CMPSCI 250: Introduction to Computation

Lecture #27: Games and Adversary Search
David Mix Barrington
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Games and Adversary Search

- Review: A* Search
- Modeling Two-Player Games
- When There is a Game Tree
- The Determinacy Theorem
- Searching a Game Tree
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Review: A* Search

- The A* Search depends on a **heuristic function**, which is a **lower bound** on the distance to the goal. If x is a node, and g is the nearest goal node to x, the **admissibility condition** on h is that $0 \leq h(x) \leq d(x, g)$.

- Suppose we have taken x off of the open list. The best-path distance from the start s to the goal g *through x* is $d(s, x) + d(x, g)$, and this cannot be less than $d(s, x) + h(x)$. Thus when we find a path of length k from s to x, we put x onto the open list with priority $k + h(x)$. We still record the distance $d(s, x)$ when we take x off of the open list.

- The advantage of A* over uniform-cost search is that we do not consider entries x in the closed list for which $d(s, x) + h(x)$ is greater than the actual best-path distance from s to g. This is because when we find the best path to g with length $d(s, g)$, we will put g on the open list with priority $d(s, g) + h(g) = d(s, g)$ and it will come off before any node with higher priority value.

The 15 Puzzle

- The **15-puzzle** is a 4 × 4 grid of pieces with one missing, and the goal is to put them in a certain arrangement by repeatedly sliding a piece into the hole.

- We can imagine a graph where nodes are positions and edges represent legal moves.

- In order to move from a given position to the goal, each piece must move *at least* the Manhattan distance from its current position to its goal position. The sum of all these Manhattan distances gives us an admissible, consistent heuristic for the actual minimum number of moves to reach the goal. So an A* search will be faster than a uniform-cost search.

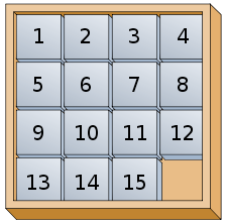


Figure from
en.wikipedia.org
“Fifteen puzzle”

Modeling Two-Player Games

- There are many kinds of games, and we are now going to look at a theory which will let us model and analyze some of them.

- You probably know that the game of **tic-tac-toe** is not very interesting to play, because if both players are familiar with the game the result is always a draw. There is a strategy for the first player, X, that allows her to always do no worse than draw. There is also a strategy for O, the second player, letting *him* do no worse than draw. If both players play these strategies, there is a draw.

- Any game that shares some particular features of tic-tac-toe is **determined** in the same way. We must have **sequential moves**, **two players**, a **deterministic** game with **no randomness**, a **zero-sum** game, and **perfect information**.

- In these cases we can model the game by a **game tree**.

When There is a Game Tree

- A game tree has a node for every possible **state** or **position** of the game. The root node represents the **start position**. A node y is a **child** of a node x if it is possible, according to the rules of the game, to get to y from x in one move. Every node is labelled by **whose turn** it is. Usually the two players alternate moves, so we can call them the **first** and **second** player (**White** and **Black**), but our analysis will not change if one player can make several moves in a row. The **leaves** of the tree represent positions where the **result** of the game is known. We label leaves with a real number indicating how much White is paid by Black, typically 1 for a White win, 0 for a draw, and -1 for a Black win, but any real number values are possible.

- To be represented by such a tree the game must be **discrete**, **deterministic**, **zero-sum**, and have **perfect information**. The tree is **finite** if there are only finitely many sequences of moves that can ever occur. We could have a finite **game graph** where nodes can be revisited, but we won't analyze these here.

The Determinacy Theorem

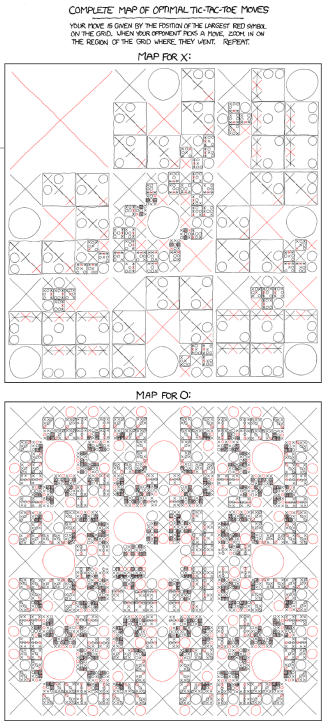
- Each leaf has a **game value**, the real number we defined above. We can inductively assign a game value to *every node* of the tree, by the following rules. The value val(s) of a final position is the label we gave it. If White is to move in position s, val(s) is the *maximum* value of any child of s. If Black is to move in position s, val(s) is the *minimum* value of any child of s.

- The **Determinacy Theorem** says that (1) any game given by a finite tree has a game value v (the value of the root given by the definition above), (2) White has a strategy that guarantees her a result of *at least* v, and (3) Black has a strategy that guarantees him that the result will be *at most v.* Thus v is the result if both plays play optimally.

- We prove that for each node x in the tree, each player has a strategy that gets them a result of val(x) or a result that is better for them. If x is a leaf of the tree this is obvious. If it is White's move she can move to the child with value val(x), and by the IH get at least this result. It's just the same if Black is to move.

How to PlayTic-Tac-Toe

- The chart to the right, if it were big enough to read, would tell you **complete strategies** for each player guaranteeing a result of 0 or better.
- The X strategy starts with moving to the top left, then has a reply to each of the eight O moves that could follow, then a reply to each of the six possible O responses to that move, and so on. The desired moves are in red.
- The O strategy must have responses to all nine initial X moves, then to all seven X responses to each of those moves, and so on. The messiest parts of the chart is where the game goes for all nine moves.



xkcd.com/832

Searching a Game Tree

- The Determinacy Theorem only tells us that these optimal strategies exist, not that they are possible to implement.

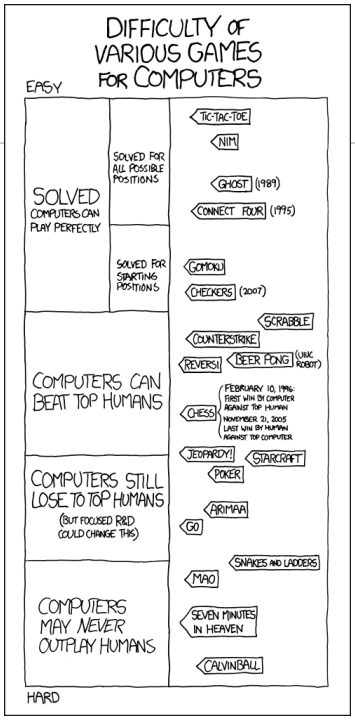
- If it is possible to **calculate the game value** of any node, then choosing the right move is easy. And we have a recursive algorithm to compute the game value, so what is the problem? The tree could be *really really big*.

- An exhaustive **adversary search** computes the exact value. If we can't do that, we need an **estimate** of the game value. In Chess, for example, we can evaluate material and some positional facts to get a good idea whether one position is better than another. We can then use **finite lookahead**, playing a game that ends in k moves, where the payoff is the estimated value of the position at the end of those k moves.

- **Alpha-beta pruning**, which we won't do, is a way to improve the search. But the required time is still usually exponential in the number of moves to go.

Examples of Games

- In 1965, my father's M.S. thesis was to build a tic-tac-toe program that learned from its mistakes. By 1992, at least, students in CMPSCI 187 could build a winning program that exhaustively searched the game tree on every move.
- There is either a winning strategy for White in Chess, or a drawing strategy for Black. But no one knows which is true. Current Chess programs just do a better job of searching and evaluating positions.
- Checkers is easier than Chess, and Go is harder. Calvinball (from *Calvin and Hobbes*) allows rules to be changed at will.



xkcd.com/1002