distinct* numbers in this set, using induction.

  In the nearby village of Figili, they have an identical situation except the ages of the children in the set are $\{1, 2, 3, 5, 8, \ldots\}$, that is, all the Fibonacci numbers except $F(0) = 0$ and $F(1) = 1$. Prove that for all positive naturals n, by induction, that n can be expressed as a sum of *distinct* numbers in this set, with the added condition that the sum may not include two *consecutive* numbers. (For example, the sum could not include both 3 and 5, or both 5 and 8.)

Question 3 (25): The next three questions all concern finite-state machines. Unlike in most previous other exams, each question is about a different language.

- (a, 10) Starting from the given λ-NFA, find the corresponding NFA using the Killing λ-Moves algorithm. Draw the λ-NFA and NFA and show all the steps. Your NFA should have three a-moves and five b-moves.

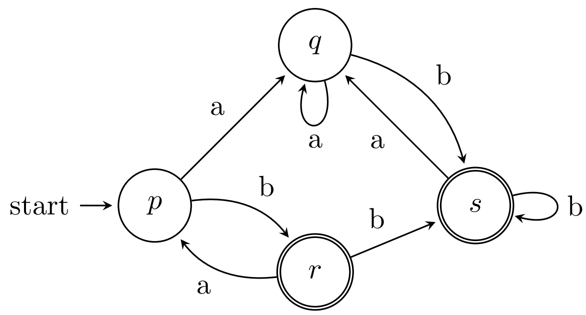
  $S = \{\iota, S_1, S_2, f\}$, start state ι, $F = \{f\}$, $\Sigma = \{a, b\}$, and transition relation
  $\Delta = \{< \iota, \lambda, S_1 >, < \iota, a, S_2 >, < S_1, b, S_2 >, < S_1, a, f >, < S_2, b, S_1 >, < S_2, \lambda, f >\}$.

- (b, 10) Starting from the NFA you found in part (a), apply the Subset Construction algorithm to find the corresponding DFA.

- (c, 5) Determine whether the DFA that you found in part (b) is minimal. Justify your answer.

Question 4 (10): Find a λ-NFA for the given regular expression $[((a+b)^*ab)+(ba+a)^*]^*$. Follow the rules of construction from the lecture. (This means including all the λ-moves, even if the λ-NFA could be simplifed by removing them.)

Question 5 (15): Apply the State Elimination Algorithm to find a regular expression for the DFA given in the diagram below.



Question 6 (20): The following are ten true/false questions, with no explanation needed or wanted, no partial credit for wrong answers, and no penalty for guessing. Some of them refer to the scenarios of the other problems, and/or the entities defined on the supplemental sheet.

- (a) The regular expresssions $(a+b)^*$ and $a(a^*+b^*)+b(a^*+b^*)$ do not denote the same language.

- (b) It is not the case that there are some languages for which we can design an NFA but we cannot design a DFA.

- (c) Let G be a directed graph, let s be a node in G, and set every edge weight equal to 1. Then a Uniform Cost Search from s will act the same as applying BFS from s.

- (d) In Question 1(c), the premises used there do not give you enough information to determine all the truth values of the predicate X.

- (e) Consider a one-tape Turing machine M with tape alphabet $\{a, b, \Box\}$, transition function δ, and states including p and q. If the current configuration of M is $\Box abqa\Box b$, and $\delta(q, a) = \{p, \Box, L\}$, then the next configuration of M is $\Box apb\Box\Box a$.

- (f) If L and L' are both Turing-recognizable languages, then it could be that $L \cap L'$ is not also Turing recognizable.

- (g) If X and Y are finite sets of the same size, and f is a function from X to Y, then f is one-to-one if and only if it is onto.

- (h) If m and n are any two positive naturals, then there exists a natural t such that any natural p with $p \geq t$ can be written as $mx + ny$, where x and y are naturals.

- (i) Consider any Hasse diagram as an undirected graph. Then it may have a cycle.

- (j) Let G be a weighted undirected graph, with positive edge weights, and let s and t be nodes of G. Let x be the shortest path from s to t found using uniform-cost search. Let y be the result of an A^* search with s as start node and t as goal node, using a consistent and admissible heuristic function. Then y cannot be strictly smaller than x, no matter what the heuristic.

Supplemental Sheet for COMPSCI 250 Final Exam, 24 May 2023

Question 1: Dave's dogs Blaze and Rhonda often see animals during their daily walks. They walk through five locations, the Neighborhood, the Farm, the Woods, the Park, and the Village.

The relation X is a subset of $D \times A \times L$, where $D = \{b, r\}$ is the set of dogs Blaze and Rhonda, $A = \{O, R, S\}$ is the set of species they may have seen, Opossum, Rabbit, or Squirrel, and $L = \{f, n, p, v, w\}$ is the set of five locations. The predicate $X(d, a, \ell)$ means "dog g observed an animal of species a in location ℓ". (Sorry that "Rhonda" and "Rabbit" have the same letter — we will use r for the dog and R for the species.)

The four statements for translations and the dog proof are:

- **Statement I:** An Opossum was seen in exactly one location, both dogs saw it, and it was the only location in which Blaze saw both a Rabbit and a Squirrel.
- **Statement II:** $\forall y : X(r, s, y)$
- **Statement III:** Wherever Rhonda saw a Rabbit, Blaze also saw a Rabbit in the same location.
- **Statement IV:** $\neg(X(r, S, f) \leftrightarrow X(b, S, f))$

The variables for the Boolean proof are:

Let $q_1 = X(b, R, n)$, $q_2 = X(b, R, p)$, $q_3 = X(b, R, v)$, and $q_4 = X(b, R, w)$.

The premises for the Boolean proof are:

$(q_2 \vee q_3) \rightarrow \neg q_1$,
$\neg q_4 \rightarrow (q_1 \wedge \neg q_3)$, and
$q_4 \rightarrow (q_1 \wedge q_2)$.

Question 3: The λ-NFA used there is: $S = \{\iota, S_1, S_2, f\}$, start state ι, $F = \{f\}$, $\Sigma = \{a, b\}$, and transition relation
$\Delta = \{< \iota, \lambda, S_1 >, < \iota, a, S_2 >, < S_1, b, S_2 >, < S_1, a, f >, < S_2, b, S_1 >, < S_2, \lambda, f >\}$.